MORE ON GO-COMPACT AND GO-(m, n)-COMPACT SPACES

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Abstract

Balachandran [1] introduced the notion of GO-compactness by involving g-open sets. Quite recently, Caldas et al. in [8] and [9] investigated this class of compactness and characterized several of its properties. In this paper, we further investigate this class of compactness and obtain several more new properties. Moreover, we introduce and study the new class of GO-(m, n)-compact spaces.

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1 Introduction and Preliminaries

In 1970, Levine [13] introduced the concept of generalized closed sets of a topological space. This concept has great influence on several researchers around the world and several research papers with interesting results in different respects came to existence (see, [1], [3], [4], [5], [6], [10], [11], [12], [14]). Recently, Caldas et al. in [8] and [9] investigated the notion of GO-compactness and obtained several fundamental properties. Caldas and Jafari [4] also introduced and investigated the concepts of g-US spaces, g-convergence, sequential GO-compactness, sequential g-continuity and sequential g-sub-continuity. It is the purpose of this paper to further investigate the notion of GO-compactness and also to introduce and study the new class of GO-(m, n)-compact spaces.

Throughout the present paper (X, τ) and (Y, σ) (or simply X and Y) denote topological spaces. Let A be a subset of X. We denote the interior and the closure of a set A by

$Int(A)$ and $Cl(A)$, respectively. $A \subset X$ is called a generalized closed set (briefly g-closed set) of X [13] if $Cl(A) \subset G$ holds whenever $A \subset G$ and G is open in X. The union of two g-closed sets is a g-closed set. A subset A of X is called a g-open set of X, if its complement A^c is g-closed in X. The intersection of all g-closed sets containing a set A is called the g-closure of A [12] and is denoted by $gCl(A)$. If $A \subset X$, then $A \subset gCl(A) \subset Cl(A)$. The collection of all g-closed (resp. g-open) subsets of X will be denoted by $GC(X)$ (resp. $GO(X)$). We set $GC(X, x) = \{V \in GC(X) : x \in V\}$ for $x \in X$. We define similarly $GO(X, x)$. The g-interior of a subset A of X is the union of all g-open sets of X contained in A and is denoted by $Int_g(A)$. Let p be a point of X and N be a subset of X. Then N is called a g-neighborhood of p in X [3] if there exists a g-open set O of X such that $p \in O \subset N$.

A space X is GO-compact if every g-open cover of X has a finite subcover. Since every open set is a g-open set, it follows that every GO-compact space is compact. But, the converse may be false and this is shown by Caldas et al. in [9] and we bring it here for the convenience of the interested reader. Let $X = \{x\} \cup \{x_i : i \in I\}$ where the indexed set I is uncountable. Let $\tau = \{\emptyset, \{x\}, X\}$ be the topology on X. Evidently, X is a compact space. However, it is not a GO-compact space because $\{\{x, x_i\} : i \in I\}$ is a g-open covering of X but it has no finite subcover. A subset A of a space X is said to be GO-compact if A is GO-compact as a subspace of X. If the product space of two no-empty spaces is GO-compact, then each factor space is GO-compact [1]. If A is g-open in X and B is g-open in Y, then $A \times B$ is g-open in $X \times Y$ [13]. A function $f : X \to Y$ is said to be g-continuous [1] if the inverse image of every closed set in Y is g-closed in X.

2 Properties of GO-compact Spaces

Definition 1 *Let (X, τ) be topological space, $x \in X$ and $\{x_s, s \in S\}$ be a net of X. We say that a net $\{x_s, s \in S\}$ g-converges to x if for each g-open set U containing x, there exists an element $s_0 \in S$ such that $s \geq s_0$ implies $x_s \in U$.*

Definition 2 *Let (X, τ) be a topological space, $\mathcal{F} = \{F_i : i \in I\}$ be a filterbase of X and $x \in X$. A filterbase $\mathcal{F}$ g-converges to x if there exists a member $F_i \in \mathcal{F}$ such that $F_i \subseteq U$ for each g-open set U containing x.*

Proposition 2.1 *If $x \in U$ and $U \in GC(X, \tau)$, then there exists a net $\{x_i\}_{i \in I}$ that g-converges to x and $x_i \in U$ for each $i \in I$.*

Proof. Suppose that $x \in U$ and $U \in GC(X, \tau)$ which means $U = gCl(U)$. This means that if $x \in N$ and $N \in GO(X, \tau)$ then $N \bigcap U \neq \emptyset$. It follows that there exists an element $x_N \in N \bigcap U$. This implies that $\{x_N\}_{N \in I}$ g-converges to x.

Proposition 2.2 *Let $\{x_i\}_{i \in I}$ be a net in (X, τ) and $U \in GC(X, \tau)$, where $x_i \in U$ for each $i \in I$. If $\{x_i\}_{i \in I}$ g-converges to x, then $x \in U$.*

Proof. Assume that $\{x_i\}_{i \in I}$ g- converges to x and x does not belong to U. Then there exists a g-open set N such that $x \in N$ and $N \bigcap U = \emptyset$. This means that there exists $i_0 \in I$ such that $x_i \in N$ for each $i \geq i_0$. Then x_i is not an element of U for each $i \geq i_0$. But this is a contradiction and therefore, $x \in U$.

Definition 3 *A point y is a g-cluster point of $\{x_i\}_{i \in I}$ if for each $i_0 \in I$ and $U \in GO(X, \tau)$ such that $y \in U$, there exists an $i' \geq i_0$ such that $x_{i'} \in U$.*

Proposition 2.3 *Let $(s_i)_{i \in I}$ be an ultranet and y be a g-cluster point of the net. Then the ultranet $(s_i)_{i \in I}$ g-converges to y.*

Proof. Suppose that $(s_i)_{i \in I}$ is an ultranet in a topological space (X, τ) and y be a g-cluster point of the net, $(s_i)_{i \in I}$. Suppose that, $(s_i)_{i \in I}$ doesn't g-converge to y. This means that there exists $U \in GO(X, \tau)$ such that $y \in U$ and s_i is not an element of U for any $i \in I$. So y is not a g-cluster point of $(s_i)_{i \in I}$.

Proposition 2.4 *Let $(s_i)_{i \in I}$ be a net in a topological space (X, τ). Then $y \in X$ is a g-cluster point of $(s_i)_{i \in I}$, if and only if a subnet of $(s_i)_{i \in I}$ g-converges to y.*

Proof. Let $(s_i)_{i\in I}$ have a subnet $(s_{k_j})_{j\in J}$ that g-converges to y and J be a directed set. Now suppose that $y \in X$ is not a g-cluster point of $(s_i)_{i\in I}$. This means that there exists $U \in GO(X,\tau)$ and $i_0 \in I$ such that, s_{i_1} is not an element of U for every $i_1 \geq i_0$. Then $(s_{k_j})_{j\in J}$ doesn't g-converge to y.

Conversely assume that y is a g-cluster point of $(s_i)_{i\in I}$.
$J = \{(i,U) : i \in I, y \in U, U \in GO(X,\tau)$ and $s_i \in U\}$ is a partially ordered set where $(i,U) \leq (i',U')$, if $i \leq i'$ and $U' \subset U$.
(i) $(i,U) \leq (i,U)$ for every $(i,U) \in J$. Because, $i \leq i$ and $U \subset U$ for every $i \in I$ and $U \in GO(X,\tau)$.
(ii) Let $(i,U) \leq (i',U')$ and $(i',U') \leq (i,U)$. Then,$i \leq i'$, $U' \subset U$ and $i' \leq i$, $U \subset U'$. This follows that $i = i'$, $U' = U$. Then, $(i',U') = (i,U)$.
(iii) Let $(i,U),(i',U')$ and $(i'',U'') \in J$ such that $(i,U) \leq (i',U')$ and $(i',U') \leq (i'',U'')$. Since I is a directed set, $i \leq i''$ where $i \leq i'$ and $i' \leq i''$. Also, we know that $U'' \subset U$ where $U' \subset U$ and $U'' \subset U'$. Then, $(i,U) \leq (i'',U'')$ where $i \leq i''$ and $U'' \subset U$. Consequently, J is a partially ordered set.

Now let $(i,U),(i',U') \in J$. Then $U \bigcap U' \in GO(X,\tau)$. We know that $U \bigcap U' \subset U$, $U \bigcap U' \subset U'$ and $y \in U \bigcap U'$. ¿Since y is a g-cluster point of $(s_i)_{i\in I}$, there exists $i'' \in I$ such that $i \leq i''$, $i' \leq i''$ and $s_{i''} \in U \bigcap U'$. Then $(i',U') \leq (i'',U \bigcap U')$ and $(i,U) \leq (i'',U \bigcap U')$. This means that J is a directed set. Define $k : J \to I$ by $k(i,A) = i$.

(a) $(i,U) \leq (i',U')$ means that $i \leq i'$. Then $k(i,U) \leq k(i',U')$.
(b) Let $i,i' \in I$ and $U \in GO(X,\tau)$ which contains y. Then there exists $i'' \in I$ such that $i \leq i''$, $i' \leq i''$ and $s_{i''} \in U$. This means that $(i'',U) \in J$, $i \leq k(i'',U)$ and $i' \leq k(i',U)$. It follows that $\{s_{k(i,U)}\}_{(i,U)\in J}$ is a subnet of $\{s_i\}_{i\in I}$.

Consider the set $U \in GO(X,\tau)$ which contains y. There exists $i_0 \in I$ such that $s_{i_0} \in U$. Then $(i_0,U) \in J$. For every $(i,V) \in J$ that $(i_0,U) \leq (i,V)$, $V \subset U$ and $s_i \in V$. It follows that $s_{k(i,V)} \in U$ for every $(i_0,U) \leq (i,V)$. So the subnet, $\{s_{k(i,U)}\}_{(i,U)\in J}$, g-converges to y.

Proposition 2.5 *Let (X,τ) be topological space. Then the following statements are equivalent:*

(i) (X, τ) *is GO-compact.*

(ii) For any family $\mathcal{K}$ *of* g*-closed subsets of* X *such that* $\bigcap_{K\in\mathcal{K}} K = \emptyset$*, there exists a finite subfamily* $\mathcal{L} \subset \mathcal{K}$ *such that* $\bigcap_{L\in\mathcal{L}} L = \emptyset$*.*

(iii) $\bigcap_{K\in\mathcal{K}} K \neq \emptyset$ *for any family* $\mathcal{K}$ *of* g*-closed subsets of* X *such that* $\bigcap_{L\in\mathcal{L}} L \neq \emptyset$ *where* $\mathcal{L} \subset \mathcal{K}$ *is a finite subfamily.*

Proof. (i) $\Rightarrow$ (ii): Let (X, τ) be GO-compact and $\mathcal{K}$ be a family of g-closed subsets such that $\bigcap_{K\in\mathcal{K}} K = \emptyset$. Then $[\bigcap_{K\in\mathcal{K}} K]^c = [\emptyset]^c$. This means that $\bigcup_{K\in\mathcal{K}} K^c = X$. There exists a finite subfamily $\mathcal{L} \subset \mathcal{K}$ such that $\bigcup_{L\in\mathcal{L}} L^c = X$ where $\bigcap_{L\in\mathcal{L}} L = \emptyset$.

(ii) $\Rightarrow$ (iii): Let $\mathcal{K}$ be a family of g-closed subsets of X. From the assumption if $\bigcap_{K\in\mathcal{K}} K = \emptyset$, then there exists a finite subfamily $\mathcal{L} \subset \mathcal{K}$ such that $\bigcap_{L\in\mathcal{L}} L = \emptyset$. This means that if $\mathcal{K}$ doesn't have any finite subfamily $\mathcal{L}$ such that $\bigcap_{L\in\mathcal{L}} L = \emptyset$, then $\bigcap_{K\in\mathcal{K}} K \neq \emptyset$.

(iii) $\Rightarrow$ (ii): Let $\mathcal{K}$ be a family of g- closed subsets of X. From the assumption, if $\bigcap_{L\in\mathcal{L}} L \neq \emptyset$ for any subfamily $\mathcal{L} \subset \mathcal{K}$ then $\bigcap_{K\in\mathcal{K}} K \neq \emptyset$. This means that, if $\bigcap_{K\in\mathcal{K}} K = \emptyset$ then there exists at least one subfamily $\mathcal{L} \subset \mathcal{K}$ such that $\bigcap_{L\in\mathcal{L}} L = \emptyset$.

(ii) $\Rightarrow$ (i): Let $\{U_i\}_{i\in I}$ be a g-open cover of X. Then, $\bigcup_{i\in I} U_i = X$. This means that $\bigcap_{i\in I} U_i^c = \emptyset$ and $U_i^c \in GC(X, \tau)$ for each $i \in I$. From the assumption, there exists a finite subfamily $J \subset I$ such that $\bigcap_{j\in J} U_j^c = \emptyset$. So $\bigcup_{j\in J} U_j = X$. Therefore (X, τ) is GO-compact.

Proposition 2.6 *Show that* (X, τ) *is GO-compact if and only if every net has at least one* g*-cluster point in the topological space.*

Proof. Let (X, τ) be GO-compact and $\{x_i\}_{i\in I}$ be any net in this space. Let's consider a family $gCl(B_j)$ of subsets where $B_j = \{x_i \mid j \leq i\}$. Then, $gCl(B_j) \in GC(X, \tau)$ for any $j \in I$ and the intersection of finitely many of $gCl(B_j)$ is nonempty. From the Proposition 2.5, we know that $\bigcap_{j\in I} gCl(B_j) \neq \emptyset$ for (X, τ) is GO-compact. Let $y \in \bigcap_{j\in I} gCl(B_j)$. Then $y \in gCl(B_j)$ for any $j \in I$. Consider $y \in U, U \in GO(X, \tau)$ and $r \in I$. Then $U \bigcap B_r \neq \emptyset$. So $U \bigcap B_k \neq \emptyset$ for any $k \in I$ such that $k \geq r$. Consequently y is a g-cluster point of $\{x_i\}_{i\in I}$.

Now suppose that every net in (X, τ) has at least one g-cluster point. Let $\{F_i\}_{i\in I}$ be a family of g-closed sets where intersection of finitely many of $F_i's$ is nonempty. Consider the set $J = \{\bigcap_{j=1}^n G_{i_j} \mid \{G_{i_j}\}_{j=1}^n \subset \{F_i\}_{i\in I}\}$ and the relation $\leq$, where $A \leq B$ whenever $B \subset A$ and $A, B \in J$.

(i) $A \subset A$ for every set $A \in J$. This means that $A \leq A$ for every set $A \in J$.

(ii) We know that if $A \supset B$ and $B \supset A$ then $A = B$. So $A \leq B$ and $B \leq A$ then $A = B$.

(iii) We know that if $C \supset B$ and $B \supset A$ then $C \supset A$. So, if $C \leq B$ and $B \leq A$ then $C \leq A$. This means that $(J, \leq)$ is a directed set and partially ordered.

Consider the function $s : J \to X$ such that $s(A) \in A$ for every $A \in J$. Then $\{s_A\}_{A\in J}$ is a net in X and by the assumption has a g-cluster point. Let y be the g-cluster point of $\{s_A\}_{A\in J}$. We know that if $A \in J$ and $F_k \leq A$, then $A \subset F_k$ where $F_k \in \{F_i\}_{i\in I}$. So $s_B \in F_k$ whenever $A \leq B$. Then, $\{s_A\}_{A\in J}$ is residually in F_k. From Proposition 2.4, since y is a g-cluster point of $\{s_A\}_{A\in J}$, a subnet of $\{s_A\}_{A\in J}$ g-converges to y. Since $\{s_A\}_{A\in J}$ is residually in F_k for each k, such a subnet would be residually in F_k for each k. From Proposition 2.2, $y \in F_k$ for each k. So $\bigcap_{i\in I} F_i \neq \emptyset$. Consequently (X, τ) is *GO*-compact from proposition 2.5.

Proposition 2.7 *A topological space (X, τ) is GO-compact if and only if every ultranet in it is g-convergent .*

Proof. Let (X, τ) be *GO*-compact and $\{s_i\}_{i\in I}$ be an ultranet in (X, τ). By Proposition 2.6, $\{s_i\}_{i\in I}$ has at least one g-cluster point. From Proposition 2.3, $\{s_i\}_{i\in I}$ g-converges to its g-cluster point. So, $\{s_i\}_{i\in I}$ is g-convergent.

Conversely let every ultra net in (X, τ) be g-convergent. Consider a net $\{s_i\}_{i\in I}$ in (X, τ). Since every net has a subnet which is an ultranet, so there exists a subnet of $\{s_i\}_{i\in I}$ which is an ultranet. This ultranet g-converges to a point and this point is g-cluster point of $\{s_i\}_{i\in I}$.

3 *GO*-(m, n)-compact Spaces

We begin with the following notions which will be used in the sequel.

Definition 4 *A space (X, τ) is said to be GO-(m, n)-compact if from every g-open covering $\{U_i \mid i \in I\}$ of X whose cardinality I, denoted by Card I, is at most n, one can select a subcovering $\{U_{i_j} \mid j \in J\}$ of X whose Card J is at most m.*

Definition 5 *A subset A of a space (X, τ) is said to be a GO-(m, n)-compact subspace if the subspace A is GO-(m, n)-compact.*

Definition 6 *A space (X, τ) is said to be completely GO-(m, n)-compact if every subspace of X is GO-(m, n)-compact.*

Remark 3.1 *Observe that a GO-$(1, n)$-compact space is a GO-n-compact space and GO-$(1, \infty)$-compact space is the usual GO-compact space. A GO-$(1, \omega)$-compactness is GO-compactness in the Fréchet sense and a GO-(ω, ∞)-compact space is a GO-Lindelöf space.*

Definition 7 *A family $\{U_i \mid i \in I\}$ of subsets of a set X is said to have the m-intersection property if every subfamily of cardinality at most m has a non-void intersection.*

Theorem 3.2 *A space (X, τ) is GO-(m, n)-compact if and only if every family $\{P_i\}$ of g-closed sets $P_i \subseteq X$ having the m-intersection property also has the n-intersection property.*

Proof. The proof is a consequence of the following equivalent statements:

(1) X is GO-(m, n)-compact;

(2) If $\{U_i \mid i \in I\}$ is a g-open cover of X such that Card $I \leq n$, then there is a subcover $\{U_{i_j} : j \in J\}$ of X such that Card $J \leq m$;

(3) If $\{U_i \mid i \in I\}$ is a family of g-open sets such that Card $I \leq n$ and every subfamily $\{U_{i_j}\}$ of Card $J \leq m$ has the property $X - (\cup_i U_{i_j}) \neq \emptyset$, then $X - (\cup_{i \in I} U_i) \neq \emptyset$,

(4) If $\{U_i \mid i \in I\}$ is a family of g-open sets such that $X - (\cup_{j \in J} U_{i_j}) \neq \emptyset$ whenever Card $J \leq m$, then $X - (\cup_{j \in J} U_{i_j}) \neq \emptyset$ whenever Card $J \leq n$;

(5) If $\{P_i \mid i \in I\}$ is a family of g-closed sets having the m-intersection property then $\{P_i\}$ has also the n-intersection property.

Theorem 3.3 *If a space X is GO-(m, n)-compact and if Y is a g-closed subset of X, then Y is a GO-(m, n)-compact subspace of X.*

Proof. Suppose that $\{U_i \mid i \in I\}$ is a g-open cover of Y such that Card $I \leq n$. By adding $U_j = X - Y$, we obtain a g-open cover of X with cardinality at most n. By eliminating U_j, we have a subcover of $\{U_i\}$ whose cardinality is at most m.

Theorem 3.4 *If X is a space such that every g-open subset of X is a GO-(m,n)-compact subspace of X, then X is completely GO-(m,n)-compact.*

Proof. Let $Y \subset X$ and $\{U_i \mid i \in I\}$ be a g-open cover of Y such that Card $I \leq n$. Then the family $\{U_i \mid i \in I\}$ is a g-open cover of the g-open set $\cup_i U_i$. Then there is a subfamily $\{U_{i_j} \mid j \in J\}$ of Card $J \leq m$ which covers $\cup_i U_i$. This subfamily also covers the set Y and so Y is *GO*-(m,n)-compact.

Theorem 3.5 *Let X be a space and $\{Y_k \mid k \in K\}$ be a family of subsets of X. If every Y_k is GO-(m,n)-compact for some $m \geq$ Card K, then $U_{k\in K}Y_k$ is a GO-(m,n)-compact subspace of X.*

Proof. If $\{U_i \mid i \in I\}$ is a g-open cover of $Y = \cup_k Y_k$, then it is a g-open cover of Y_k for every $k \in K$. If Card $I \leq n$, then $\{U_i\}$ contains a subfamily $\{U_{i_{j_k}} \mid j_k \in J_k\}$ for which $Car J_k \leq m$ and is a covering of Y_k. The union of these families is a g-open subfamily of $\{U_i\}$ which covers Y and its cardinality is at most m.

Definition 8 *A point $x \in X$ is said to be an m-g-accumulation point of a set S in X if for every g-open set U_x containing x, we have Card $(U_x \cap S) > m$.*

Observe that if $m = 0, 1$ or ω, then the relation Card $(U_x \cap S) > m$ means that $U_x \cap S \neq \emptyset$, not finite or not countable.

Theorem 3.6 *Let X be a space and S a subset of X of cardinality greater than m (i.e. $S \subset X$ and Card $S > m$). If X is GO-(m,n)-compact for some $n > m$, then S has a g-accumulation point in X. If X is GO-(m,∞)-compact, then S has an m-g-accumulation point in X.*

Proof. Assume that $S \subset X$ of cardinality at most n which has no g-accumulation points in X. Then for each $x \in X$, there is an open set U_x such that at most one point of S belongs to U_x. Suppose U is the union of all sets U_x which contain no points of S. Let U_s denote the union of all sets U_x which contain the point $s \in S$. Then U and U_s are g-open sets. Therefore $\{U, U_s\}$ is a g-open cover of X of cardinality at most n. If X is GO-(m,n)-compact, then this cover contains a subcover of cardinality at most n. If X is GO-(m,n)-compact, then this cover contains a subcover of cardinality at most m. But this subcover must contain every U_s since $s \in S$ is covered only by U_s. Thus Card $S \leq m$. If the cardinality of a set S is greater than m, then S has at least one g-accumulation point in X. The two other cases can be proved by the same token with a little modification.

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