Early Evaluation and Effectiveness of Social Distancing Measures for Controlling COVID-19 Outbreaks

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26/04/2020†

Abstract

Based on real data, we study the effectiveness and we propose an early evaluation method for COVID-19 social distancing measures. Version v2 posted on 26/03/20. Version v3 posted on 26/04/20. In version v3 sections 7 and 8 have been added leaving unchanged previous sections.

Key Words: virology, systems simulation.

1 Introduction

One of the most simple model in Virology is the SIR (Susceptible, Infected, Removed) Model, mostly used for didactics but still very effective. A key parameter of the Model is R_0, the basic reproduction number which represent the average number of people an infected transmits the virus to. This Parameter is affected by social distancing measure taken by governments to block the infection. However, the model is not designed to take into account a variable R_0. In this paper we try to somehow introduce this variable in the model.

2 The SIR Model

The SIR model studies the evolution of the number of people $S(t)$, $I(t)$ and $R(t)$ which are respectively the Susceptible (the ones that can be can be infected), the Infected and the Removed (the ones that have been immunised by the virus or vaccine).

Of course in stationary hypothesis, which is when the period of the outbreak is small enough to consider the total population N constant, we have $S+I+R = N$.

The SIR model is described by the following equations:

$$\begin{cases} x' = -\beta xy \\ y' = \beta xy - \gamma y \end{cases} \quad where \quad \begin{matrix} x(t) = \frac{S(t)}{N} \\ y(t) = \frac{I(t)}{N} \end{matrix} \tag{1}$$

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†Posted at: https://vixra.org/abs/2003.0554 - Current version: v3 - April 2020

where we define also $R_0 = \frac{\beta}{\gamma}$ and $T_i = \frac{1}{\gamma}$ where T_i is the effective period in which an infected person passes the virus to R_0 other people (in average). For the COVID-19, first estimations of the parameters from various organizations around the world are T_i between 5 and 7 days and R_0 between 2 and 3 people.

With these two parameter the model is fully defined and R_0 is basically responsible for the pick and the total infected at the end of the outbreak while T_i is basically responsible for the duration of it.

If the initial conditions are $x(0) = x_0$ and $y(0) \approx 0$, which is the case for COVID-19, then it is possible to expand y to the first order in the equations which became linear in y. By using $x = x_0$ in we get:

$$y' = (\beta x_0 - \gamma)y \tag{2}$$

which has the following solution:

$$y = y_0 e^{rt};\ \ with\ \ r = \beta x_0 - \gamma = (\frac{\beta x_0}{\gamma} - 1)\gamma = (R_0 x_0 - 1)\gamma \tag{3}$$

We get easily:

$$R_0 = \frac{1}{x_0}\left[1 + rT_i\right] \tag{4}$$

If R_0 is constant, also r does not change and the initial part of the solution $y(t)$ is a constant slope line in a plot where the vertical axis is in a logarithmic scale. Assuming to have the value for T_i, R_0 can be estimated using the slope of the linear regression of the above curve.

Finally, assuming that T_i does not depend from social distancing measures in place from the government and R_0 changes because of them, we may think to evaluate R_0 with the following function D_0:

$$D_0 = \frac{1}{x_0}\left[1 + \frac{d}{dt} ln[y(t)] T_i\right] \tag{5}$$

Where $y(t)$ are real measured data and for the first outbreak we may choose safely $x_0 = 1$. Note that if only a constant fraction of infected are detected, the logarithm transform the multiplicative constant in an additive constant and the derivative take it to zero. For the reason the estimation of D_0 is not affected in that case. Note also that when $D_0 = 1$, we see a peak in the active cases of the outbreak.

However, there are some problem with D_0, and this will be discussed in the following paragraph.

3 R_0 Varying with Time

We turn now our attention to a real case which is the outbreak of CoVd-19 of the beginning of 2020. When social distancing measure are taken by the government, R_0 changes as a step function, from a day to he following, between a value R_i, previous measures, to a value R_f, post measures. However, analysis of real data shows that the estimation of D_0 from Eq. [5] goes down slowly. Fig. 1 shows D_0 evaluated on real data from the Chinese outbreak (source [2]).

From the figure D_0 looks having an exponential decreasing trend (i.e. $D_0(t) = R_i + (R_f - R_i)(1 - e^{-\frac{t}{\tau}})$. We make the hypothesis that D_0 has an inertia in

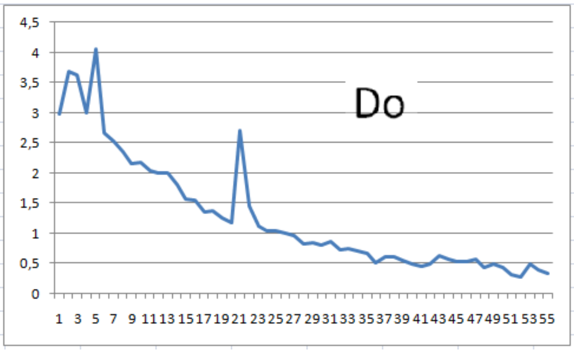

Figure 1: D_0 in China estimated on real data

changing and it is like the output of a first order dynamic systems with transfer function:

$$D_0(s) = \frac{1}{1 + s\tau} R_0(s) \tag{6}$$

which responds to the step function $R_0 = R_i + (R_f - R_i)u(t)$ where $u(t)$ is the Heaviside function.

We propose to modify the model, in order to take into account the delay of the system to respond to a change in R_0, as shown in Fig. 2.

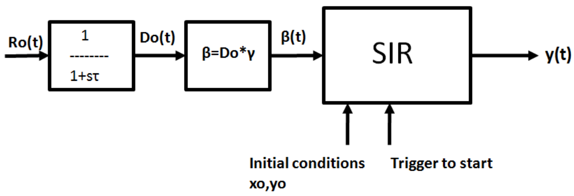

Figure 2: Simulation with variable R_0

4 Evaluation of the Time Constant

A first approximation of the value of the time constant τ can be done directly from the plot of D_0. For example, from real data of the Chinese outbreak in Wuhan of Jan. 2020 (source [2]), after the lockdown D_0 has gone from a value of about $R_i = 2.53$ to a value of about $R_f = 0.5$ (we will see later that this value is much lower). Since the time constant (assuming a first order system) is the time required for D_0 to decrease by 63.2% of the interval $(R_i - R_f)$, the time constant can be evaluates as the number of days required to D_0 to get to a value of about 1.26.

From the data and based on the above consideration we get a time constant of exactly:

$$\tau = 14\, days \tag{7}$$

5 Evaluation of R_0 in the Early Days

We want to evaluate R_0 from the early days after social separating measures have been applied to see if it has been effective. Given the above assumption of a first order system behaviour, the value of $D_0(t)$ is function of the three parameter, R_i, R_f and τ as follows:

$$f_{D_0}(t, R_i, R_f, \tau) = R_i + (R_f - R_i)\left(1 - e^{-\frac{t}{\tau}}\right) \tag{8}$$

Given the real data $D_0(t)$, the above three parameters can be evaluated mimimising the functional:

$$J(R_i, R_f, \tau) = \sum_{n=days\,of\,data} |D_0(t_n) - f_{D_0}(t_n, R_i, R_f, \tau)|^2\, dt \tag{9}$$

6 Characterization of Outbreaks

We have evaluated the above three parameters using real data from the Chinese outbreak in Wuhan (source [2]) and for the Italian outbreak (source [1]) of the beginning of 2020 and we have found as follows:

Chinese outbreak:

$$\begin{array}{lcll} R_i & = & 2.533 & [people] \\ R_f & = & 0.290 & [people] \\ \tau & = & 15.805 & [days] \\ T_p & = & 20 & [days] \quad \text{time of peak from lockdown} \\ T_i & = & 6 & [days] \quad \text{assumed value} \end{array} \tag{10}$$

Fig. 3 shows the comparison between real data and relevant characterizing curve:

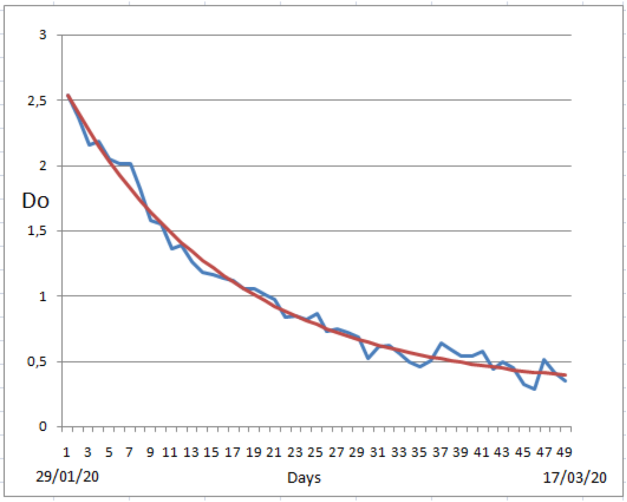

Figure 3: Chinese COVID-19 Outbreak

Italian outbreak:

$$\begin{array}{lllll} R_i & = & 2.367 & [people] & \\ R_f & = & 0.822 & [poeple] & \\ \tau & = & 17.271 & [days] & \\ T_p & = & 39 & [days] & \text{time of peak from lockdown} \\ T_i & = & 6 & [days] & \text{assumed value} \end{array} \qquad (11)$$

Fig. 4 shows the comparison between real data and the relevant characterizing curve:

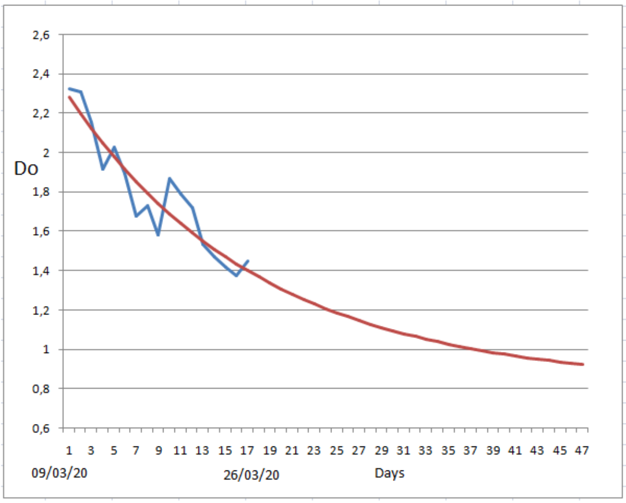

Figure 4: Italian COVID-19 Outbreak

In the above figure, real data (blue line) are reported till the data of writing of this paper (i.e. 26/03/20). The red line shows the theoretical evolution of the outbreak if the trend of the data continues to follow the same pattern. It has to be noted that the real data contains a lot of noise which has a major effect of the final parameters.

Comparing the two examples above we note that according the the data, Chinese measure have been more effective leading to an final R_0 much lower. This because R_0 is affected by β which in turn is affected by the probability for people to meet. Throughout the paper we have assumed that T_i and therefore γ are not affected by this measures.

Finally, in Fig. 5, we propose a plot of the theoretical evolution of the active cases $I(t)$ for the outbreak in Italy (red line) versus real data when available (blue line).

We note that, according to the data, Italy is quite marginal in having an effective value for R_f, so marginal that, further fluctuation of the measured D_0 value, could even put in danger the possibility to reach an early peak.

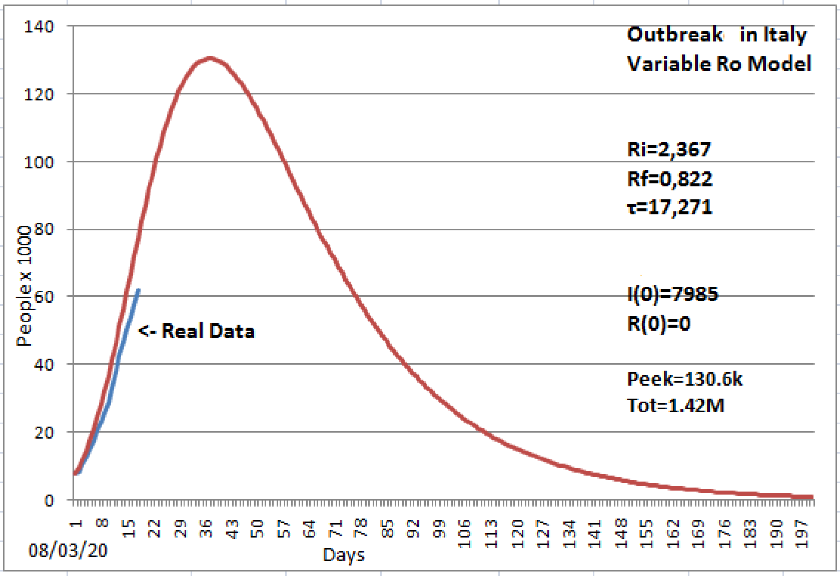

Figure 5: Theoretical Evolution of Italian Outbreak

7 Final Trend of the Outbreak

This section has been added on the 26/04/20 in the version v3 of this paper leaving sections up to 6 unchanged. In this paragraph we compare the trend of the Italian outbreak with the forecast performed one month earlier detailed in the previous section. The forecast was for a peak in Italy after 39 days from Lockdown and therefore it was foreseen on the 16/04/20. The real peak happened in Italy on the 20/04/20.

The forecast was performed 24 day before real peak and the error was of only 4 days. In Fig. 6, we propose a plot of the theoretical evolution of the active cases $I(t)$ for the outbreak in Italy (red line) versus real data when available (blue line). The plot has been done using the updated data (compare with fig. 5 above).

The new parameters evaluated with data the updated at time of writing are the following:

Italian outbreak:

$$\begin{array}{llll} R_i & = & 2.258 & [people] \\ R_f & = & 0.908 & [poeple] \\ \tau & = & 17.007 & [days] \\ T_i & = & 6 & [days] \quad \text{assumed value} \end{array} \tag{12}$$

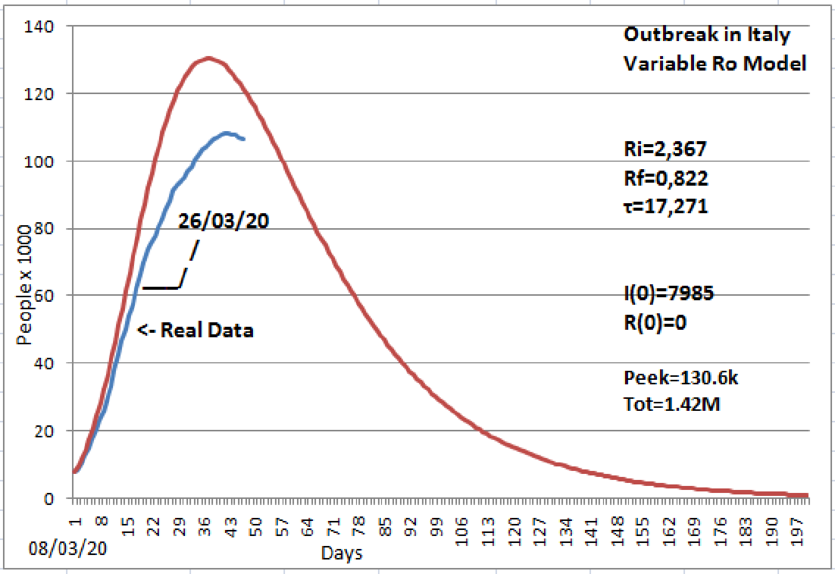

Figure 6: Theoretical Vs Real Data for Italian Outbreak at 24/04/20

8 Outbreak in UK

This section has been added on the 26/04/20 in the version v3 of this paper leaving section up to 6 unchanged. We provide a forecast for the outbreak in UK with data available at time of writing (i.e. 24/04/20).

UK outbreak:

$$\begin{array}{llll} R_i & = & 2.552 & [people] \\ R_f & = & 0.907 & [people] \\ \tau & = & 19.148 & [days] \\ T_p & = & 52 & [days] \quad \text{time of peak from lockdown} \\ T_i & = & 6 & [days] \quad \text{assumed value} \end{array} \tag{13}$$

The peak in is foreseen at 52 days from Lockdown which is on the 14/05/20.

The above parameters have been calculated from the trend of D_0 in UK with data updated at the time of writing (26/02/20). This time, the derivative to evaluate D_o have been calculated on 3 points and a final average on 3 points has been performed to smooth the curve and remove spikes due to measure error. Note the weekly periodicity of the data. Fig. 7 shows the comparison between real data and relevant characterizing curve:

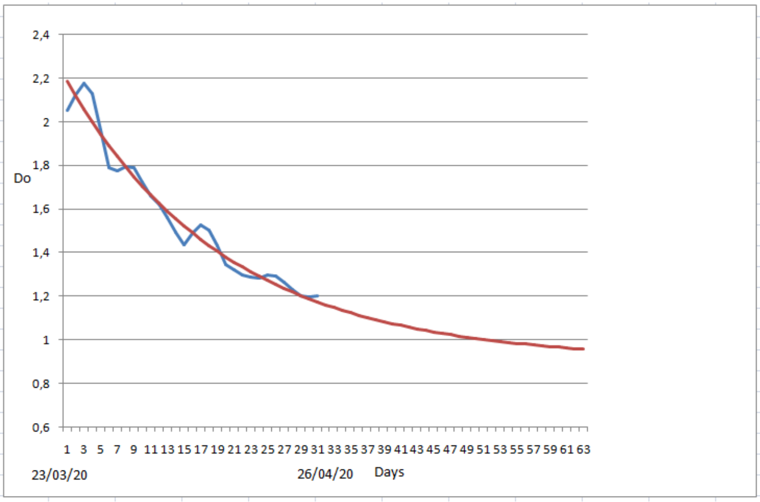

Figure 7: UK COVID-19 Outbreak

Finally, in Fig. 8, we propose a plot of the theoretical evolution of the active cases $I(t)$ for the outbreak in UK (red line). Real data in the plot is not visible (they would be on a blue line) because at the moment it is perfectly covered by simulation data.

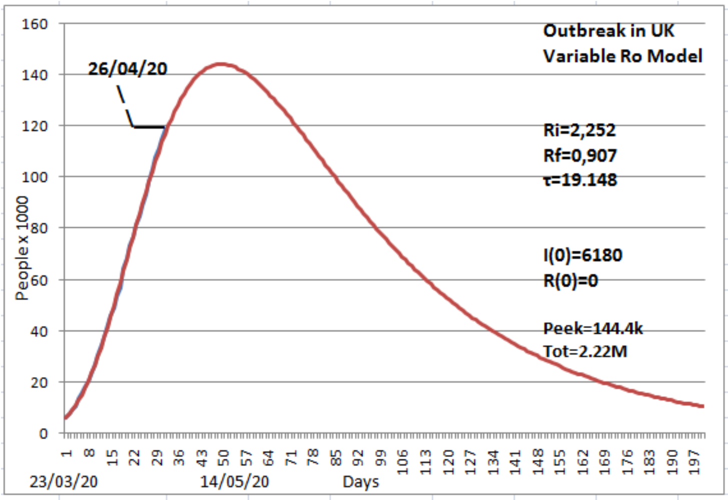

Figure 8: Theoretical Evolution of UK Outbreak

References

[1] Italian outbreak *www.ilsole24ore.com*.

[2] Chinese outbreak data *www.worldometers.info*.