On a triangle with two parallel sides

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Abstract. We consider the side lengths of a triangle with two parallel sides by division by zero.

Keywords. triangle with parallel sides, division by zero.

1. Introduction

Let us consider a triangle ABC in the plane such that $a = |BC|$, $b = |CA|$ and $c = |AB|$. Let θ_a (resp. θ_b) be the angle between $\overrightarrow{BA}$ and $\overrightarrow{AC}$ (resp. $\overrightarrow{BC}$) (see Figure 1). In this note we fix the points A, B and the angle θ_b, and consider the side lengths of parallel sides of ABC in the case $\theta_a = \theta_b$ (see Figure 2). We use the definition of division by zero [1, 2]

$$\frac{z}{0} = 0 \textit{ for any real number } z. \tag{1}$$

We use a rectangular coordinate system such that A and B have coordinates $(p, 0)$ and $(q, 0)$, respectively, where we assume $p = c + q$ and the point C lies on the region $y \geq 0$.

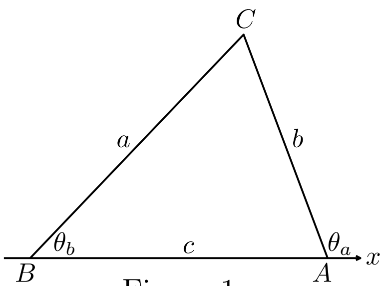

Figure 1.

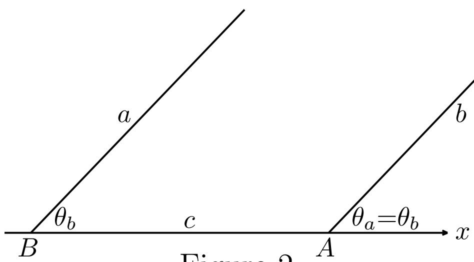

Figure 2.

2. Side length

The point of intersection of the lines expressed by the equations $y \cos\theta_a = (x - p)\sin\theta_a$ and $y\cos\theta_b = (x - q)\sin\theta_b$ coincides with the point C, and has coordinates

$$\left(\frac{p\sin\theta_a\cos\theta_b - q\sin\theta_b\cos\theta_a}{\sin(\theta_a - \theta_b)}, \frac{c\sin\theta_a\sin\theta_b}{\sin(\theta_a - \theta_b)}\right). \tag{2}$$

Therefore we get

$$a = \frac{c\sin\theta_b}{\sin(\theta_a - \theta_b)}, \quad b = \frac{c\sin\theta_a}{\sin(\theta_a - \theta_a)}. \tag{3}$$

If $\theta_a = \theta_b$, then $\sin(\theta_a - \theta_b) = 0$, and we get $a = b = 0$ by (1). Therefore *the side length of the parallel sides of a triangle equals 0.*

Notice that the y-coordinate in (2) also shows that the height corresponding to the base AB equals 0 if $\theta_a = \theta_b$. Also (2) shows that the point C coincides with the origin $(0, 0)$ if $\theta_a = \theta_b$.

References

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[2] S. Saitoh, Division by zero calculus (draft).