"Vasile Alecsandri" University of Bacău
Faculty of Sciences
Scientific Studies and Research
Series Mathematics and Informatics
Vol. 27(2017), No. 1, 33-48

# SEMIOPEN SETS IN IDEAL MINIMAL SPACES

S. JAFARI, N. RAJESH AND R. SARANYA

**Abstract.** In this paper, we present and study the concepts of semiopen sets and their related notions in ideal minimal spaces.

## 1. Introduction

In 2001, Popa and Noiri [8] introduced the notion of minimal structure which is a generalization of a topology on a given nonempty set. Among others, they introduced the notion of $m$-continuous function as a function defined between a minimal structure and a topological space. They showed that the $m$-continuous functions have properties similar to those of continuous functions between topological spaces. Let $X$ be a topological space and $A \subset X$. The closure of $A$ and the interior of $A$ are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subfamily $m$ of the power set $P(X)$ of a nonempty set $X$ is called a minimal structure [8] on $X$ if $\emptyset$ and $X$ belong to $m$. By $(X, m)$, we denote a nonempty set $X$ with a minimal structure $m$ on $X$. The members of the minimal structure $m$ are called $m$-open sets [8], and the pair $(X, m)$ is called an $m$-space. The complement of $m$-open set is said to be $m$-closed [8].

**Keywords and phrases:** Ideal minimal spaces, $m$-semi-$\mathcal{I}$-open sets, $m$-semi-$\mathcal{I}$-closed sets.
**(2010) Mathematics Subject Classification:** 54D10.

The concept of ideals in topological spaces has been introduced and studied by Kuratowski [5] and Vaidyanathasamy [10]. An ideal $\mathcal{I}$ on a topological space $(X, \tau)$ is a nonempty collection of subsets of $X$ which satisfies (i) $A \in \mathcal{I}$ and $B \subset A$ implies $B \in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. Given a minimal space $(X, m)$ with an ideal $\mathcal{I}$ on $X$ and if $\mathcal{P}(X)$ is the set of all subsets of $X$, a set operator $(.)^*_m$: $\mathcal{P}(X) \to \mathcal{P}(X)$, called the local minimal function [9] of $A$ with respect to $m$ and $\mathcal{I}$, is defined as follows: for $A \subset X$, $A^*_m(m, \mathcal{I}) = \{x \in X | U \cap A \notin \mathcal{I}$ for every $U \in m(x)\}$, where $m(x) = \{U \in m | x \in U\}$. The set operator $m\operatorname{Cl}^*(.)$ is called a minimal $*$-closure and is defined as $m\operatorname{Cl}^*(A) = A \cup A^*_m$ for $A \subset X$. The minimal structure $m^*(m, \mathcal{I})$ called the $*$-minimal, is finer than $m$ and $m\operatorname{Int}^*(A)$ denotes the interior of $A$ in $m^*(m, \mathcal{I})$.

## 2. Preliminaries

**Definition 2.1.** [8] *Given $A \subset X$, the $m$-interior of $A$ and the $m$-closure of $A$ are defined by* $m\operatorname{Int}(A) = \cup\{W/W \in m, W \subseteq A\}$ *and* $m\operatorname{Cl}(A) = \cap\{F/A \subseteq F, X \setminus F \in m\}$, *respectively.*

**Theorem 2.2.** *Let $(X, m)$ be an $m$-space, and $A$, $B$ subsets of $X$. Then $x \in m\operatorname{Cl}(A)$ if and only if $U \cap A \neq \varnothing$ for every $U \in m$ containing $x$. And satisfying the following properties:*

(i) $m\operatorname{Cl}(m\operatorname{Cl}(A)) = m\operatorname{Cl}(A)$.
(ii) $m\operatorname{Int}(m\operatorname{Int}(A)) = m\operatorname{Int}(A)$.
(iii) $m\operatorname{Int}(X \setminus A) = X \setminus m\operatorname{Cl}(A)$.
(iv) $m\operatorname{Cl}(X \setminus A) = X \setminus m\operatorname{Int}(A)$.
(v) *If* $A \subset B$ *then* $m\operatorname{Cl}(A) \subset m\operatorname{Cl}(B)$.
(vi) $m\operatorname{Cl}(A \cup B) \subset m\operatorname{Cl}(A) \cup m\operatorname{Cl}(B)$.
(vii) $A \subset m\operatorname{Cl}(A)$ *and* $m\operatorname{Int}(A) \subset A$.

**Definition 2.3.** *A subset $A$ of a minimal space $(X, m)$ is said to be $m$-semiopen* [6] *if $A \subset m\operatorname{Cl}(m\operatorname{Int}(A))$.*

**Definition 2.4.** *A function $f : (X, m) \to (Y, \tau)$ is said to be $m$-semicontinuous* [6] *if the inverse image of every open set of $Y$ is $m$-semiopen in $(X, m)$.*

**Definition 2.5.** *A subset $S$ of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be semi-$\mathcal{I}$-open* [4] *if $S \subset \operatorname{Int}(\operatorname{Cl}^*(S))$.*

**Definition 2.6.** *A subset $A$ of an ideal minimal space $(X, m, \mathcal{I})$ is said to be*

(i) *$m$-$\alpha$-$\mathcal{I}$-open* [3] *if $A \subset m\operatorname{Int}(m\operatorname{Cl}^*(m\operatorname{Int}(A)))$.*

(ii) *m-pre-$\mathcal{I}$-open* [2] *if* $A \subset m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A))$.
(iii) *m-$\delta$-$\mathcal{I}$-open* [1] *if* $m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A)) \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A))$.
(iv) *strongly m-$\beta$-$\mathcal{I}$-open* [1] *if* $A \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A)))$.

**Definition 2.7.** *A function* $f : (X, m) \to (Y, \tau)$ *is said to be:*

(i) *m-pre-$\mathcal{I}$-continuous* [2] *if the inverse image of every open set of Y is m-pre-$\mathcal{I}$-open in X.*
(ii) *m-$\alpha$-$\mathcal{I}$-continuous* [3] *if the inverse image of every open set of Y is m-$\alpha$-$\mathcal{I}$-open in X.*
(iii) *m-$\delta$-$\mathcal{I}$-continuous* [3] *if the inverse image of every open set of Y is m-$\delta$-$\mathcal{I}$-open in X.*

## 3. $m$-SEMI-$\mathcal{I}$-OPEN SETS

**Definition 3.1.** *A subset A of an ideal minimal space* $(X, m, \mathcal{I})$ *is said to be m-semi-$\mathcal{I}$-open if and only if* $A \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A))$.
*The family of all m-semi-$\mathcal{I}$-open sets of* $(X, m, \mathcal{I})$ *is denoted by* $S\mathcal{I}O(X, m)$. *Moreover, the family of all m-semi-$\mathcal{I}$-open sets of* $(X, m, \mathcal{I})$ *containing x is denoted by* $ms\mathcal{I}O(X, x)$.

**Remark 3.2.** *Let* $\mathcal{I}$ *and* $\mathcal{J}$ *be two ideals on X. If* $\mathcal{I} \subset \mathcal{J}$, *then* $S\mathcal{I}O(X, m) \subset S\mathcal{J}O(X, m)$.

**Proposition 3.3.** (i) *Every m-$\alpha$-$\mathcal{I}$-open set is m-semi-$\mathcal{I}$-open.*
(ii) *Every m-semi-$\mathcal{I}$-open set is m-semiopen.*
(iii) *Every m-semi-$\mathcal{I}$-open set is m-$\delta$-$\mathcal{I}$-open.*

*Proof.* The proof follows from the definitions. □

The following examples show that the converses of Proposition 3.3 is not true in general.

**Example 3.4.** *Let* $X = \{a, b, c\}$ $m = \{\emptyset, \{a\}, \{b\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{a\}\}$. *Then the set* $\{b, c\}$ *is m-semi-$\mathcal{I}$-open but it is not m-$\alpha$-$\mathcal{I}$-open; the set* $\{a, c\}$ *is m-semiopen but it is not m-semi-$\mathcal{I}$-open and the set* $\{c\}$ *is m-$\delta$-$\mathcal{I}$-open but not m-semi-$\mathcal{I}$-open.*

**Example 3.5.** *Let* $X = \{a, b, c\}$ $m = \{\emptyset, \{a\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{a\}\}$. *Then the set* $\{b\}$ *is m-$\delta$-$\mathcal{I}$-open but it is not m-semi-$\mathcal{I}$-open.*

**Remark 3.6.** *It is clear that m-semi-$\mathcal{I}$-openness and m-pre-$\mathcal{I}$-openness are independent notions as it is shown in the following example.*

**Example 3.7.** *Let $X = \{a, b, c\}$ $m = \{\emptyset, \{a, b\}, \{a, c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then the set $\{b, c\}$ is m-pre-$\mathcal{I}$-open but it is not m-semi-$\mathcal{I}$-open. Let $(X, m)$ be as in Example 3.4, the set $\{b, c\}$ is m-semi-$\mathcal{I}$-open but it is not m-pre-$\mathcal{I}$-open*

**Proposition 3.8.** *Let $(X, m, \{\emptyset\})$ be an ideal minimal space and $A \subset X$. The subset $A$ is m-semi-$\mathcal{I}$-open if and only if $A$ is m-semiopen.*

*Proof.* The proof follows from the fact that if $\mathcal{I} = \{\emptyset\}$, then $A^*_m = m\,\mathrm{Cl}(A)$ [9]. □

**Proposition 3.9.** *A subset $A$ of an ideal minimal space $(X, m, \mathcal{I})$ is m-semi-$\mathcal{I}$-open if and only if $m\,\mathrm{Cl}^*(A) = m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A))$.*

*Proof.* Let $A \in \mathcal{SIO}(X, m)$. Then we have $A \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A))$. Then $m\,\mathrm{Cl}^*(A) \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A))$ and hence $m\,\mathrm{Cl}^*(A) = m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A))$. The converse is obvious. □

**Remark 3.10.** *The intersection of two m-semi-$\mathcal{I}$-open sets need not be m-semi-$\mathcal{I}$-open as it can be seen from the following example.*

**Example 3.11.** *Let $X = \{a, b, c, d\}$, $m = \{\varnothing, \{a, b\}, \{a, c\}, X\}$ and $\mathcal{I} = \{\varnothing, \{a\}\}$. Then the sets $\{a, b\}$ and $\{a, c\}$ are m-semi-$\mathcal{I}$-open sets of $(X, m, \mathcal{I})$ but their intersection $\{a\}$ is not an m-semi-$\mathcal{I}$-open set of $(X, m, \mathcal{I})$.*

However, we have the following

**Theorem 3.12.** *If $\{A_\alpha\}_{\alpha\in\Omega}$ is a family of m-semi-$\mathcal{I}$-open sets in $(X, m, \mathcal{I})$, then $\bigcup_{\alpha\in\Omega} A_\alpha$ is m-semi-$\mathcal{I}$-open in $(X, m, \mathcal{I})$.*

*Proof.* Since $\{A_\alpha : \alpha \in \Omega\} \subset \mathcal{SIO}(X, m)$, then $A_\alpha \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A_\alpha))$ for every $\alpha \in \Omega$. Thus, $\underset{\alpha\in\Omega}{\cup} A_\alpha \subset \underset{\alpha\in\Omega}{\cup} m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A_\alpha)) \subset m\,\mathrm{Cl}^*(\underset{\alpha\in\Omega}{\cup} m\,\mathrm{Int}(A_\alpha)) = m\,\mathrm{Cl}^*(m\,\mathrm{Int}(\underset{\alpha\in\Omega}{\cup} A_\alpha))$. Therefore, we obtain $\underset{\alpha\in\Omega}{\cup} A_\alpha \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(\underset{\alpha\in\Omega}{\cup} A_\alpha))$. Hence any union of $m$-semi-$\mathcal{I}$-open sets is $m$-semi-$\mathcal{I}$-open. □

**Theorem 3.13.** *Let $(X, m, \mathcal{I})$ be an ideal minimal space. Then a subset $A$ of $X$ is m-semi-$\mathcal{I}$-open if and only if it is both m-$\delta$-$\mathcal{I}$-open and strong m-$\beta$-$\mathcal{I}$-open.*

*Proof.* Let $A$ be an $m$-semi-$\mathcal{I}$-open set, then we have $A \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A)) \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A)))$. This shows that $A$ is strong $m$-$\beta$-$\mathcal{I}$-open. Moreover, $m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A)) \subset m\,\mathrm{Cl}^*(A) \subset$

$m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A))$. Therefore, $A$ is $m$-$\delta$-$\mathcal{I}$-open. Conversely, let $A$ be $m$-$\delta$-$\mathcal{I}$-open and strong $m$-$\beta$-$\mathcal{I}$-open set, then we have $m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A)) \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A))$. Thus we obtain that $m\,\mathrm{Cl}^*(m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A))) \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A))$. Since $A$ is strong $m$-$\beta$-$\mathcal{I}$-open, we have $A \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A))) \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A))$ and $A \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(A))$. Hence $A$ is an $m$-semi-$\mathcal{I}$-open set. □

**Definition 3.14.** *In an ideal minimal space $(X, m, \mathcal{I})$, $A \subset X$ is said to be m-semi-$\mathcal{I}$-closed if $X \backslash A$ is m-semi-$\mathcal{I}$-open in $X$.*

**Theorem 3.15.** *A subset $A$ is an m-semi-$\mathcal{I}$-closed set in an ideal minimal space $(X, m, \mathcal{I})$ if and only if $m\,\mathrm{Cl}(m\,\mathrm{Int}^*(A)) \subset A$.*

*Proof.* Straightforward. □

**Theorem 3.16.** *If $A$ is an m-semi-$\mathcal{I}$-closed set in an ideal minimal space $(X, m, \mathcal{I})$, then $m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A)) \subset A$.*

*Proof.* Since $A \in S\mathcal{I}C(X, m)$, $X \backslash A \in S\mathcal{I}O(X, m)$. Hence, $X \backslash A \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(X \backslash A)) \subset m\,\mathrm{Cl}(m\,\mathrm{Int}(X \backslash A)) = X \backslash (m\,\mathrm{Int}(m\,\mathrm{Cl}(A))) \subset X \backslash (m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A)))$. Therefore, we obtain $m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A)) \subset A$. □

**Proposition 3.17.** *Let $(X, m, \mathcal{I})$ be an ideal minimal space. If a subset $A$ of $X$ is m-$\beta$-$\mathcal{I}$-closed and m-$\delta$-$\mathcal{I}$-open, then it is m-semi-$\mathcal{I}$-closed.*

*Proof.* The proof follows from the definitions. □

**Proposition 3.18.** *A subset $A$ of an ideal minimal space $(X, m, \mathcal{I})$ is m-semi-$\mathcal{I}$-closed if and only if $m\,\mathrm{Int}(m\,\mathrm{Cl}^*(A)) = m\,\mathrm{Int}(A)$.*

*Proof.* Obvious. □

**Theorem 3.19.** *An arbitrary intersection of m-semi-$\mathcal{I}$-closed sets is always m-semi-$\mathcal{I}$-closed.*

*Proof.* It follows from Theorems 3.12 and 3.16. □

**Definition 3.20.** *Let $(X, m, \mathcal{I})$ be an ideal minimal space, $S$ a subset of $X$ and $x$ be a point of $X$. Then*

(i) *$x$ is called an m-semi-$\mathcal{I}$-interior point of $S$ if there exists $V \in S\mathcal{I}O(X, m)$ such that $x \in V \subset S$.*

ii) *the set of all m-semi-$\mathcal{I}$-interior points of $S$ is called m-semi-$\mathcal{I}$-interior of $S$ and is denoted by $ms\mathcal{I}\,\mathrm{Int}(S)$.*

**Theorem 3.21.** *Let $A$ and $B$ be subsets of $(X, m, \mathcal{I})$. Then the following properties hold:*

(i) $ms\mathcal{I}\operatorname{Int}(A) = \cup\{T : T \subset A$ *and* $A \in S\mathcal{I}O(X,m)\}$.
(ii) $ms\mathcal{I}\operatorname{Int}(A)$ *is the largest m-semi-$\mathcal{I}$-open subset of* $X$ *contained in* $A$.
(iii) $A$ *is m-semi-$\mathcal{I}$-open if and only if* $A = ms\mathcal{I}\operatorname{Int}(A)$.
(iv) $ms\mathcal{I}\operatorname{Int}(ms\mathcal{I}\operatorname{Int}(A)) = ms\mathcal{I}\operatorname{Int}(A)$.
(v) *If* $A \subset B$, *then* $ms\mathcal{I}\operatorname{Int}(A) \subset ms\mathcal{I}\operatorname{Int}(B)$.
(vi) $ms\mathcal{I}\operatorname{Int}(A) \cup ms\mathcal{I}\operatorname{Int}(B) \subset ms\mathcal{I}\operatorname{Int}(A \cup B)$.
(vii) $ms\mathcal{I}\operatorname{Int}(A \cap B) \subset ms\mathcal{I}\operatorname{Int}(A) \cap ms\mathcal{I}\operatorname{Int}(B)$.

*Proof.* (i). Let $x \in \cup\{T : T \subset A$ and $A \in S\mathcal{I}O(X,m)\}$. Then, there exists $T \in mS\mathcal{I}O(X,x)$ such that $x \in T \subset A$ and hence $x \in ms\mathcal{I}\operatorname{Int}(A)$. This shows that $\cup\{T : T \subset A$ and $A \in S\mathcal{I}O(X,m)\}$ $\subset ms\mathcal{I}\operatorname{Int}(A)$. For the reverse inclusion, let $x \in ms\mathcal{I}\operatorname{Int}(A)$. Then there exists $T \in mS\mathcal{I}O(X,x)$ such that $x \in T \subset A$. we obtain $x \in \cup\{T : T \subset A$ and $A \in mS\mathcal{I}O(X)\}$. This shows that $ms\mathcal{I}\operatorname{Int}(A) \subset$ $\cup\{T : T \subset A$ and $A \in S\mathcal{I}O(X,m)\}$. Therefore, we obtain $ms\mathcal{I}\operatorname{Int}(A)$ $= \cup\{T : T \subset A$ and $A \in S\mathcal{I}O(X,m)\}$.
The proof of (ii)-(v) are obvious.
(vi). Clearly, $ms\mathcal{I}\operatorname{Int}(A) \subset ms\mathcal{I}\operatorname{Int}(A \cup B)$ and $ms\mathcal{I}\operatorname{Int}(B) \subset ms\mathcal{I}\operatorname{Int}(A \cup B)$. Then we obtain $ms\mathcal{I}\operatorname{Int}(A) \cup ms\mathcal{I}\operatorname{Int}(B) \subset ms\mathcal{I}\operatorname{Int}(A \cup B)$.
(vii). Since $A \cap B \subset A$ and $A \cap B \subset B$, by (v), we have $ms\mathcal{I}\operatorname{Int}(A \cap B) \subset ms\mathcal{I}\operatorname{Int}(A)$ and $ms\mathcal{I}\operatorname{Int}(A \cap B) \subset ms\mathcal{I}\operatorname{Int}(B)$. Then $ms\mathcal{I}\operatorname{Int}(A \cap B) \subset ms\mathcal{I}\operatorname{Int}(A) \cap ms\mathcal{I}\operatorname{Int}(B)$. □

**Definition 3.22.** *Let* $(X,m,\mathcal{I})$ *be an ideal minimal space,* $S$ *a subset of* $X$ *and* $x$ *be a point of* $X$. *Then*
(i) $x$ *is called an m-semi-$\mathcal{I}$-cluster point of* $S$ *if* $V \cap S \neq \emptyset$ *for every* $V \in mS\mathcal{I}O(X,x)$.
(ii) *the set of all m-semi-$\mathcal{I}$-cluster points of* $S$ *is called m-semi-$\mathcal{I}$-closure of* $S$ *and is denoted by* $ms\mathcal{I}\operatorname{Cl}(S)$.

**Theorem 3.23.** *Let* $A$ *and* $B$ *be subsets of* $(X,m,\mathcal{I})$. *Then the following properties hold:*
(i) $ms\mathcal{I}\operatorname{Cl}(A) = \cap\{F : A \subset F$ *and* $F \in S\mathcal{I}C(X,m)\}$.
(ii) $ms\mathcal{I}\operatorname{Cl}(A)$ *is the smallest m-semi-$\mathcal{I}$-closed subset of* $X$ *containing* $A$.
(iii) $A$ *is m-semi-$\mathcal{I}$-closed if and only if* $A = ms\mathcal{I}\operatorname{Cl}(A)$.
(iv) $ms\mathcal{I}\operatorname{Cl}(ms\mathcal{I}\operatorname{Cl}(A)) = ms\mathcal{I}\operatorname{Cl}(A)$.
(v) *If* $A \subset B$, *then* $ms\mathcal{I}\operatorname{Cl}(A) \subset ms\mathcal{I}\operatorname{Cl}(B)$.
(vi) $ms\mathcal{I}\operatorname{Cl}(A \cup B) = ms\mathcal{I}\operatorname{Cl}(A) \cup ms\mathcal{I}\operatorname{Cl}(B)$.
(vii) $ms\mathcal{I}\operatorname{Cl}(A \cap B) \subset ms\mathcal{I}\operatorname{Cl}(A) \cap ms\mathcal{I}\operatorname{Cl}(B)$.

*Proof.* (i). Suppose that $x \notin ms\mathcal{I}\,\mathrm{Cl}(A)$. Then there exists $V \in S\mathcal{I}O(X,m)$ such that $V \cap A \neq \emptyset$. Since $X \backslash V$ is $m$-semi-$\mathcal{I}$-closed set containing $A$ and $x \notin X \backslash V$, we obtain $x \notin \cap\{F : A \subset F$ and $F \in S\mathcal{I}C(X,m)\}$. Then there exists $F \in S\mathcal{I}C(X,m)$ such that $A \subset F$ and $x \notin F$. Since $X \backslash V$ is $m$-semi-$\mathcal{I}$-closed set containing $x$, we obtain $(X \backslash F) \cap A = \emptyset$. This shows that $x \notin ms\mathcal{I}\,\mathrm{Cl}(A)$. Therefore, we obtain $ms\mathcal{I}\,\mathrm{Cl}(A) = \cap\{F : A \subset F$ and $F \in S\mathcal{I}C(X,m)\}$.
The other proofs are obvious. □

**Theorem 3.24.** *Let $(X,m,\mathcal{I})$ be an ideal minimal space and $A \subset X$. A point $x \in ms\mathcal{I}\,\mathrm{Cl}(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in mS\mathcal{I}O(X,x)$.*

*Proof.* Suppose that $x \in ms\mathcal{I}\,\mathrm{Cl}(A)$. We shall show that $U \cap A \neq \emptyset$ for every $U \in mS\mathcal{I}O(X,x)$. Suppose that there exists $U \in mS\mathcal{I}O(X,x)$ such that $U \cap A = \emptyset$. Then $A \subset X \backslash U$ and $X \backslash U$ is $m$-semi-$\mathcal{I}$-closed. Since $A \subset X \backslash U$, $ms\mathcal{I}\,\mathrm{Cl}(A) \subset ms\mathcal{I}\,\mathrm{Cl}(X \backslash U)$. Since $x \in ms\mathcal{I}\,\mathrm{Cl}(A)$, we have $x \in ms\mathcal{I}\,\mathrm{Cl}(X \backslash U)$. Since $X \backslash U$ is $m$-semi-$\mathcal{I}$-closed, we have $x \in X \backslash U$; hence $x \notin U$, which is a contradicition that $x \in U$. Therefore, $U \cap A \neq \emptyset$. Conversely, suppose that $U \cap A \neq \emptyset$ for every $U \in mS\mathcal{I}O(X,x)$. We shall show that $x \in ms\mathcal{I}\,\mathrm{Cl}(A)$. Suppose that $x \notin ms\mathcal{I}\,\mathrm{Cl}(A)$. Then there exists $U \in mS\mathcal{I}O(X,x)$ such that $U \cap A = \emptyset$. This is a contradicition to $U \cap A \neq \emptyset$; hence $x \in ms\mathcal{I}\,\mathrm{Cl}(A)$. □

**Theorem 3.25.** *Let $(X,m,\mathcal{I})$ be an ideal minimal space and $A \subset X$. Then the following propeties hold:*

(i) *$ms\mathcal{I}\,\mathrm{Int}(X \backslash A) = X \backslash ms\mathcal{I}\,\mathrm{Cl}(A)$;*
(i) *$ms\mathcal{I}\,\mathrm{Cl}(X \backslash A) = X \backslash ms\mathcal{I}\,\mathrm{Int}(A)$.*

*Proof.* (i). Let $x \in ms\mathcal{I}\,\mathrm{Cl}(A)$. Since $x \notin ms\mathcal{I}\,\mathrm{Cl}(A)$, there exists $V \in ms\mathcal{I}O(X,x)$ such that $V \cap A \neq \emptyset$; hence we obtain $x \in ms\mathcal{I}\,\mathrm{Int}(X \backslash A)$. This shows that $X \backslash ms\mathcal{I}\,\mathrm{Cl}(A) \subset ms\mathcal{I}\,\mathrm{Int}(X \backslash A)$. Let $x \in ms\mathcal{I}\,\mathrm{Int}(X \backslash A)$. Since $ms\mathcal{I}\,\mathrm{Int}(X \backslash A) \cap A = \emptyset$, we obtain $x \notin ms\mathcal{I}\,\mathrm{Cl}(A)$; hence $x \in X \backslash ms\mathcal{I}\,\mathrm{Cl}(A)$. Therefore, we obtain $ms\mathcal{I}\,\mathrm{Int}(X \backslash A) = X \backslash ms\mathcal{I}\,\mathrm{Cl}(A)$.
(ii). Follows from (i). □

**Definition 3.26.** *A subset $B_x$ of an ideal minimal space $(X,m,\mathcal{I})$ is said to be an $m$-semi-$\mathcal{I}$-neighbourhood of a point $x \in X$ if there exists an $m$-semi-$\mathcal{I}$-open set $U$ such that $x \in U \subset B_x$.*

**Theorem 3.27.** *A subset of an ideal minimal space $(X, m, \mathcal{I})$ is m-semi-$\mathcal{I}$-open if and only if it is an m-semi-$\mathcal{I}$-neighbourhood of each of its points.*

*Proof.* Let $G$ be an $m$-semi-$\mathcal{I}$-open set of $X$. Then by definition, it is clear that $G$ is an $m$-semi-$\mathcal{I}$-neighbourhood of each of its points, since for every $x \in G$, $x \in G \subset G$ and $G$ is $m$-semi-$\mathcal{I}$-open. Conversely, suppose $G$ is an $m$-semi-$\mathcal{I}$-neighbourhood of each of its points. Then for each $x \in G$, there exists $S_x \in mS\mathcal{I}O(X)$ such that $S_x \subset G$. Then $G = \bigcup\{S_x : x \in G\}$. Since each $S_x$ is $m$-semi-$\mathcal{I}$-open, $G$ is $m$-semi-$\mathcal{I}$-open in $(X, m, \mathcal{I})$. □

## 4. Semi-$\mathcal{I}$-continuous functions

**Definition 4.1.** *A function $f : (X, m, \mathcal{I}) \to (Y, \tau)$ is said to be m-semi-$\mathcal{I}$-continuous if the inverse image of every open set of $Y$ is m-semi-$\mathcal{I}$-open in $X$.*

**Proposition 4.2.**
- (i) *Every m-$\alpha$-$\mathcal{I}$-continuous function is m-semi-$\mathcal{I}$-continuous but not conversely.*
- (ii) *Every m-semi-$\mathcal{I}$-continuous function is m-semicontinuous but not conversely.*
- (iii) *Every m-semi-$\mathcal{I}$-continuous function is m-$\delta$-$\mathcal{I}$-continuous but not conversely.*
- (iv) *m-semi-$\mathcal{I}$-continuity and m-pre-$\mathcal{I}$-continuity are independent.*

*Proof.* The proof follows from Proposition 3.3, Examples 3.4 and 3.7. □

**Theorem 4.3.** *For a function $f : (X, m, \mathcal{I}) \to (Y, \tau)$, the following statements are equivalent:*
- (i) *$f$ is m-semi-$\mathcal{I}$-continuous;*
- (ii) *For each point $x$ in $X$ and each open set $F$ of $Y$ such that $f(x) \in F$, there is an m-semi-$\mathcal{I}$-open set $A$ in $X$ such that $x \in A$, $f(A) \subset F$;*
- (iii) *The inverse image of each closed set of $Y$ is m-semi-$\mathcal{I}$-closed in $X$;*
- (iv) *For each subset $A$ of $X$, $f(ms\mathcal{I}\,\mathrm{Cl}(A)) \subset \mathrm{Cl}(f(A))$;*
- (v) *For each subset $B$ of $Y$, $ms\mathcal{I}\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}(B))$;*
- (vi) *For each subset $C$ of $Y$, $f^{-1}(\mathrm{Int}(C)) \subset ms\mathcal{I}\,\mathrm{Int}(f^{-1}(C))$.*

*Proof.* (i)⇒(ii): Let $x \in X$ and $F$ be an open set of $Y$ containing $f(x)$. By (i), $f^{-1}(F)$ is $m$-semi-$\mathcal{I}$-open in $X$. Let $A = f^{-1}(F)$. Then $x \in A$ and $f(A) \subset F$.

(ii)⇒(i): Let $F$ be an open set in $Y$ and let $x \in f^{-1}(F)$. Then $f(x) \in F$. By (ii), there is an $m$-semi-$\mathcal{I}$-open set $U_x$ in $X$ such that $x \in U_x$ and $f(U_x) \subset F$. Then $x \in U_x \subset f^{-1}(F)$. Hence $f^{-1}(F)$ is $m$-semi-$\mathcal{I}$-open in $X$.
(i)⇔(iii): This follows due to the fact that for any subset $B$ of $Y$, $f^{-1}(Y \backslash B) = X \backslash f^{-1}(B)$.
(iii)⇒(iv): Let $A$ be a subset of $X$. Since $A \subset f^{-1}(f(A))$ we have $A \subset f^{-1}(\mathrm{Cl}(f(A)))$. Since $\mathrm{Cl}(f(A))$ is closed in $Y$, by (iii) $f^{-1}(\mathrm{Cl}(f(A)))$ is $m$-semi-$\mathcal{I}$-closed in $X$. Then $ms\mathcal{I}\,\mathrm{Cl}(A) \subset f^{-1}(\mathrm{Cl}(f(A)))$. Then $f((ms\mathcal{I}\,\mathrm{Cl}(A))) \subset \mathrm{Cl}(f(A))$.
(iv)⇒(iii): Let $F$ be any closed subset of $Y$. Then $f(ms\mathcal{I}\,\mathrm{Cl}(f^{-1}(F))) \subset \mathrm{Cl}(f(f^{-1}(F))) = \mathrm{Cl}(F) = F$. Therefore, $ms\mathcal{I}\,\mathrm{Cl}(f^{-1}(F)) \subset f^{-1}(F)$. Consequently, $f^{-1}(F)$ is $m$-semi-$\mathcal{I}$-closed in $X$.
(iv)⇒(v): Let $B$ be any subset of $Y$. Now, $f(ms\mathcal{I}\,\mathrm{Cl}(f^{-1}(B))) \subset \mathrm{Cl}(f(f^{-1}(B))) \subset \mathrm{Cl}(B)$. Consequently, $ms\mathcal{I}\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}(B))$.
(v)⇒(iv): Let $B = f(A)$ where $A$ is a subset of $X$. Then, $ms\mathcal{I}\,\mathrm{Cl}(A) \subset ms\mathcal{I}\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}(B)) = f^{-1}(\mathrm{Cl}(f(A)))$. This shows that $f(ms\mathcal{I}\,\mathrm{Cl}(A)) \subset \mathrm{Cl}(f(A))$.
(i)⇒(vi): Let $B$ be an open set in $Y$. Observe that $f^{-1}(\mathrm{Int}(B))$ is $m$-semi-$\mathcal{I}$-open in $X$ and we have $f^{-1}(\mathrm{Int}(B)) \subset ms\mathcal{I}\,\mathrm{Int}(f^{-1}\,\mathrm{Int}(B)) \subset ms\mathcal{I}\,\mathrm{Int}(f^{-1}(B))$.
(vi)⇒(i): Let $B$ be an open set in $Y$. Then $\mathrm{Int}(B) = B$ and $f^{-1}(B) \subset f^{-1}(\mathrm{Int}(B)) \subset ms\mathcal{I}\,\mathrm{Int}(f^{-1}(B))$. Hence we have $f^{-1}(B) = ms\mathcal{I}\,\mathrm{Int}(f^{-1}(B))$. This shows that $f^{-1}(B)$ is $m$-semi-$\mathcal{I}$-open in $X$. □

**Theorem 4.4.** *Let $f : (X, m, \mathcal{I}) \to (Y, \tau)$ be an $m$-semi-$\mathcal{I}$-continuous function. Then for each subset $V$ of $Y$, $f^{-1}(\mathrm{Int}(V)) \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(f^{-1}(V)))$.*

*Proof.* Let $V$ be any subset of $Y$. Then $\mathrm{Int}(V)$ is open in $Y$ and so $f^{-1}(\mathrm{Int}(V))$ is $m$-semi-$\mathcal{I}$-open in $X$. Hence $f^{-1}(\mathrm{Int}(V)) \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(f^{-1}(\mathrm{Int}(V)))) \subset m\,\mathrm{Cl}^*(m\,\mathrm{Int}(f^{-1}(V)))$. □

**Corollary 4.5.** *Let $f : (X, m, \mathcal{I}) \to (Y, \tau)$ be an $m$-semi-$\mathcal{I}$-continuous function. Then for each subset $V$ of $Y$, $m\,\mathrm{Int}(m\,\mathrm{Cl}^*(f^{-1}(V))) \subset f^{-1}(\mathrm{Cl}(V))$.*

**Theorem 4.6.** *Let $f : (X, m, \mathcal{I}) \to (Y, \tau)$ be a bijection. Then $f$ is $m$-semi-$\mathcal{I}$-continuous if and only if $\mathrm{Int}(f(U)) \subset f(ms\mathcal{I}\,\mathrm{Int}(U))$ for each subset $U$ of $X$.*

*Proof.* Let $U$ be any subset of $X$. Then by Theorem 4.3, $f^{-1}(\mathrm{Int}(f(U))) \subset ms\mathcal{I}\,\mathrm{Int}(f^{-1}(f(U)))$. Since $f$ is a bijection, $\mathrm{Int}(f(U)) = f(f^{-1}(\mathrm{Int}(f(U)))) \subset f(ms\mathcal{I}\,\mathrm{Int}(U))$. Conversely, let $V$ be any subset of $Y$. Then $\mathrm{Int}(f(f^{-1}(V))) \subset f(ms\mathcal{I}\,\mathrm{Int}(f^{-1}(V)))$. It follows from the bijectivity of $f$ that $\mathrm{Int}(V) = \mathrm{Int}(f(f^{-1}(V))) \subset f(ms\mathcal{I}\,\mathrm{Int}(f^{-1}(V)))$; hence $f^{-1}(\mathrm{Int}(V) \subset ms\mathcal{I}\,\mathrm{Int}(f^{-1}(V))$. Therefore, by Theorem 4.3, $f$ is $m$-semi-$\mathcal{I}$-continuous. □

**Definition 4.7.** *The graph $G(f)$ of a function $f : (X, m, \mathcal{I}) \to (Y, \tau)$ is said to be $m$-semi-$\mathcal{I}$-closed in $X \times Y$ if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in mS\mathcal{I}O(X, x)$ and an open set $V$ of $Y$ containing $y$ such that $(U \times V) \cap G(f) = \emptyset$.*

**Lemma 4.8.** *The graph $f : (X, m, \mathcal{I}) \to (Y, \tau)$ is $m$-semi-$\mathcal{I}$-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exists $U \in mS\mathcal{I}O(X, x)$ and an open set $V$ of $Y$ containing $y$ such that $f(U) \cap V = \emptyset$.*

*Proof.* The proof is an immediate consequence of Definition 4.7. □

**Theorem 4.9.** *If $f : (X, m, \mathcal{I}) \to (Y, \tau)$ is an $m$-semi-$\mathcal{I}$-continuous function and $(Y, \tau)$ is $T_2$, then $G(f)$ is $m$-semi-$\mathcal{I}$-closed.*

*Proof.* Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$. Since $Y$ is $T_2$, there exists an open set $V$ of $Y$ such that $f(x) \in V$ and $y \notin V$. Since $f$ is $m$-semi-$\mathcal{I}$-continuous, there exists $U \in mS\mathcal{I}O(X, x)$ such that $f(U) \subset V$. Therefore, $f(U) \cap V = \emptyset$. Therefore, by Lemma 4.8, $G(f)$ is $m$-semi-$\mathcal{I}$-closed. □

**Definition 4.10.** *An ideal minimal space $(X, m, \mathcal{I})$ is said to be an $m$-semi-$\mathcal{I}$-$T_2$ space if for each pair of distinct points $x, y \in X$, there exist $U, V \in mS\mathcal{I}O(X)$ containing $x$ and $y$, respectively, such that $U \cap V = \emptyset$.*

**Theorem 4.11.** *If $f : (X, m, \mathcal{I}) \to (Y, \tau)$ is an $m$-semi-$\mathcal{I}$-continuous injection and $Y$ is a $T_2$ space, then $(X, m, \mathcal{I})$ is a $m$-semi-$\mathcal{I}$-$T_2$ space.*

*Proof.* The proof follows from the definitions. □

**Theorem 4.12.** *If $f : (X, m, \mathcal{I}) \to (Y, \tau)$ is an injective $m$-semi-$\mathcal{I}$-continuous function with an $m$-semi-$\mathcal{I}$-closed graph, then $X$ is an $m$-semi-$\mathcal{I}$-$T_2$ space.*

*Proof.* Let $x_1$ and $x_2$ be any distinct points of $X$. Then $f(x_1) \neq f(x_2)$, so $(x_1, f(x_2)) \in (X \times Y)\setminus G(f)$. Since the graph $G(f)$ is $m$-semi-$\mathcal{I}$-closed, there exist an $m$-semi-$\mathcal{I}$-open set $U$ containing $x_1$ and $V \in$

$\tau$ containing $f(x_2)$ such that $f(U) \cap V = \emptyset$. Since $f$ is $m$-semi-$\mathcal{I}$-continuous, $f^{-1}(V)$ is an $m$-semi-$\mathcal{I}$-open set containing $x_2$ such that $U \cap f^{-1}(V) = \emptyset$. Hence $X$ is $m$-semi-$\mathcal{I}$-$T_2$. □

**Definition 4.13.** *An ideal minimal space $(X, m, \mathcal{I})$ is said to be $m$-semi-$\mathcal{I}$-connected if $X$ cannot be expressed as the union of two nonempty disjoint $m$-semi-$\mathcal{I}$-open sets.*

**Theorem 4.14.** *A $m$-semi-$\mathcal{I}$-continuous image of an $m$-semi-$\mathcal{I}$-connected space is connected.*

*Proof.* The proof is clear. □

**Lemma 4.15.** [7] *For any function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, $f(\mathcal{I})$ is an ideal on $Y$.*

**Definition 4.16.** *A subset $K$ of an ideal minimal space $(X, m, \mathcal{I})$ is said to be $m$-semi-$\mathcal{I}$-compact relative to $X$, if for every cover $\{U_\lambda : \lambda \in \Lambda\}$ of $K$ by $m$-semi-$\mathcal{I}$-open sets of $X$, there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $K \setminus \bigcup\{U_\lambda : \lambda \in \Lambda_0\} \in \mathcal{I}$. The space $(X, m, \mathcal{I})$ is said to be $m$-semi-$\mathcal{I}$-compact if $X$ is $m$-semi-$\mathcal{I}$-compact subset of $X$.*

**Definition 4.17.** *A subset $K$ of an ideal minimal space $(X, m, \mathcal{I})$ is said to be countable $m$-semi-$\mathcal{I}$-compact relative to $X$, if for every cover $\{U_\lambda : \lambda \in \Lambda\}$ of $K$ by countable $m$-semi-$\mathcal{I}$-open sets of $X$, there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $K \setminus \bigcup\{U_\lambda : \lambda \in \Lambda_0\} \in \mathcal{I}$. The space $(X, m, \mathcal{I})$ is said to be countable $m$-semi-$\mathcal{I}$-compact if $X$ is countable $m$-semi-$\mathcal{I}$-compact subset of $X$.*

**Definition 4.18.** *A subset $K$ of an ideal minimal space $(X, m, \mathcal{I})$ is said to be $m$-semi-$\mathcal{I}$-Lindelöf relative to $X$, if for every cover $\{U_\lambda : \lambda \in \Lambda\}$ of $K$ by $m$-semi-$\mathcal{I}$-open sets of $X$, there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $K \setminus \bigcup\{U_\lambda : \lambda \in \Lambda_0\} \in \mathcal{I}$. The space $(X, m, \mathcal{I})$ is said to be $m$-semi-$\mathcal{I}$-Lindelöf if $X$ is $m$-semi-$\mathcal{I}$-Lindelöf subset of $X$.*

**Theorem 4.19.** *If $f : (X, m, \mathcal{I}) \to (Y, \sigma)$ is an $m$-semi-$\mathcal{I}$-continuous surjection and $(X, m, \mathcal{I})$ is $m$-semi-$\mathcal{I}$-compact, then $(Y, \sigma, f(\mathcal{I}))$ is $f(\mathcal{I})$-compact.*

*Proof.* Let $\{V_\lambda : \lambda \in \Lambda\}$ be an open cover of $Y$. Then $\{f^{-1}(V_\lambda) : \lambda \in \Lambda\}$ is an $m$-semi-$\mathcal{I}$-open cover of $X$ and hence, there exist a finite subset $\Lambda_0$ of $\lambda$ such that $X \setminus \bigcup\{f^{-1}(V_\lambda) : \lambda \in \Lambda_0\} \in \mathcal{I}$. Since $f$ is surjective, $Y \setminus \bigcup\{V_\lambda : \lambda \in \Lambda_0\} = f(X \setminus \bigcup\{f^{-1}(V_\lambda) : \lambda \in \Lambda_0\}) \in \mathcal{I}$. Therefore, $(Y, \sigma, f(\mathcal{I}))$ is $f(\mathcal{I})$-compact. □

**Theorem 4.20.** *If $f : (X, m, \mathcal{I}) \to (Y, \sigma)$ is an $m$-semi-$\mathcal{I}$-continuous surjection and $(X, m, \mathcal{I})$ is $m$-semi-$\mathcal{I}$-Lindelöf, then $(Y, \sigma, f(\mathcal{I}))$ is $f(\mathcal{I})$-Lindelöf.*

*Proof.* The proof is similar to previous theorem. □

**Theorem 4.21.** *If $f : (X, m, \mathcal{I}) \to (Y, \sigma)$ is an $m$-semi-$\mathcal{I}$-continuous surjection and $(X, m, \mathcal{I})$ is countable $m$-semi-$\mathcal{I}$-compact, then $(Y, \sigma, f(\mathcal{I}))$ is countable $f(\mathcal{I})$-compact.*

*Proof.* The proof is similar to previous theorem. □

**Definition 4.22.** *A function $f : (X, \tau) \to (Y, m, \mathcal{I})$ is said to be:*

(i) *$m$-semi-$\mathcal{I}$-open if $f(U)$ is an $m$-semi-$\mathcal{I}$-open set of $Y$ for every open set $U$ of $X$.*
(ii) *$m$-semi-$\mathcal{I}$-closed if $f(U)$ is an $m$-semi-$\mathcal{I}$-closed set of $Y$ for every closed set $U$ of $X$.*

**Theorem 4.23.** *For a function $f : (X, \tau) \to (Y, m, \mathcal{I})$, the following statements are equivalent:*

(i) *$f$ is $m$-semi-$\mathcal{I}$-open;*
(ii) *$f(\mathrm{Int}(U)) \subset ms\mathcal{I}\,\mathrm{Int}(f(U))$ for each subset $U$ of $X$;*
(iii) *$\mathrm{Int}(f^{-1}(V)) \subset f^{-1}(ms\mathcal{I}\,\mathrm{Int}(V))$ for each subset $V$ of $Y$.*

*Proof.* *(i)* ⇒ *(ii)*: Let $U$ be any subset of $X$. Then $\mathrm{Int}(U)$ is an open set of $X$. Then $f(\mathrm{Int}(U))$ is an $m$-semi-$\mathcal{I}$-open set of $Y$. Since $f(\mathrm{Int}(U)) \subset f(U)$, $f(\mathrm{Int}(U)) = ms\mathcal{I}\,\mathrm{Int}(f(\mathrm{Int}(U))) \subset ms\mathcal{I}\,\mathrm{Int}(f(U))$.
*(ii)* ⇒ *(iii)*: Let $V$ be any subset of $Y$. Then $f^{-1}(V)$ is a subset of $X$. Hence $f(\mathrm{Int}(f^{-1}(V))) \subset ms\mathcal{I}\,\mathrm{Int}(f(f^{-1}(V))) \subset ms\mathcal{I}\,\mathrm{Int}(V))$. Then $\mathrm{Int}(f^{-1}(V)) \subset f^{-1}(f(\mathrm{Int}(f^{-1}(V)))) \subset f^{-1}(ms\mathcal{I}\,\mathrm{Int}(V))$.
*(iii)* ⇒ *(i)*: Let $U$ be any open set of $X$. Then $\mathrm{Int}(U) = U$ and $f(U)$ is a subset of $Y$. Now, $V = m\,\mathrm{Int}(V) \subset \mathrm{Int}(f^{-1}(f(V))) \subset f^{-1}(ms\mathcal{I}\,\mathrm{Int}(f(V)))$. Then $f(V) \subset f(f^{-1}(ms\mathcal{I}\,\mathrm{Int}(f(V)))) \subset ms\mathcal{I}\,\mathrm{Int}(f(V))$ and $ms\mathcal{I}\,\mathrm{Int}(f(V)) \subset f(V)$. Hence $f(V)$ is a $m$-semi-$\mathcal{I}$-open set of $Y$; hence $f$ is $m$-semi-$\mathcal{I}$-open. □

**Theorem 4.24.** *Let $f : (X, \tau) \to (Y, m, \mathcal{I})$ be a function. Then $f$ is an $m$-semi-$\mathcal{I}$-closed function if and only if for each subset $V$ of $X$, $ms\mathcal{I}\,\mathrm{Cl}(f(V)) \subset f(\mathrm{Cl}(V))$.*

*Proof.* Let $f$ be an $m$-semi-$\mathcal{I}$-closed function and $V$ any subset of $X$. Then $f(V) \subset f(\mathrm{Cl}(V))$ and $f(\mathrm{Cl}(V))$ is an $m$-semi-$\mathcal{I}$-closed set of $Y$. We have $ms\mathcal{I}\,\mathrm{Cl}(f(V)) \subset ms\mathcal{I}\,\mathrm{Cl}(f(\mathrm{Cl}(V))) = f(\mathrm{Cl}(V))$. Conversely, let $V$ be an open set of $X$. Then $f(V) \subset ms\mathcal{I}\,\mathrm{Cl}(f(V)) \subset f(\mathrm{Cl}(V)) =$

$f(V)$; hence $f(V)$ is an $m$-semi-$\mathcal{J}$-closed subset of $Y$. Therefore, $f$ is an $m$-semi-$\mathcal{I}$-closed function. □

**Theorem 4.25.** *Let $f : (X, \tau) \to (Y, m, \mathcal{I})$ be a function. Then $f$ is an $m$-semi-$\mathcal{I}$-closed function if and only if for each subset $V$ of $Y$, $f^{-1}(ms\mathcal{I}\operatorname{Cl}(V)) \subset \operatorname{Cl}(f^{-1}(V))$.*

*Proof.* Let $V$ be any subset of $Y$. Then by Theorem 4.24, $ms\mathcal{I}\operatorname{Cl}(V) \subset f(\operatorname{Cl}(f^{-1}(V)))$. Since $f$ is bijection, $f^{-1}(ms\mathcal{I}\operatorname{Cl}(V)) = f^{-1}(ms\mathcal{I}\operatorname{Cl}(f(f^{-1}(V)))) \subset f^{-1}(f(\operatorname{Cl}(f^{-1}(V)))) = \operatorname{Cl}(f^{-1}(V))$. Conversely, let $U$ be any subset of $X$. Since $f$ is bijection, $ms\mathcal{I}\operatorname{Cl}(f(U)) = f(f^{-1}(ms\mathcal{I}\operatorname{Cl}(f(U))) \subset f(\operatorname{Cl}(f^{-1}(f(U)))) = f(\operatorname{Cl}(U))$. Therefore, by Theorem 4.24, $f$ is an $m$-semi-$\mathcal{I}$-closed function. □

**Theorem 4.26.** *Let $f : (X, \tau) \to (Y, m, \mathcal{I})$ be an $m$-semi-$\mathcal{I}$-open function. If $V$ is a subset of $Y$ and $U$ is a closed subset of $X$ containing $f^{-1}(V)$, then there exists an $m$-semi-$\mathcal{I}$-closed set $F$ of $Y$ containing $V$ such that $f^{-1}(F) \subset U$.*

*Proof.* Let $V$ be any subset of $Y$ and $U$ a closed subset of $X$ containing $f^{-1}(V)$, and let $F = Y\backslash(f(X\backslash V))$. Then $f(X\backslash V) \subset f(f^{-1}(X\backslash V)) \subset X\backslash V$ and $X\backslash U$ is an open set of $X$. Since $f$ is $m$-semi-$\mathcal{I}$-open, $f(X\backslash U)$ is an $m$-semi-$\mathcal{I}$-open set of $Y$. Hence $F$ is an $m$-semi-$\mathcal{I}$-closed set of $Y$ and $f^{-1}(F) = f^{-1}(Y\backslash(f(X\backslash U)) \subset U$. □

**Theorem 4.27.** *Let $f : (X, \tau) \to (Y, m, \mathcal{I})$ be an $m$-semi-$\mathcal{I}$-closed function. If $V$ is a subset of $Y$ and $U$ is an open subset of $X$ containing $f^{-1}(V)$, then there exists $m$-semi-$\mathcal{I}$-open set $F$ of $Y$ containing $V$ such that $f^{-1}(F) \subset U$.*

*Proof.* The proof is similar to the Theorem 4.26. □

**Theorem 4.28.** *Let $f : (X, \tau) \to (Y, m, \mathcal{I})$ be an $m$-semi-$\mathcal{I}$-open function. Then for each subset $V$ of $Y$, $f^{-1}(m\operatorname{Int}(m\operatorname{Cl}^*(V)) \subset \operatorname{Cl}(f^{-1}(V)$.*

*Proof.* Let $V$ be any subset of $Y$. Then $\operatorname{Cl}(f^{-1}(V)$ is a closed set of $X$ containing $f^{-1}(V)$. Since $f$ is $m$-semi-$\mathcal{I}$-open, by Theorem 4.26, there is an $m$-semi-$\mathcal{I}$-open set $F$ of $Y$ containing $V$ such that $f^{-1}(m\operatorname{Int}(m\operatorname{Cl}^*(V)) \subset m\operatorname{Int}(m\operatorname{Cl}^*(F)) \subset f^{-1}(F) \subset \operatorname{Cl}(f^{-1}(V))$. □

**Theorem 4.29.** *Let $f : (X, \tau) \to (Y, m, \mathcal{I})$ be a bijection such that for each subset $V$ of $Y$, $f^{-1}(m\operatorname{Int}(m\operatorname{Cl}^*(V)) \subset \operatorname{Cl}(f^{-1}(V))$. Then $f$ is an $m$-semi-$\mathcal{I}$-open function.*

*Proof.* Let $U$ be an open subset of $X$. Then $f(X\backslash U)$ is a subset of $Y$ and $f^{-1}(m\operatorname{Int}(m\operatorname{Cl}^*(f(X\backslash U)))) \subset \operatorname{Cl}(f^{-1}(f(X\backslash U))) = \operatorname{Cl}(X\backslash U) = X\backslash U$, and so $m\operatorname{Int}(m\operatorname{Cl}^*(f(X\backslash U))) \subset f(X\backslash U)$. Hence $f(X\backslash U)$ is an $m$-semi-$\mathcal{I}$-closed set of $Y$ and $f(U) = X\backslash(f(X\backslash U))$ is a $m$-semi-$\mathcal{I}$-open set of $Y$. Therefore, $f$ is an $m$-semi-$\mathcal{I}$-open function. □

## 5. $(m_1, m_2)$-SEMI-$\mathcal{I}$-IRRESOLUTE FUNCTIONS

**Definition 5.1.** *A function $f : (X, m_1, \mathcal{I}) \to (Y, m_2, \mathcal{J})$ is said to be $(m_1, m_2)$-semi-$\mathcal{I}$-irresolute if the inverse image of every $m_2$-semi-$\mathcal{J}$-open set of $Y$ is $m_1$-semi-$\mathcal{I}$-open in $X$.*

**Theorem 5.2.** *Let $f : (X, m_1, \mathcal{I}) \to (Y, m_2, \mathcal{J})$ be a function, then*

(1) *$f$ is $(m_1, m_2)$-semi-$\mathcal{I}$-irresolute;*
(2) *the inverse image of each $m_2$-semi-$\mathcal{J}$-closed subset of $Y$ is $m_1$-semi-$\mathcal{I}$-closed in $X$;*
(3) *for each $x \in X$ and each $V \in S\mathcal{J}O(Y, m_2)$ containing $f(x)$, there exists $U \in S\mathcal{I}O(X, m_1)$ containing $x$ such that $f(U) \subset V$.*

*Proof.* The proof is obvious from that fact that the arbitrary union of $m$-semi-$\mathcal{I}$-open subsets is $m$-semi-$\mathcal{I}$-open. □

**Theorem 5.3.** *Let $f : (X, m_1, \mathcal{I}) \to (Y, m_2, \mathcal{J})$ be a function, then*

(1) *$f$ is $(m_1, m_2)$-semi-$\mathcal{I}$-irresolute;*
(2) *$m_1 s\mathcal{I}\operatorname{Cl}(f^{-1}(V)) \subset f^{-1}(m_2 s\mathcal{J}\operatorname{Cl}(V))$ for each subset $V$ of $Y$;*
(3) *$f(m_1 s\mathcal{I}\operatorname{Cl}(U) \subset m_2 s\mathcal{J}\operatorname{Cl}(f(U))$ for each subset $U$ of $X$.*

*Proof.* (1) ⇒ (2): Let $V$ be any subset of $Y$. Then $V \subset m_2 s\mathcal{J}\operatorname{Cl}(V)$ and $f^{-1}(V) \subset f^{-1}(m_2 s\mathcal{I}\operatorname{Cl}(V))$. Since $f$ is $(m_1, m_2)$-semi-$\mathcal{I}$-irresolute, $f^{-1}(m_2 s\mathcal{J}\operatorname{Cl}(V))$ is an $m_1$-semi-$\mathcal{I}$-closed subset of $X$. Hence $m_1 s\mathcal{I}\operatorname{Cl}(f^{-1}(V)) \subset m_1 s\mathcal{I}\operatorname{Cl}(f^{-1}(m_2 s\mathcal{J}\operatorname{Cl}(V))) = f^{-1}(m_2 s\mathcal{J}\operatorname{Cl}(V))$.
(2) ⇒ (3): Let $U$ be any subset of $X$. Then $f(U) \subset m_2 s\mathcal{J}\operatorname{Cl}(f(U))$ and $m_1 s\mathcal{I}\operatorname{Cl}(U) \subset m_1 s\mathcal{I}\operatorname{Cl}(f^{-1}(f(U))) \subset f^{-1}(m_2 s\mathcal{J}\operatorname{Cl}(f(U)))$. This implies that $f(m_1 s\mathcal{I}\operatorname{Cl}(U)) \subset f(f^{-1}(m_2 s\mathcal{J}\operatorname{Cl}(f(U)))) \subset m_2 s\mathcal{J}\operatorname{Cl}(f(U))$.
(3) ⇒ (1): Let $V$ be an $m_2$-semi-$\mathcal{J}$-closed subset of $Y$. Then $f(m_1 s\mathcal{I}\operatorname{Cl}(f^{-1}(V)) \subset m_1 s\mathcal{I}\operatorname{Cl}(f^{-1}(f(V))) \subset m_1 s\mathcal{I}\operatorname{Cl}(V) = V$. This implies that $m_1 s\mathcal{I}\operatorname{Cl}(f^{-1}(V)) \subset f^{-1}(f(m_1 s\mathcal{I}\operatorname{Cl}(f^{-1}(V)))) \subset f^{-1}(V)$. Therefore, $f^{-1}(V)$ is an $m$-semi-$\mathcal{I}$-closed subset of $X$ and consequently $f$ is an $(m_1, m_2)$-semi-$\mathcal{I}$-irresolute function. □

**Theorem 5.4.** *A function* $f : (X, m_1, \mathcal{I}) \to (Y, m_2, \mathcal{J})$ *is an* $(m_1, m_2)$*-semi-*$\mathcal{I}$*-irresolute if and only if* $f^{-1}(m_2 s\mathcal{J}\,\mathrm{Int}(V)) \subset m_1 s\mathcal{I}\,\mathrm{Int}(f^{-1}(V))$ *for each subset* $V$ *of* $Y$*.*

*Proof.* Let $V$ be any subset of $Y$. Then $ms\mathcal{J}\,\mathrm{Int}(V) \subset V$. Since $f$ is $(m_1, m_2)$-semi-$\mathcal{I}$-irresolute, $f^{-1}(m_2 s\mathcal{J}\,\mathrm{Int}(V))$ is an $m$-semi-$\mathcal{I}$-open subset of $X$. Hence $f^{-1}(m_2 s\mathcal{J}\,\mathrm{Int}(V)) = m_1 s\mathcal{I}\,\mathrm{Int}(f^{-1}(m_2 s\mathcal{J}\,\mathrm{Int}(V))) \subset m_1 \mathcal{I} s\,\mathrm{Int}(f^{-1}(V))$. Conversely, let $V$ be an $m_2$-semi-$\mathcal{J}$-open subset of $Y$. Then $f^{-1}(V) = f^{-1}(m_2 s\mathcal{J}\,\mathrm{Int}(V)) \subset m_1 s\mathcal{I}\,\mathrm{Int}(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is an $m_1$-semi-$\mathcal{I}$-open subset of $X$ and consequently $f$ is an $(m_1, m_2)$-semi-$\mathcal{I}$-irresolute function. □

## References


[1] S. Jafari and N. Rajesh, *Some subsets of ideal minimal spaces* (under preparation).

[2] S. Jafari and N. Rajesh, *Preopen sets in ideal minimal spaces*, Questions Answers Gen. Topology 29 (2011), no. 1, 81-90.

[3] S. Jafari, R. Saranya and N. Rajesh, *Properties of $\alpha$-open sets in ideal minimal spaces* (submitted).

[4] E. Hatir and T. Noiri, On semi-$\mathcal{I}$-open sets and semi-$\mathcal{I}$-continuous functions, *Acta Math. Hungar.*, **107**(4)(2005), 345-353.

[5] K. Kuratowski, Topology, *Academic semiss, New York*, (1966).

[6] W. K. Min, $m$-Semiopen Sets And $M$-Semicontinuous Functions On Spaces With Minimal Structures,*Honam Math. J.*, **(31) (2) (2009)**, 239-245.

[7] R. L. Newcomb, Topologies which are compact modulo an ideal, Ph.D. Thesis, University of California, USA (1967).

[8] V. Popa and T. Noiri, On the definition of some generalized forms of continuity under minimal conditions, *Mem. Fac. Sci. Kochi. Univ. Ser. Math.*, **(22) (2001)**, 9-19.

[9] O. B. Ozbakir and E. D. Yildirim, On some closed sets in ideal minimal spaces, *Acta Math. Hungar.*, **125(3) (2009)**, 227-235.

[10] R. Vaidyanathaswamy, The localisation theory in set topology, *Proc. Indian Acad. Sci.*, **20**(1945), 51-61.

College of Vestsjaelland South,
Herrestraede 11, 4200 Slagelse
Denmark.
e-mail:jafaripersia@gmail.com

Department of Mathematics, Rajah Serfoji Govt. College
Thanjavur-613005 Tamilnadu,
India.
e-mail: nrajesh_topology@yahoo.co.in

1/82, South Street, Pillaiyarnatham
Thnangudi, Thanjavur 613 601 Tamilnadu,
India.
e-mail: sara.rengaraj@gmail.com