

Basque language and the Graphical law

Anindya Kumar Biswas*

Department of Physics;

North-Eastern Hill University,

Mawkynroh-Umshing, Shillong-793022.

(Dated: January 22, 2020)

Abstract

We study a Basque to English dictionary. We draw in the natural logarithm scale, number of words starting with a letter vs rank of the letter, both normalised. We find that the graphs are closer to the curves of reduced magnetisation vs reduced temperature for various approximations of Ising model.

* anindya@nehu.ac.in

I. INTRODUCTION

”Gekhane dekhibe chhai,
uraiya dekho tai,
paile paite paro,
parosho ratan.”

—Bengali proverb.

When the author was a student in class eight, he got a class-mate by the nickname ”buru”. Since then, he was wondering where from the word originated. The road ended when he stumbled on the ”Etymological Dictionary of Basque”, [1], written by the late R. L. Trask. ”Buru” is a two thousand years old Basque word, meaning headman. The dictionary is a treasure trove. Surname of a colleague of the author is Askari. ”Askari” in Basque means lunch or, tea. We often overhear the word ”zaldibaji”. In Basque ”zaldi” means horse. Interestingly, in Basque, ”Bilarri” means ear, ”biki” means twin or, pair of twins, ”bihar” means tomorrow, ”biru” means thread or, fibre, ”diru” means money, ”giro” is atmosphere, ”jagon” is guard, ”kabra” stands for a spiny red fish, ”kopar” means ”basin”, ”handi” is big, ”lur” is earth, ”patar” is hard liquor. Surprisingly, ”judu” refers to Jew.

Basque people originated from Aquitanian tribes populating the coast of South of France and North of Spain. Luis Michelena chronicled Basque language, [1]. Developing on Luis Michelena, the late R. L. Trask constructed the dictionary of Basque.

Basque is an subject-Object-Verb language. It does not have grammatical gender or, noun classification. According to L. Michelena, there are nine dialects of Basque: Bizkaian, Gipuzkoan, High Navarese, Aezkoan, Salazarese, Ronclaise, Lapurdian, Low Navarese and Zuberoan.

The language has five vowels: i, e, a, o, u. The consonants are split in fortis: (p), t, k, tz, ts, N, L, P and lenis: b, d, g, z, s, n, l, r. There are five diphthongs: ai, ei, oi, au, eu [1].

The Basque alphabet is composed of twenty two letters. The number of words starting with the letters ala , [1], are as follows:

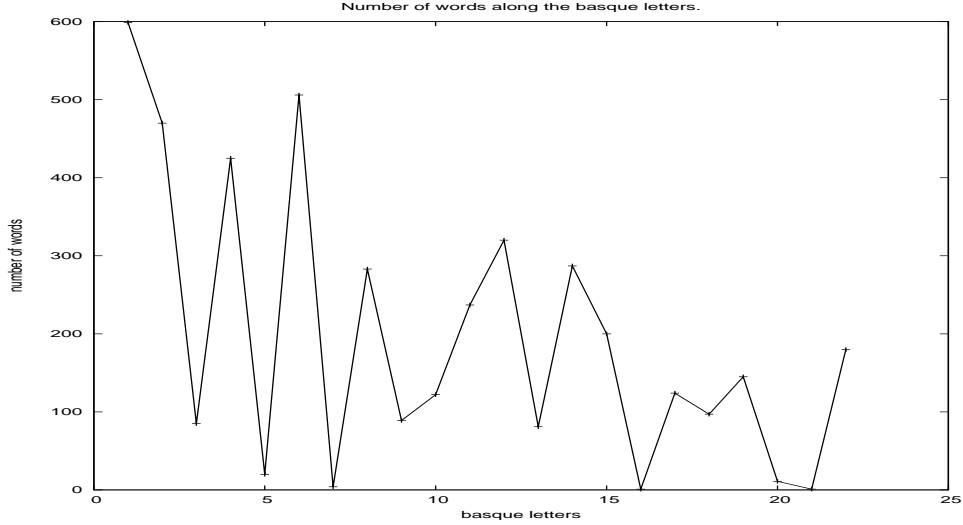


FIG. 1. Vertical axis is number of words and horizontal axis is respective letters. Letters are represented by the number in the alphabet or, dictionary sequence. ([1]).

letter	A	B	D	E	F	G	H	I	J	K	L	M	N	O	P	R	S	T	U	X	Y	Z
number	599	470	85	425	20	506	4	283	89	122	237	320	81	287	200	1	124	97	145	11	1	180

The largest number of words, 599 to be specific, start with the letter "A". The next block of words numbering 506 with the letter "G" as the initial. We draw number of words vs. sequence number of letters in the fig.1.

In recent works, [2], the present author took a trip to probe into the word (and verb,adverb,adjective) contents along the letters in a language. The letters were arranged in ascending order of their ranks from the rank one. The letter with the highest number of words starting with, was taken as of rank one. For a natural language, a dictionary from it to English, was a natural choice for that type of study. The author has found that behind each language which was subjected to preliminary study, there is a curve of magnetisation. From that the author has conjectured that behind any written natural language there are curves of magnetisation, for words, verbs, adverbs and adjectives respectively. The graphical law was found also to exist in the contemporary chinese usages, [2]. Moreover, the curve drawn for the Arabian language was found to be very close to Onsager exact solution i.e. reduced magnetisation vs. reduced temperature, of two dimensional Ising model,[3]. We have not

studied Hebrew or, Persian. We hope, Hebrew or, Persian like Arabian will also be close to Onsager solution. It happened so that Ising, was a Jew, [4], who investigated the model by his name but due to his advisor Lenz. An alloy has been found, [5], to follow Onsager solution In 1974.

Moreover, we looked into, [6], dictionaries of five disciplines of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of graphical law behind bengali language,[7].

Name of a railway station in the Bardhaman district of West Bengal is Guskara. Guskara is place predominantly inhabited by farming people. If we remove the "G" from the word, we come across Uskara or, Euskara, which happens to be another name of Basque language. "Mihidana" is a famous sweet from Bardhaman. Mihi is as well a Basque word meaning tongue. A block in Sundarban goes by the name Gosaba. If we remove G from the front, we obtain "osaba" which is a Basque word meaning stepfather. Tamal is a bengali name, which in Basque means misfortune. Makal is a derogatory epithet in bengali language, in Basque it stands for weak. Obi is a bengali name, in Basque it means cavity. "durduri" in both bengali language and Basque means restless or, nervous. "esan" is a bengali word representing south-west, in Basque it means south. "faltsu" in Basque means false, in bengali language "faltu" means also false. "garai" is a surname in Bardhaman. In Basque, though it is rare in use means high. Road to Orissa used to be through Bardhaman. Oriya language has lot of similarities with the bengali language hundred and fifty years back. Two common words in Oriya are "toki" meaning girl and "baina" meaning brother. In Basque "toki" means place, "baina" means but. A prominent network of seafaring businessmen mostly hailing from Orissa coast, in the medieval era was "Sadhavas".

Hence, we ponder on whether Basque language has something to do with bengali language. Both are Subject-Object-Verb languages. Speakers of both languages are divided into two countries, united by sea. Bengalis and basques are passionate about football, food,art. Both have the history of seafaring. Both languages bear the influence of non-native speakers, willy-nilly.

As we go along this article, we will find that the same kind of magnetisation curve(s) almost underlie both the languages. We describe how a graphical law is hidden within in the Basque language, in this article. We organise the paper as follows. We explain our method of study in the section III after giving an introduction to the the standard curves of magnetisation

of Ising model in the section II. In the ensuing section, section IV, we narrate our graphical results. We describe how natural logarithm of number of words arranged in descending order, normalised by different normalisers when plotted against the respective rank are fit with lines of magnetisations. We also plot for each normaliser, normalised natural logarithm of Basque and bengali languages words. Then we conclude about the existence of the graphical law in the section V. In that section we try to find out relationship of Basque language with other languages on the basis of underlying magnetisation curves. We end up through acknowledgement section VI and bibliography.

II. MAGNETISATION

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like paramagnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of

one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\sum_i \sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1 + L)$ and $N_- = \frac{N}{2}(1 - L)$. Magnetisation or, net magnetic moment, M is $\mu\sum_i \sigma_i$ or, $\mu(N_+ - N_-)$ or, μNL , $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[3], the lattice of spins is $-J\sum_{n,n}\sigma_i\sigma_j - \mu B\sum_i \sigma_i$, where n.n refers to nearest neighbour pairs.

The difference $\Delta\epsilon$ of energy if we flip an up spin to down spin is, [8], $2J\gamma\bar{\sigma} + 2\mu B$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\Delta\epsilon}{k_B T})$. In the Bragg-Williams approximation,[9], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$\ln \frac{1+L}{1-L} = 2 \frac{\gamma J L + \mu B}{k_B T} = 2 \frac{L + \frac{\mu B}{\gamma J}}{\frac{T}{\gamma J/k_B}} = 2 \frac{L + c}{\frac{T}{T_c}} \quad (1)$$

where, $c = \frac{\mu B}{\gamma J}$, $T_c = \gamma J/k_B$. $\frac{T}{T_c}$ is referred to as reduced temperature.

Plot of L vs $\frac{T}{T_c}$ or, reduced magnetisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [8]. W. L. Bragg was a professor of Hans Bethe. Rudlof Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudlof Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

In the approximation scheme which is improvement over the Bragg-Williams, due to Bethe-Peierls, [10], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}} \quad (2)$$

$\ln \frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe datas generated from the equation(1) and the equation(2) and curves of magnetisation plotted on the basis of those datas.

A. Reduced magnetisation vs reduced temperature datas

BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). Bethe(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.2. Empty spaces in the table mean corresponding point pairs were not used for plotting a line.

BW	BW(c=0.01)	Bethe(4)	reduced magnetisation
0	0	0	1
0.435	0.439	0.563	0.978
0.439	0.443	0.568	0.977
0.491	0.495	0.624	0.961
0.501	0.507	0.630	0.957
0.514	0.519	0.648	0.952
0.559	0.566	0.654	0.931
0.566	0.573	0.7	0.927
0.584	0.590	0.7	0.917
0.601	0.607	0.722	0.907
0.607	0.613	0.729	0.903
0.653	0.661	0.770	0.869
0.659	0.668	0.773	0.865
0.669	0.676	0.784	0.856
0.679	0.688	0.792	0.847
0.701	0.710	0.807	0.828
0.723	0.731	0.828	0.805
0.732	0.743	0.832	0.796
0.756	0.766	0.845	0.772
0.779	0.788	0.864	0.740
0.838	0.853	0.911	0.651
0.850	0.861	0.911	0.628
0.870	0.885	0.923	0.592
0.883	0.895	0.928	0.564
0.899	0.918		0.527
0.904	0.926	0.941	0.513
0.946	0.968	0.965	0.400
0.967	0.998	0.965	0.300
0.987		1	0.200
0.997		1	0.100
1	1	1	0

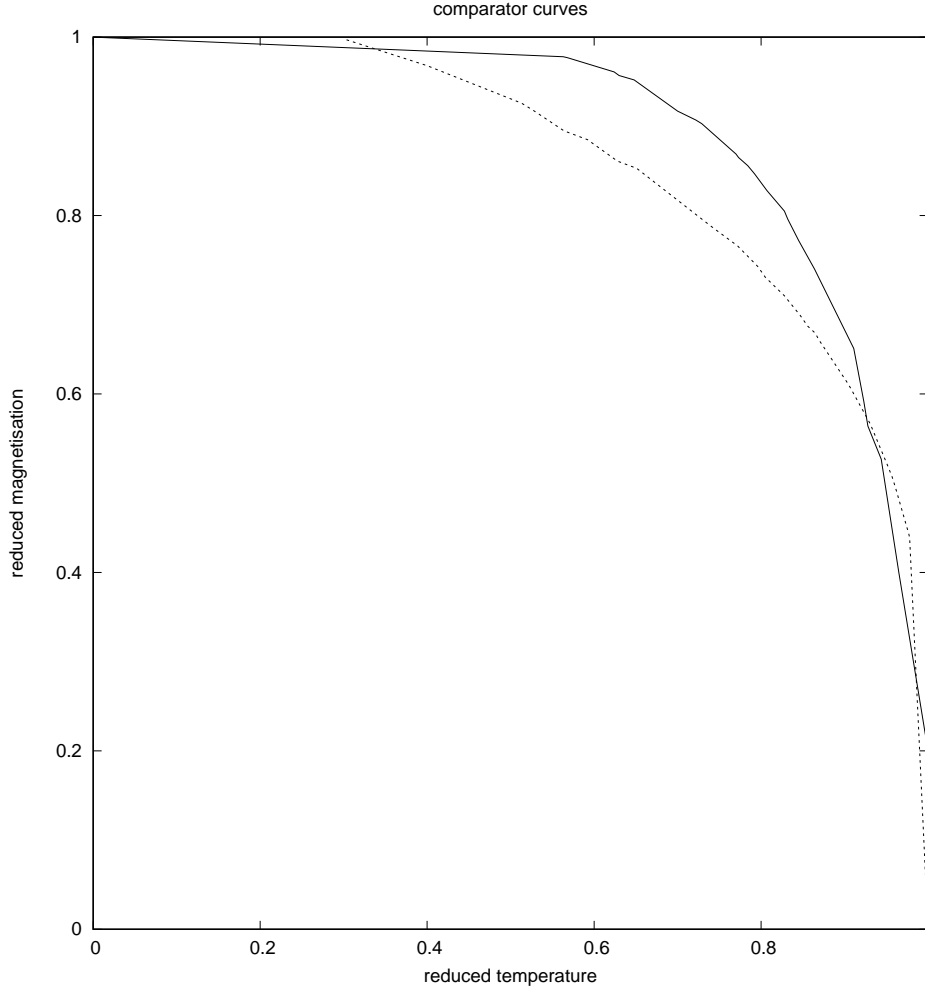


FIG. 2. Reduced magnetisation vs reduced reduced temperature curves for Bragg-Williams approximation, in presence of little magnetic field and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours (outer one).

III. METHOD OF STUDY

We take the Basque-English dictionary,[1]. Then we count the words, one by one from the beginning to the end, starting with different letters. When words belonging to different dialects with the same meaning are given, we have counted one for the set. We assort the letters according to the number of words, in the descending order, denoted by f and the respective rank, denoted by k . k is a positive integer starting from one. Moreover, we attach a limiting rank, k_{lim} , and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty one and the limiting number of words is one. As a result both

$\frac{\ln f}{\ln f_{max}}$ and $\frac{\ln k}{\ln k_{lim}}$ varies from zero to one. Then we plot $\frac{\ln f}{\ln f_{max}}$ against $\frac{\ln k}{\ln k_{lim}}$.

We then ignore the letters with the highest and then next highest number of words and redo the plot, normalising the $\ln f$ s with next-to-maximum $\ln f_{nextmax}$, and starting from $k = 2$; next-to-next-to-maximum $\ln f_{nextnextmax}$, and starting from $k = 3$; next-to-next-to-next-to-maximum $\ln f_{nextnextnextmax}$, and starting from $k = 4$.

IV. RESULTS

A. $\ln f$ normalised by $\ln f_{max}$:

k	lnk	lnk/ $\ln k_{lim}$	f	lnf	lnf/ $\ln f_{max}$
1	0	0	599	6.40	1
2	0.69	0.227	506	6.23	0.973
3	1.10	0.362	470	6.15	0.961
4	1.39	0.457	425	6.05	0.945
5	1.61	0.530	320	5.77	0.902
6	1.79	0.589	287	5.66	0.884
7	1.95	0.641	283	5.65	0.883
8	2.08	0.684	237	5.47	0.855
9	2.20	0.724	200	5.30	0.828
10	2.30	0.757	180	5.19	0.811
11	2.40	0.789	145	4.98	0.778
12	2.48	0.816	124	4.82	0.753
13	2.56	0.842	122	4.80	0.750
14	2.64	0.868	97	4.57	0.714
15	2.71	0.891	89	4.49	0.702
16	2.77	0.911	85	4.44	0.694
17	2.83	0.931	81	4.39	0.686
18	2.89	0.951	20	3.00	0.469
19	2.94	0.967	11	2.40	0.375
20	3.00	0.987	4	1.39	0.217
21	3.04	1	1	0	0

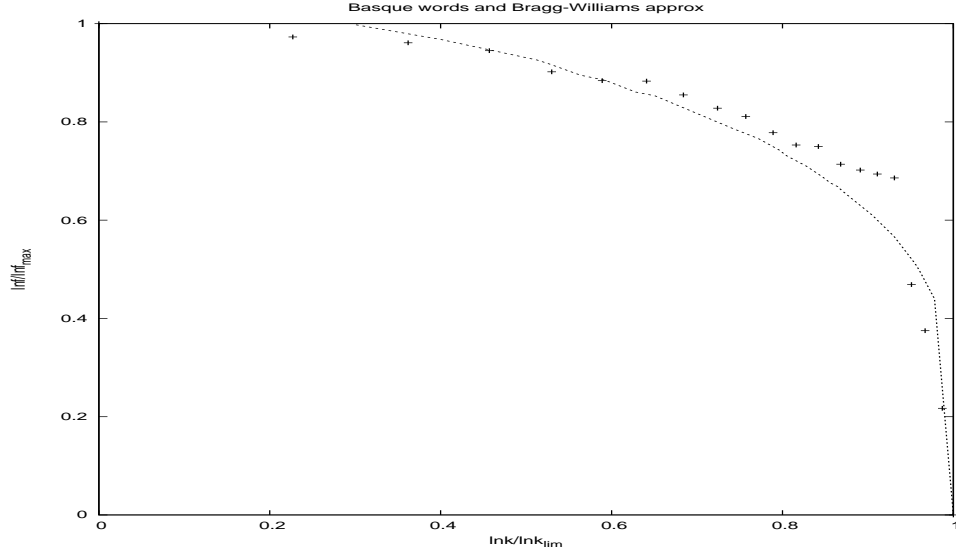


FIG. 3. Vertical axis is $\frac{\ln f}{\ln f_{max}}$ and horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the Basque language components as represented by the titles. For words fit curve is Bragg-Williams in presence of little magnetic field.

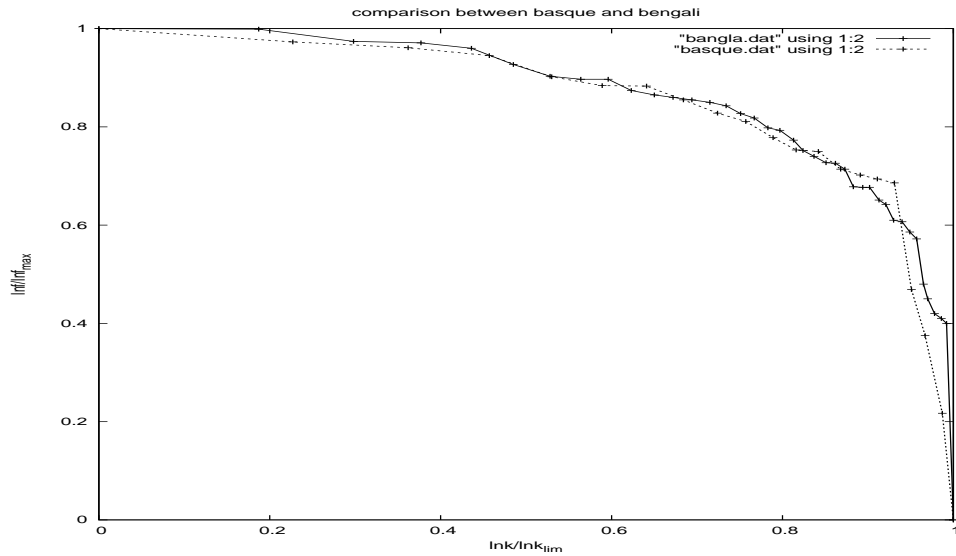


FIG. 4. Vertical axis is $\frac{\ln f}{\ln f_{max}}$ and horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points on dashed line represent the Basque language words and + points on solid line stand for bengali words.

B. $\ln f$ normalised by $\ln f_{next-max}$:

k	lnk	lnk/ $\ln k_{lim}$	f	lnf	lnf/ $\ln f_{next-max}$
2	0.69	0.227	506	6.23	1
3	1.10	0.362	470	6.15	0.987
4	1.39	0.457	425	6.05	0.971
5	1.61	0.530	320	5.77	0.926
6	1.79	0.589	287	5.66	0.909
7	1.95	0.641	283	5.65	0.907
8	2.08	0.684	237	5.47	0.878
9	2.20	0.724	200	5.30	0.851
10	2.30	0.757	180	5.19	0.833
11	2.40	0.789	145	4.98	0.799
12	2.48	0.816	124	4.82	0.774
13	2.56	0.842	122	4.80	0.770
14	2.64	0.868	97	4.57	0.734
15	2.71	0.891	89	4.49	0.721
16	2.77	0.911	85	4.44	0.713
17	2.83	0.931	81	4.39	0.705
18	2.89	0.951	20	3.00	0.482
19	2.94	0.967	11	2.40	0.385
20	3.00	.987	4	1.39	0.223
21	3.04	1	1	0	0

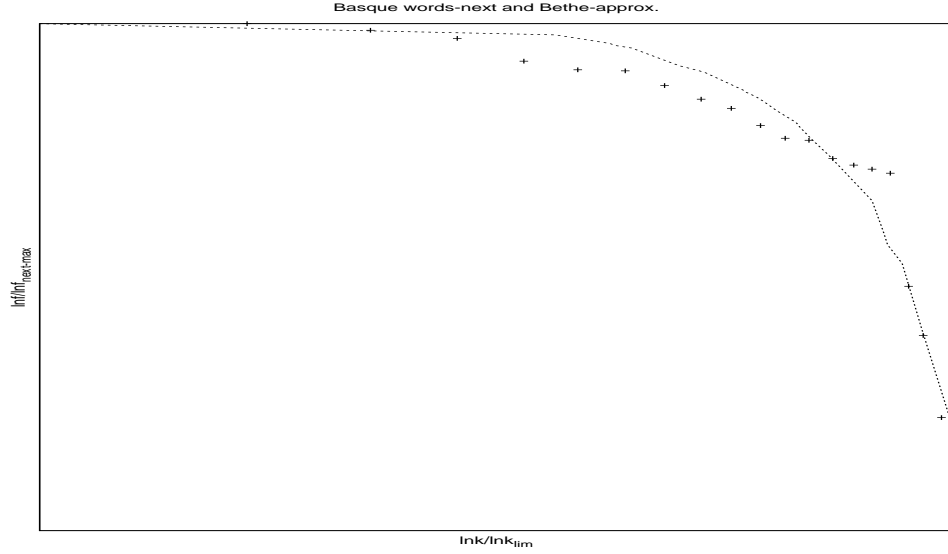


FIG. 5. The + points represent the components as represented by the titles of the Basque language. Vertical axis is $\frac{\ln f}{\ln f_{next-max}}$ and horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. Comparator curve is Bethe-Peierls line for $\gamma = 4$ or, four nearest neighbours.

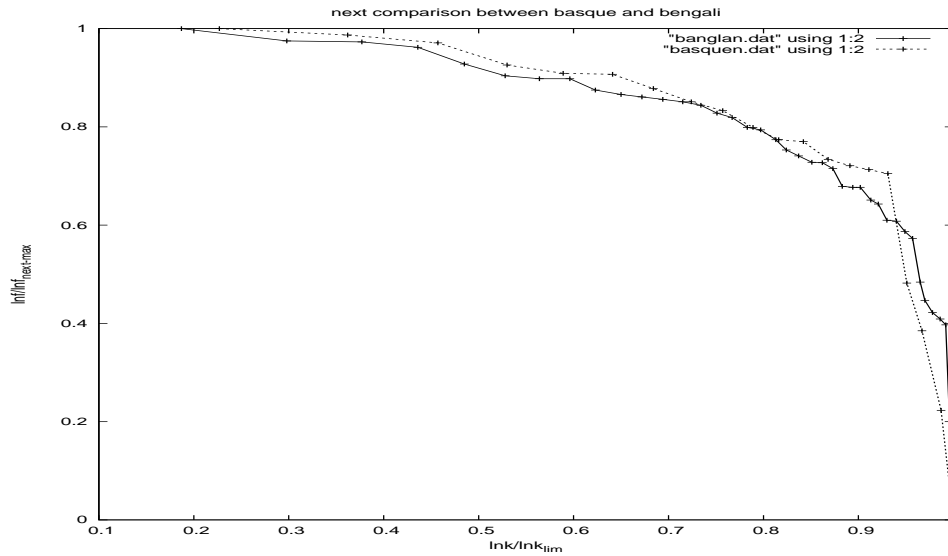


FIG. 6. Vertical axis is $\frac{\ln f}{\ln f_{next-max}}$ and horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points on dashed line represent the Basque language words and + points on solid line stand for bengali words.

C. $\ln f$ normalised by $\ln f_{next-next-max}$:

k	lnk	lnk/ $\ln k_{lim}$	f	lnf	lnf/ $\ln f_{next-next-max}$
3	1.10	0.362	470	6.15	1
4	1.39	0.457	425	6.05	0.984
5	1.61	0.530	320	5.77	0.938
6	1.79	0.589	287	5.66	0.920
7	1.95	0.641	283	5.65	0.919
8	2.08	0.684	237	5.47	0.889
9	2.20	0.724	200	5.30	0.862
10	2.30	0.757	180	5.19	0.844
11	2.40	0.789	145	4.98	0.810
12	2.48	0.816	124	4.82	0.784
13	2.56	0.842	122	4.80	0.780
14	2.64	0.868	97	4.57	0.743
15	2.71	0.891	89	4.49	0.730
16	2.77	0.911	85	4.44	0.722
17	2.83	0.931	81	4.39	0.714
18	2.89	0.951	20	3.00	0.488
19	2.94	0.967	11	2.40	0.390
20	3.00	0.987	4	1.39	0.226
21	3.04	1	1	0	0

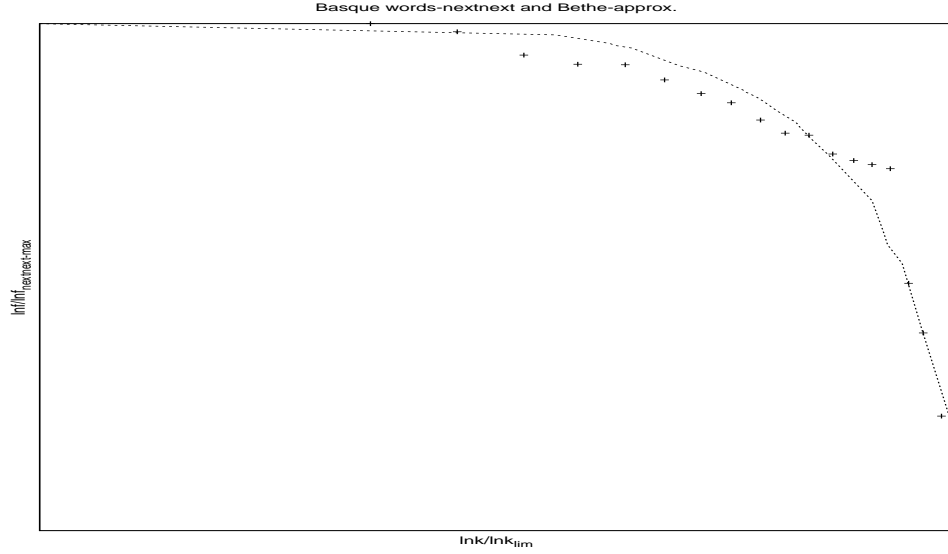


FIG. 7. The + points represent the components as represented by the titles of the Basque language. Vertical axis is $\frac{\ln f}{\ln f_{\text{nextnext-max}}}$ and horizontal axis is $\frac{\ln k}{\ln k_{\text{lim}}}$. Fit curve for words is Bethe-Peierls line for $\gamma = 4$ or, four nearest neighbours.

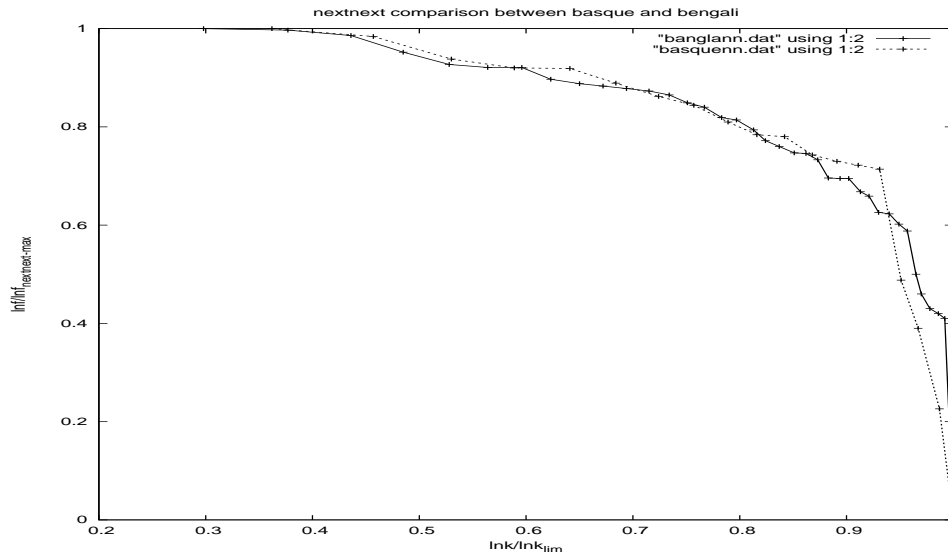


FIG. 8. Vertical axis is $\frac{\ln f}{\ln f_{\text{nextnext-max}}}$ and horizontal axis is $\frac{\ln k}{\ln k_{\text{lim}}}$. The + points on dashed line represent the Basque language words and + points on solid line stand for bengali words.

D. $\ln f$ normalised by $\ln f_{next-next-next-max}$:

k	$\ln k$	$\ln k / \ln k_{lim}$	f	$\ln f$	$\ln f / \ln f_{nextnextnext-max}$
4	1.39	0.457	425	6.05	1
5	1.61	0.530	320	5.77	0.954
6	1.79	0.589	287	5.66	0.936
7	1.95	0.641	283	5.65	0.934
8	2.08	0.684	237	5.47	0.904
9	2.20	0.724	200	5.30	0.876
10	2.30	0.757	180	5.19	0.858
11	2.40	0.789	145	4.98	0.823
12	2.48	0.816	124	4.82	0.797
13	2.56	0.842	122	4.80	0.793
14	2.64	0.868	97	4.57	0.755
15	2.71	0.891	89	4.49	0.742
16	2.77	0.911	85	4.44	0.734
17	2.83	0.931	81	4.39	0.726
18	2.89	0.951	20	3.00	0.496
19	2.94	0.967	11	2.40	0.397
20	3.00	0.987	4	1.39	0.230
21	3.04	1	1	0	0

V. CONCLUSION

From the figures (fig.3, fig.5, fig.7, fig.9), we observe that dispersion is the least for the first figure i.e. $\frac{\ln f}{\ln f_{max}}$ vs $\frac{\ln k}{\ln k_{lim}}$ with the fit curve being Bragg-Williams line with little magnetic field.

The associated correspondance with the Ising model is,

$$\frac{\ln f}{\ln f_{max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$\ln k \longleftrightarrow T.$$

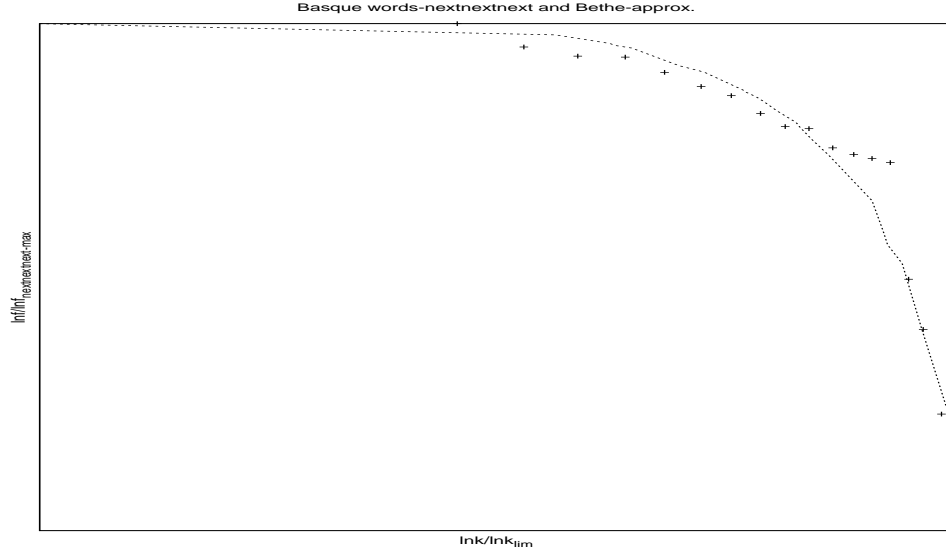


FIG. 9. The + points represent the components as represented by the titles of the Basque language. Vertical axis is $\frac{\ln f}{\ln f_{\text{nextnextnext-max}}}$ and horizontal axis is $\frac{\ln k}{\ln k_{\text{lim}}}$. Visual match is Bethe-Peierls line for $\gamma = 4$ or, four nearest neighbours.

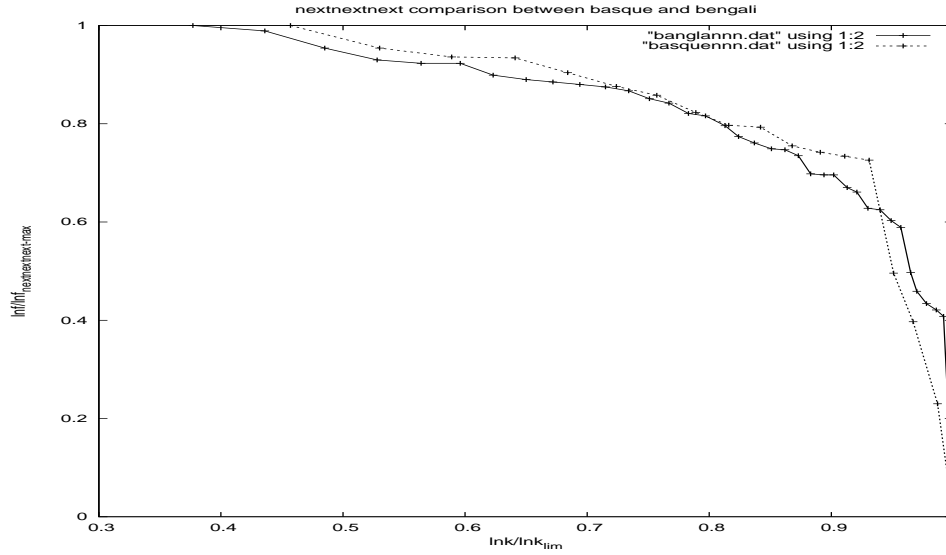


FIG. 10. Vertical axis is $\frac{\ln f}{\ln f_{\text{nextnextnext-max}}}$ and horizontal axis is $\frac{\ln k}{\ln k_{\text{lim}}}$. The + points on dashed line represent the Basque language words and + points on solid line stand for bengali words.

k corresponds to temperature in an exponential scale, [11]. Hence, Basque language follows graphical law. Basque language can be identified with Bragg-Williams line with little magnetic field.

Bragg-Williams approximation assumes existence of the same order down the scale i.e. equality of the long and short range order. Matching with Bragg-Williams approximation line with the Basque language tends to imply that the language probably is homogeneous down the societal scale.

As temperature decreases, i.e. lnk decreases, f increases. The letters which are recording higher entries compared to those which have lesser entries are at lower temperature. As basque language expands, the letters like which get enriched more and more, fall at lower and lower temperatures. This is a manifestation of cooling effect as was first observed in [12] in another way.

A. Discussion

Languages which are matched by Bragg-Williams curve in presence of little magnetic field, in the $\frac{lnf}{lnf_{max}}$ level, are Bengali, Urdu, South African English, Chinese, Lakher-Mara.

Tibetan and Urdu, in the $\frac{lnf}{lnf_{next-max}}$ level, are matched by the Bethe-Peierls curve for four neighbours.

Italiano, French, Turkmen, South African English, Hindi, Urdu, Bengali, in the $\frac{lnf}{lnf_{nextnext-max}}$ level, are fit by the Bethe-Peierls curve for four neighbours.

The Bethe-Peierls curve for four neighbours underlie, in the $\frac{lnf}{lnf_{nextnextnext-max}}$ level, Italiano, French, Hindi, Bengali, Khasi, Lotha languages. Incidentally, Khasi does not have the letter c like that in Basque.

VI. ACKNOWLEDGEMENT

The author came to know of Basque language by reading BBC news article. We have used gnuplot for drawing the figures.

-
- [1] R. L. Trask, Etymological Dictionary of Basque, edited for the web publication by Max W. Wheeler, University of Sussex 2008, © the estate of the late R. L. Trask.
- [2] Anindya Kumar Biswas, "Graphical Law beneath each written natural language", arXiv:1307.6235v3[physics.gen-ph]. A preliminary study of words of dictionaries of twenty six

languages, more accurate study of words of dictionary of Chinese usage and all parts of speech of dictionary of Lakher(Mara) language and of verbs, adverbs and adjectives of dictionaries of six languages are included.

- [3] E. Ising, Z.Physik 31,253(1925).
- [4] S. M. Bhattacharjee and A. Khare, "Fifty Years of the Exact solution of the Two-dimensional Ising Model by Onsager", arXiv:cond-mat/9511003v2.
- [5] K. I. Keda and K. Hirakawa, Solid Stat. Comm. 14 (1974) 529.
- [6] Anindya Kumar Biswas, "A discipline of knowledge and the graphical law ", IJARPS Volume 1(4), p 21, 2014; viXra: 1908:0090[Linguistics].
- [7] Anindya Kumar Biswas, "Bengali language and Graphical law ", viXra: 1908:0090[Linguistics].
- [8] R. K. Pathria, Statistical Mechanics, p400-403, 1993 reprint, Pergamon Press,© 1972 R. K. Pathria.
- [9] W. L. Bragg and E. J. Williams, Proc. Roy. Soc. A, vol.145, p. 699(1934);
- [10] Kerson Huang, Statistical Mechanics, second edition, John Wiley and Sons(Asia) Pte Ltd..
- [11] Sonntag, Borgnakke and Van Wylen, Fundamentals of Thermodynamics, p206-207, fifth edition, John Wiley and Sons Inc.
- [12] Alexander M. Petersen, Joel N. Tenenbaum, Shlomo Havlin, H. Eugene Stanley, and Matjaž Perc, "Languages cool as they expand: Allometric scaling and the decreasing need for new words", Sci. Rep.2(2012) 943, arXiv:1212.2616v1. and references therein.