# NANO $\widetilde{G}\alpha$-CLOSED SETS IN NANO TOPOLOGICAL SPACES

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**Abstract**

The basic objective of this paper is to introduce and investigate the properties of Nano $\widetilde{g}\alpha$-closed sets in Nano topological spaces.
**Keywords.** $N\widetilde{g}\alpha$-closed set, $N\widehat{g}$-closed set, $N^{*}g$-closed set and $N^{\sharp}gs$-closed set.
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## 1. Introduction

Levine [4] introduced the concept of generalized closed sets in 1970. The concept of $\widetilde{g}\alpha$-closed sets was introduced by R. Devi et al. [2]. In 2013, the notion of Nano topological space with respect to the subset $X$ of a universe $U$ was introduced by Lellis Thivagar [3] which was defined in terms of approximations and boundary region of a universe using an equivalence relation on it. He has defined Nano closed sets, Nano-interior and Nano-closure of a set and also established certain weak forms of Nano open sets such as Nano $\alpha$-open sets, Nano semi-open sets and Nano pre-open sets. In this paper, we introduce a new class of sets on Nano topological spaces called Nano $\widetilde{g}\alpha$-closed sets and discuss the relation of this new sets with existing ones.

## 2. Preliminaries

**Definition 2.1** [3] Let $U$ be a non-empty finite set of objects called the universe and $R$ be an equivalence relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(U, R)$ is said to be the approximation space. Let $X \subseteq U$, then

(i) The lower approximation of $X$ with respect to $R$ is the set of all objects, which can be for certain classified as $X$ with respect to $R$ and its denoted by $L_R(X)$ and $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x \in U$.

(ii) The upper approximation of $X$ with respect to $R$ is the set of all objects, which can be possibly classified as $X$ with respect to $R$ and it is denoted by $U_R(X)$ which $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.

(iii) The boundary region of $X$ with respect to $R$ is the set of all objects which can be classified neither as $X$ nor as not $X$ with respect to $R$ and it is denoted by $B_R(X)$ and $B_R(X) = U_R(X) - L_R(X)$.

**Property 2.2.** [3] If $(U, R)$ is an approximation space and $X, Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.

(ii) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$.

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.

(iv) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.

(v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$, whenever $X \subseteq Y$.

(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$.

(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$.

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

**Definition 2.3.** [3] Let $U$ be the universe, $R$ be an equivalence relation on $U$ and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then by property 2.2, $\tau_R(X)$ satisfies the following axioms:

(i) $U$ and $\phi \in \tau_R(X)$.

(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

$\tau_R(X)$ is a topology on $U$ called the Nano topology on $U$ with respect to $X$. We denote the Nano topological space by $(U, \tau_R(X))$ as . The elements of $\tau_R(X)$ are called Nano open sets.

**Remark 2.4.** [3] If $\tau_R(X)$ is the Nano topology on $U$ with respect to $X$, the the set $B = \{U, L_R(X), U_R(X)\}$ is the basis for $\tau_R(X)$.

**Definition 2.5.** [3] If $(U, \tau_R(X))$ is a Nano topological space with respect to $X$, where $X \subseteq U$ and if $A \subseteq U$, then the Nano interior of $A$ is defined as the union of all Nano open subsets of $A$ and it is denoted by $Nint(A)$. That is, $Nint(A)$ is the largest Nano open subset of $A$. The Nano closure of $A$ is defined as the intersection of all Nano closed sets containing $A$ and it is denoted by $Ncl(A)$. $Ncl(A)$ is the smallest Nano closed set containing $A$.

**Definition 2.6.** [3] A Nano topological space $(U, \tau_R(X))$ is said to be extremely disconnected if the Nano closure of each Nano open set is Nano open.

**Definition 2.7.** [3] Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then $A$ is said to be

(i) Nano semi open if $A \subseteq Ncl(Nint(A))$

(ii) Nano $\alpha$-open if $A \subseteq Nint(Ncl(Nint(A)))$

The Nano $\alpha$-closure of $A$ is defined as the intersection of all Nano $\alpha$-closed sets containing $A$ and it is denoted by $N\alpha cl(A)$. This means that $N\alpha cl(A)$ is the smallest Nano $\alpha$-closed set containing $A$. The Nano semi-closure of $A$ is defined as the intersection of all Nano semi-closed sets containing $A$ and it is denoted by $Nscl(A)$. $Nscl(A)$ is the smallest Nano semi closed set containing $A$. We recall the following definitions which are useful in the sequel.

**Definition 2.8.** Let $U$ be a Nano topological space. A subset $A$ of $U$ is called

(a) Nano semi-generalized closed (briefly $Nsg$-closed) set [1] if $Nscl(A) \subseteq V$, whenever $A \subseteq V$ and $V$ is Nano semi-open in $U$,

(b) Nano generalized semi-closed (briefly $Ngs$-closed) set [1] if $Nscl(A) \subseteq V$, whenever $A \subseteq V$ and $V$ is Nano open in $U$,

(c) Nano $\alpha$-generalized closed (briefly $N\alpha g$-closed) set [5] if $N\alpha cl(A) \subseteq V$, whenever $A \subseteq V$ and $V$ is Nano open in $U$,

(d) Nano generalized $\alpha$-closed (briefly $Ng\alpha$-closed) set [5] if $N\alpha cl(A) \subseteq V$, whenever $A \subseteq V$ and $V$ is $N\alpha$-open in $U$.

## 3. Properties of Nano $\widetilde{g}\alpha$-closed sets

We introduce the following definitions.

**Definition 3.1.** Let $U$ be a Nano topological space. A subset $A$ of $U$ is called

(a) Nano $\widehat{g}$-closed set (briefly $N\widehat{g}$-closed) if $Ncl(A) \subseteq V$, whenever $A \subseteq V$ and $V$ is Nano semi-open in $U$,

(b) Nano $^*g$-closed set (briefly $N^*g$-closed) if $Ncl(A) \subseteq V$, whenever $A \subseteq V$ and $V$ is $N\widehat{g}$-open in $U$,

(c) Nano $^\sharp gs$-closed set (briefly $N^\sharp gs$-closed) if $Nscl(A) \subseteq V$, whenever $A \subseteq V$ and $V$ is $N^*g$-open in $U$.

In this section, we define and study the forms of Nano $\tilde{g}\alpha$-closed sets.

**Definition 3.2.** Let $U$ be a Nano topological space. A subset $A$ of $U$ is called Nano $\widetilde{g}\alpha$-closed set (briefly $N\widetilde{g}\alpha$-closed) if $N\alpha cl(A) \subseteq V$, whenever $A \subseteq V$ and $V$ is $N^\sharp gs$-open in $U$.

**Theorem 3.3.**

(a) Every $N\alpha$-closed set is a $N\widetilde{g}\alpha$-closed set.

(b) Every $N\widetilde{g}\alpha$-closed set is a $Ngs$-closed set.

(c) Every $N\widetilde{g}\alpha$-closed set is a $Ng\alpha$-closed set.

(d) Every $N\widetilde{g}\alpha$-closed set is a $N\alpha g$-closed set.

(e) Every $N\widetilde{g}\alpha$-closed set is a $Nsg$-closed set.

**Proof.**

(a) Let $A$ be an $N\alpha$-closed set in $U$, then $A = N\alpha cl(A)$. Let $A \subseteq V$, $V$ is $N^{\sharp}gs$-open in $U$. Since $A$ is $N\alpha$-closed, $A = \alpha cl(A) \subseteq V$. This shows that $A$ is $N\widetilde{g}\alpha$-closed set.

(b) Let $A$ be an $N\widetilde{g}\alpha$-closed set in $U$. Let $A \subseteq V$, $V$ is a nano open in $U$ which implies $V$ is an $N^{\sharp}gs$-open set. Since $A$ is $N\widetilde{g}\alpha$-closed, $Nscl(A) \subseteq N\alpha cl(A) \subseteq V$. This shows that $A$ is $Ngs$-closed set.

(c) Let $A$ be an $N\widetilde{g}\alpha$-closed set in $U$. Let $A \subseteq V$, $V$ is an $N\alpha$-open set in $U$ which implies $V$ is an $N^{\sharp}gs$-open set. Since $A$ is $N\widetilde{g}\alpha$-closed, $N\alpha cl(A) \subseteq V$. This shows that $A$ is an $Ng\alpha$-closed set.

(d) Let $A$ be an $N\widetilde{g}\alpha$-closed set in $U$. Let $A \subseteq V$, $V$ is a Nano open in $U$ which implies $V$ is an $N^{\sharp}gs$-open set. Since $A$ is $N\widetilde{g}\alpha$-closed, $N\alpha cl(A) \subseteq V$. This shows that $A$ is an $N\alpha g$-closed set.

(e) Let $A$ be an $N\widetilde{g}\alpha$-closed set in $U$. Let $A \subseteq V$, $V$ is a nano semi open in $U$ which implies $V$ is an $N^{\sharp}gs$-open set. Since $A$ is $N\widetilde{g}\alpha$-closed, $Nscl(A) \subseteq N\alpha cl(A) \subseteq V$. This shows that $A$ is an $Nsg$-closed set.

The following examples show that these implications are not reversible.

**Example 3.4.**

(a) Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Thus the Nano topology, $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. Then in a space $U$, a subset $\{a, b, c\}$ is an $N\widetilde{g}\alpha$-closed set but it is not an $N\alpha$-closed set.

(b) Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then the Nano topology, $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. Hence in a space $U$, a subset $\{a\}$ is an $Ngs$-closed set but it is not an $N\widetilde{g}\alpha$-closed set.

(c) Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b, c\}$. Then the Nano topology, $\tau_R(X) = \{U, \phi, \{a, c\}, \{b, d\}\}$. In a space $U$, a subset $\{a, b, c\}$ is an $Ng\alpha$-closed set but it is not an $N\widetilde{g}\alpha$-closed set.

(d) Let $U = \{a, b, c, d, e\}$ with $U/R = \{\{d\}, \{a, b\}, \{c, e\}\}$ and $X = \{a, d\}$. Then the Nano topology, $\tau_R(X) = \{U, \phi, \{d\}, \{a, b, d\}, \{a, b\}\}$. Then in a space $U$, a subset $\{a, c\}$ is an $N\alpha g$-closed set but it is not an $N\widetilde{g}\alpha$-closed set.

(e) Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then the Nano topology, $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. In a space $U$, a subset $\{b\}$ is an $Nsg$-closed set but it is not an $N\widetilde{g}\alpha$-closed set.

**Theorem 3.5.** Let $A$ be a subset of a Nano topological space $U$. If $A$ is $N\widetilde{g}\alpha$-closed, then $N\alpha cl(A) - A$ does not contain any non-empty $N^\sharp gs$-closed set.
**Proof.** Suppose that $A$ is $N\widetilde{g}\alpha$-closed and let $F$ be a non-empty $N^\sharp gs$-closed set with $F \subseteq N\alpha cl(A) - A$. Then $A \subseteq U - F$ and so $N\alpha cl(A) \subseteq U - F$. Hence $F \subseteq U - N\alpha cl(A)$ which is a contradiction.

**Theorem 3.6.** Let $A$ be a subset of a Nano topological space $U$. If $A$ is $N\widetilde{g}\alpha$-closed and $A \subseteq B \subseteq N\alpha cl(A)$, then $B$ is $N\widetilde{g}\alpha$-closed.
**Proof.** Let $V$ be an $N^\sharp gs$-open set of $U$ such that $B \subseteq V$. Then $A \subseteq V$. Since $A$ is $N\widetilde{g}\alpha$-closed, then $N\alpha cl(A) \subseteq V$. Now $N\alpha cl(B) \subseteq N\alpha cl(N\alpha cl(A)) \subseteq V$. Therefore $B$ is also an $N\widetilde{g}\alpha$-closed set of $U$.

**Theorem 3.7.** Let $A$ and $B$ be $N\widetilde{g}\alpha$-closed sets of a Nano topological space $U$. Then $A \cup B$ is an $N\widetilde{g}\alpha$-closed set in $U$.
**Proof.** Let $A$ and $B$ be $N\widetilde{g}\alpha$-closed sets. Let $A \cup B \subseteq V$, where $V$ is $N^\sharp gs$-open. Since $A$ and $B$ are $N\widetilde{g}\alpha$-closed sets, $N\alpha cl(A) \subseteq V$ and $N\alpha cl(B) \subseteq V$. This implies that $N\alpha cl(A \cup B) = N\alpha cl(A) \cup N\alpha cl(B) \subseteq V$ and so $N\alpha cl(A \cup B) \subseteq V$. Therefore $A \cup B$ is $N\widetilde{g}\alpha$-closed.

**Lemma 3.8.** For any space $U$, $U = U_{N^\sharp gsc} \cup U_{N\widetilde{g}\alpha o}$ holds.
**Proof.** Let $x \in U$. Suppose that $\{x\}$ is not $N^\sharp gs$-closed set in $U$. Then $U$ is a unique $N^\sharp gs$-open set containing $U - \{x\}$. Thus $U - \{x\}$ is $N\widetilde{g}\alpha$-closed in $U$ and so $\{x\}$ is $N\widetilde{g}\alpha$-open. Therefore $x \in U_{N^\sharp gsc} \cup U_{N\widetilde{g}\alpha o}$ holds.

We need more notations:

For a subset $A$ of $U$, $ker(A) = \cap\{V/V \in \tau_R(X) \text{ and } A \subseteq V\}$;
$N^\sharp GSO\text{-}ker(A) = \cap\{V/V \in N^\sharp GSO(U) \text{ and } A \subseteq V\}$.

**Theorem 3.9.** For a subset $A$ of $U$, the following conditions are equivalent.

(1) $A$ is $N\widetilde{g}\alpha$-closed in $U$.

(2) $N\alpha cl(A) \subseteq N^\sharp GSO\text{-}ker(A)$ holds.

(3) (i) $N\alpha cl(A) \cap U_{N^\sharp gsc} \subseteq A$ and (ii) $N\alpha cl(A) \cap U_{N^\sharp gso} \subseteq N^\sharp GSO\text{-}ker(A)$

holds.

**Proof.**
**(1) ⇒ (2)** Let $x \notin N^{\sharp}GSO\text{-}ker(A)$. Then there exists a set $V \in N^{\sharp}GSO(U)$ such that $x \notin V$ and $A \subseteq V$. Since $A$ is $N\widetilde{g}\alpha$-closed, $N\alpha cl(A) \subseteq V$ and so $x \notin N\alpha cl(A)$. This shows that $N\alpha cl(A) \subseteq N^{\sharp}GSO\text{-}ker(A)$.
**(2) ⇒ (1)** Let $V \in N^{\sharp}GSO(U)$ such that $A \subseteq V$. Then, we have that $N^{\sharp}GSO\text{-}ker(A) \subseteq V$ and so by (2) $N\alpha cl(A) \subseteq V$. Therefore $A$ is $N\widetilde{g}\alpha$-closed.
**(2) ⇒ (3)** (i) First we claim that $N^{\sharp}GSO\text{-}ker(A) \cap U_{N^{\sharp}gsc} \subseteq A$. Indeed, let $x \in N^{\sharp}GSO\text{-}ker(A) \cap U_{N^{\sharp}gsc}$ and assume that $x \notin A$. Since the set $U - \{x\} \in N^{\sharp}GSO(U)$ and $A \subseteq U - \{x\}$, $N^{\sharp}GSO\text{-}ker(A) \subseteq U - \{x\}$. Then we have that $x \in U - \{x\}$ and so this is a contradiction. Thus we show that $N^{\sharp}GSO\text{-}ker(A) \cap U_{N^{\sharp}gsc} \subseteq A$. By using (2), $N\alpha cl(A) \cap U_{N^{\sharp}gsc} \subseteq N^{\sharp}GSO\text{-}ker(A) \cap U_{N^{\sharp}gsc} \subseteq A$.
(ii) It is obtained by (2).
**(3) ⇒ (2)** By Lemma 3.8 and (3),

$$\begin{aligned} N\alpha cl(A) &= N\alpha cl(A) \cap U = N\alpha cl(A) \cap (U_{N^{\sharp}gsc} \cup U_{N\widetilde{g}\alpha o}) \\ &= (N\alpha cl(A) \cap U_{N^{\sharp}gsc}) \cup (N\alpha cl(A) \cap U_{N\widetilde{g}\alpha o}) \\ &= A \cup N^{\sharp}GSO\text{-}ker(A) \\ &= N^{\sharp}GSO\text{-}ker(A) \text{ holds.} \end{aligned}$$

**Theorem 3.10.** Let $U$ be a Nano topological space and $A \subseteq U$.

(a) If $A$ is $N^{\sharp}gs$-open and $N\widetilde{g}\alpha$-closed, then $A$ is $\alpha$-closed in $U$.

(b) Suppose that $U$ is an $N\alpha$-space. An $N\widetilde{g}\alpha$-closed set $A$ is $N\alpha$-closed in $U$ if and only if $N\alpha cl(A) - A$ is $N\alpha$-closed in $U$.

(c) For each $x \in U$, $\{x\}$ is $N^{\sharp}gs$-closed or $U - \{x\}$ is $N\widetilde{g}\alpha$-closed in $U$.

(d) Every subset is $N\widetilde{g}\alpha$-closed in $U$ if and only if $N^{\sharp}gs$-open set is $N\alpha$-closed.

**Proof.**

(b) (Necessity) If $A$ is $N\alpha$-closed, then $N\alpha cl(A) - A = \phi$.
(Sufficiency) Suppose that $A$ is $N\widetilde{g}\alpha$-closed and $N\alpha cl(A) - A$ is $N\alpha$-closed. Then, $N\alpha cl(A) - A$ is $N^{\sharp}gs$-closed in $U$ and by Theorem 3.5, $N\alpha cl(A) - A = \phi$. Therefore $A$ is $N\alpha$-closed in $U$.

(c) If $\{x\}$ is not $N^{\sharp}gs$-closed, then $U - \{x\}$ is not $N^{\sharp}gs$-open. Therefore $U - \{x\}$ is $N\widetilde{g}\alpha$-closed in $U$.

(d) (Necessity) Let $V$ be an $N^\sharp gs$-open set. Then we have that $N\alpha cl(V) \subseteq V$ and hence $V$ is $N\alpha$-closed.
(Sufficiency) Let $A$ be a subset and $V$ is an $N^\sharp gs$-open set such that $A \subseteq V$. Then $N\alpha cl(A) \subseteq N\alpha cl(V) = V$ and hence $A$ is $N\widetilde{g}\alpha$-closed.

## 4. Properties of Nano $\widetilde{g}\alpha$-open sets

**Definition 4.1.** A subset $A$ of a Nano topological space $U$ is called Nano $\widetilde{g}\alpha$-open set (briefly $N\widetilde{g}\alpha$-open) if $A^c$ is $N\widetilde{g}\alpha$-closed.

**Theorem 4.2.** A subset $A \subseteq U$ is $N\widetilde{g}\alpha$-open if and only if $F \subseteq N\alpha int(A)$ whenever $F$ is $N^\sharp gs$-closed set and $F \subseteq A$.
**Proof.** Let $A$ be an $N\widetilde{g}\alpha$-open set and suppose $F \subseteq A$, where $F$ is $N^\sharp gs$-closed. Then $U - A$ is $N\widetilde{g}\alpha$-closed set contained in $N^\sharp gs$-open set $U - F$. Hence $N\alpha cl(U - A) \subseteq U - F$ and $U - N\alpha int(A) \subseteq U - F$. Thus $F \subseteq N\alpha int(A)$.

Conversely, if $F$ is $N^\sharp gs$-closed set with $F \subseteq N\alpha int(A)$ and $F \subseteq A$, then $U - N\alpha int(A) \subseteq U - F$. Thus $N\alpha cl(U - A) \subseteq U - F$. Hence $U - A$ is an $N\widetilde{g}\alpha$-closed set and $A$ is an $N\widetilde{g}\alpha$-open set.

**Theorem 4.3.** Let $A$ and $B$ be subsets of a Nano topological space $U$. If $N\alpha int(A) \subseteq B \subseteq A$ and $A$ is $N\widetilde{g}\alpha$-open, then $B$ is $N\widetilde{g}\alpha$-open.
**Proof.** Let $N\alpha int(A) \subseteq B \subseteq A$. Then $A^c \subseteq B^c \subseteq N\alpha cl(A^c)$, where $A^c$ is $N\widetilde{g}\alpha$-closed and hence $B^c$ is also $N\widetilde{g}\alpha$-closed by Theorem 3.6. Therefore, $B$ is $N\widetilde{g}\alpha$-open.

**Theorem 4.4.** Let $A$ be a subset of a Nano topological space $U$. If $A$ is $N\widetilde{g}\alpha$-closed, then $N\alpha cl(A) - A$ is $N\widetilde{g}\alpha$-open.
**Proof.** Let $A$ be $N\widetilde{g}\alpha$-closed and $F$ be $N^\sharp gs$-closed such that $F \subseteq N\alpha cl(A) - A$. Then $F = \phi$ by Theorem 3.5. Therefore $F \subseteq N\alpha int(N\alpha cl(A) - A)$. Hence $N\alpha cl(A) - A$ is $N\widetilde{g}\alpha$-open.

**Definition 4.5.** Let $U$ be a Nano topological space and $x \in U$. A subset $N$ of $U$ is said to be $N\widetilde{g}\alpha$-neighbourhood of $x$ if there exists an $N\widetilde{g}\alpha$-open set $G$ such that $x \in G \subseteq N$.

**Definition 4.6.**

(a) $N\widetilde{g}\alpha int(A) = \bigcup \Big\{ B : B \text{ is } N\widetilde{g}\alpha\text{-open set and } B \subseteq A \Big\}$.

(b) $N\widetilde{g}\alpha cl(A) = \bigcap \Big\{ B : B$ is $N\widetilde{g}\alpha$-closed set and $A \subseteq B \Big\}$.

**Theorem 4.7.** Let $A$ and $B$ be subsets of $U$. Then

(a) $N\widetilde{g}\alpha int(U) = U$ and $N\widetilde{g}\alpha int(\phi) = \phi$.

(b) $N\widetilde{g}\alpha int(A) \subseteq A$.

(c) If $B$ is any $N\widetilde{g}\alpha$-open set contained in $A$, then $B \subseteq N\widetilde{g}\alpha int(A)$.

(d) If $A \subseteq B$, then $N\widetilde{g}\alpha int(A) \subseteq N\widetilde{g}\alpha int(B)$.

**Proof.**

(a) Since $U$ and $\phi$ are $N\widetilde{g}\alpha$-open sets, by definition $N\widetilde{g}\alpha int(U) = \bigcup \Big\{ B : B$ is $N\widetilde{g}\alpha$-open set and $B \subseteq U \Big\} = U$. Since $\phi$ is the only $N\widetilde{g}\alpha$-open set contained in $\phi$, $N\widetilde{g}\alpha int(\phi) = \phi$.

(b) Let $x \in Ng\tilde{\alpha}int(A) \Rightarrow x$ is a $N\widetilde{g}\alpha$ interior of $A \Rightarrow A$ is a $N\widetilde{g}\alpha$-neighbourhood of $x \Rightarrow x \in A$. Thus $N\widetilde{g}\alpha int(A) \subseteq A$.

(c) Let $B$ be any $N\widetilde{g}\alpha$-open set such that $B \subseteq A$. Let $x \in B$. Since $B$ is an $N\widetilde{g}\alpha$-open set contained in $A$, $x$ is an $N\widetilde{g}\alpha$-interior point of $A$. That is $B$ is an $N\widetilde{g}\alpha int(B)$. Hence $B \subseteq N\widetilde{g}\alpha int(A)$.

(d) Let $A$ and $B$ be subsets of $U$ such that $A \subseteq B$. Let $x \in N\widetilde{g}\alpha int(A)$. Then $x$ is an $N\widetilde{g}\alpha$-interior point of $A$ and so $A$ is $N\widetilde{g}\alpha$-neighbourhood of $x$. This implies that $x \in N\widetilde{g}\alpha int(B)$. Hence $N\widetilde{g}\alpha int(A) \subseteq N\widetilde{g}\alpha int(B)$.

**Theorem 4.8.** If a subset $A$ of a nano topological space $U$ is $N\widetilde{g}\alpha$-open, then $N\widetilde{g}\alpha int(A) = A$.
**Proof.** Let $A$ be an $N\widetilde{g}\alpha$-open subset of $U$. We know that $N\widetilde{g}\alpha int(A) \subseteq A$. Also $A$ is an $N\widetilde{g}\alpha$-open set contained in $A$. By Theorem 4.7 (c), $A \subseteq N\widetilde{g}\alpha int(A)$. Hence $N\widetilde{g}\alpha int(A) = A$.

**Theorem 4.9.** If $A$ and $B$ are subsets of $U$, then $N\widetilde{g}\alpha int(A) \cup N\widetilde{g}\alpha int(B) \subseteq N\widetilde{g}\alpha int(A \cup B)$.
**Proof.** We know that $A \subseteq A \cup B$ and $B \subseteq A \cup B$. By Theorem 4.7 (d), $N\widetilde{g}\alpha int(A) \subseteq N\widetilde{g}\alpha int(A \cup B)$ and $N\widetilde{g}\alpha int(B) \subseteq N\widetilde{g}\alpha int(A \cup B)$. This implies that $N\widetilde{g}\alpha int(A) \cup N\widetilde{g}\alpha int(B) \subseteq N\widetilde{g}\alpha int(A \cup B)$.

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