Massification of the spacetime

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Abstract: Until the early twentieth century, the three-dimensional space and one-dimensional time were considered separate beings. In 1909, German mathematician H. Minkowski connected together space and time into single idea, creating a new the four-dimensional spacetime.

In this paper we proposed the extension of this idea by the connection together the Minkowski four-dimensional spacetime and *the mass density* into **the single idea**, creating a new entity: the four-dimensional spacetime with the mass density.

1. Introduction

Until the early twentieth century, the three-dimensional space and one-dimensional time were considered separate beings. In 1909, German mathematician H. Minkowski connected together space and time into single idea, creating the four-dimensional spacetime [1]. The idea of the spacetime enjoyed success in the *Special Relativity* (SR) and the *General Relativity* (GR), correctly describing a range of physical phenomena.

In this paper we propose the extension of this idea by the connection together the Minkowski four-dimensional spacetime and *the mass density* into **the single idea**, creating a new entity: *the four-dimensional spacetime with the mass density*. We expect that this idea will allow us to solve the problem of the sources of inertia.

We assume additionally that:

- in absence of the outer gravitational field the mass density becomes *the bare mass density*,
- under influence outer gravitational field the bare mass density becomes *the effective mass density*.

The idea of the spacetime with the bare (or the effective) mass density we call *the massification of spacetime*.

2. The bare medium

The spacetime with the bare mass density ρ^{bare} is defined mathematically as follows

$$\rho_{\mu\nu}^{bare} \overset{def}{=} \rho^{bare} \cdot \eta_{\mu\nu} = \mathrm{diag}(\rho^{bare}, -\rho^{bare}, -\rho^{bare}, -\rho^{bare}) \tag{1}$$

where: $\rho_{\mu\nu}^{bare}$ *is the bare mass density tensor,* $\eta_{\mu\nu}$ is *the Minkowski tensor*, μ, ν = 0, 1, 2, 3.

In the absence of any outer fields, the spacetime with the constant bare mass density ($\rho^{bare} = const$) is the homogeneous, isotropic and time independent.

The bare mass density is no longer the scalar and becomes the tensor. Note that ρ^{bare} never reaches zero ($\rho^{bare} \neq 0$), although it may be very close. In the contrast to the vacuum, the spacetime with the bare mass density (let us call them *the bare medium*) is a never empty. So determined the bare medium is equivalent to *the field of inertia*, which is a special case of the gravitational field. The inertial field is responsible for the inertia of the body and is described by the tensor $\rho_{\mu\nu}^{bare}$.

In the bare medium the metric is defined as the Minkowski metric

$$ds^2(\eta_{\mu\nu}) = \eta_{\mu\nu} \cdot dx^\mu dx^\nu . \tag{2}$$

This metric is independent of the inertial frame of reference and well suited to describe all the physical phenomena occurring in SR. The bare medium has influences on the physical processes. The presence of bodies and their motion has no influence on the bare medium. Particles behave in accordance with *the principle of inertia*, i.e. they are at rest or moving in a straight line at constant speed with respect to the bare medium (not with respect to the spacetime itself).

During the uniform motion, clocks and roots indicate the different time and length, than at the rest. This difference results from the change of the bare mass density during the uniform motion with respect to the bare medium.

3. The effective medium

Under influence outer gravitational field the bare mass density becomes the effective mass density. The spacetime with the effective mass density let us call them *the effective medium*.

The metric of the effective medium is mathematically defined as

$$ds^2(\rho_{\mu\nu}(x)) \overset{def}{=} \frac{\rho_{\mu\nu}(x)}{\rho^{bare}} \cdot dx^\mu dx^\nu \tag{3}$$

where: $\rho_{\mu\nu}(x)$ is the symmetric and position dependent the effective mass density tensor.

The effective mass tensor $\rho_{\mu\nu}(x)$ can be positive or negative, so the metric (3) also may take positive or negative values. This tensor describes all the physical properties of the effective medium and also the mathematical relationship between the effective medium and the bare medium under the influence the gravitational field. In a some sense, $\rho_{\mu\nu}(x)$ is similar to the metric tensor $g_{\mu\nu}(x)$ with metric

$$ds^2(g_{\mu\nu}(x)) = g_{\mu\nu}(x) \cdot dx^\mu dx^\nu \tag{4}$$

in GR.

Rest and motion of all bodies takes place with respect to the effective medium, which becomes a new reference frame. Additionally the presence of bodies and their motion has influence on the effective medium. In non-inertial systems the field of inertia passes into the gravitational field.

In the absence of any outer gravitational fields the effective mass becomes the bare mass and the metric (3) becomes the metric (2).

The concept of effective mass tensor to describe gravitational phenomena, instead of usual metric tensor, for the first time, was discussed in [2].

Let's analyze the motion of the body in an effective medium and let's compare the equation of motion with the classical Newtonian equation.

4. The equation of motion in the effective medium

The Lagrangian function for the body in the effective medium has form

$$L = \frac{1}{2}\rho_{\mu\nu}(x)\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}$$

The equation of motion

$$\frac{dp_{\gamma}(x)}{d\tau} - \frac{1}{2}\frac{\partial\rho_{\mu\nu}(x)}{\partial x^{\gamma}}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau} = 0 \tag{5}$$

where: $p_{\gamma}(x) = \rho_{\mu\gamma}(x)\cdot\frac{dx^{\mu}}{d\tau}$ is *the effective density of the four-momentum*, τ is *the proper time*.

The equation of motion (5) *explicitly* refers to the effective medium, which is described by the effective mass density tensor $\rho_{\mu\nu}(x)$. So the motion of the body takes place **only** in relation to the effective medium, not to the relation of the spacetime itself or all bodies in the Universe (*Mach's Principle* [3]). The new quality of the understanding has been reached.

When $\rho_{\gamma\nu}(x)$ does not depends *explicitly* on τ, the equation (5) takes the form

$$\rho_{\mu\gamma}(x)\frac{d^2x^{\mu}}{d\tau^2} + \Gamma_{\gamma\mu\nu}\left(\rho_{\mu\nu}(x)\right)\cdot\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau} = 0 \tag{6}$$

where:

$$\Gamma_{\gamma\mu\nu}\left(\rho_{\mu\nu}(x)\right) \stackrel{def}{=} \frac{1}{2}\left(\frac{\partial\rho_{\gamma\mu}(x)}{\partial x^{\nu}} + \frac{\partial\rho_{\gamma\nu}(x)}{\partial x^{\mu}} - \frac{\partial\rho_{\mu\nu}(x)}{\partial x^{\gamma}}\right) \tag{7}$$

assuming that the condition

$$\frac{\partial\rho_{\mu\gamma}(x)}{\partial x^{\nu}}\cdot\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau} = \frac{1}{2}\left(\frac{\partial\rho_{\mu\gamma}(x)}{\partial x^{\nu}} + \frac{\partial\rho_{\nu\gamma}(x)}{\partial x^{\mu}}\right)\cdot\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}$$

is satisfied.

The equation (7) is very similar to *the Christoffel symbols of the first kind*, where instead metric tensor $g_{\mu\nu}(x)$, we have the effective mass density tensor $\rho_{\mu\nu}(x)$. This is an interesting result because the Christoffel symbols describing *the metric connection*, while the equation

$$\frac{d}{d\tau}\left(g_{\gamma\nu}(x)\frac{dx^{\nu}}{d\tau}\right)-\frac{1}{2}\frac{\partial g_{\mu\nu}(x)}{\partial x^{\gamma}}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}=0 \tag{8}$$

is *the geodesic equation* in GR.

If the surrounding bodies consist only with the bare masses, i.e. $\rho_{\mu\nu}(x)=\rho_{\mu\nu}^{bare}$, $\Gamma_{\gamma\mu\nu}\left(\rho_{\mu\nu}^{bare}\right)=0$ then the equation of motion (6) takes the form:

$$\rho_{\mu\nu}^{bare}\frac{d^2x^{\nu}}{d\tau^2}=0\,. \tag{9}$$

The body with the bare mass density $\rho_{\mu\nu}^{bare}$ is in the rest or moves in a straight line with the constant speed in the respect to the bare medium. The principle of inertia has gained a new meaning and the equation (9) determines the new inertial reference frame – *the bare medium reference frame*. This reference frame is determined by the bare medium property only.

During any change in state of motion of the body appears the inertia, which source is the spacetime with the effective mass density. The inertia becomes an intrinsic property of the massification of spacetime. The magnitude of the inertia of any body is also determined by the massification of spacetime. This is the opposite of that, than previously thought. Until now it was thought that inertia is determined by the masses of the Universe and by their distribution [4]. In our model an isolated object in the Universe always has of the inertial properties, because the spacetime with the bare mass density ρ^{bare} formed an inseparable whole. The spacetime ceased to be empty.

We analyze now the massification of spacetime for the slow motion speed and the slow rotating body in a static and a weak gravitational field.

5. A weak gravitational field approximation

In a weak gravitational field we can decompose $\rho_{\mu\nu}(x)$ to the following simple form $\rho_{\mu\nu}(x)=\rho_{\mu\nu}^{bare}+\rho_{\mu\nu}^{*}(x)$, where: $\rho_{\mu\nu}^{*}(x)<<1$ is a very small perturbation in the effective mass density tensor.

5.1. The equation of motion

At slow motion speeds, in a static, a weak and the spherically symmetric field the equation of motion (6) reduces to

$$\left(\rho^{bare}+\rho_{rr}^{*}(r)\right)\cdot\frac{d^2r}{dt^2}\cong-\frac{c^2}{2}\nabla\rho_{00}^{*}(r) \tag{10}$$

where: c is the speed of light.

The equation (10) is a little different than well-known Newton's equation of the motion for the gravity. It what currently we consider to be *the inertial mass density*, really is the sum of the bare mass density ρ^{bare} and $\rho^*_{rr}(x)$ - rr-component of the very small perturbation in the effective mass density. Note that gravitational mass density does not appear *explicitly* in the equation (10).

Does it mean that, in our model, during massification of the spacetime, *the Equivalence Principle*, underlying the GR, lost *raison d'être*?

According to *the Correspondence Principle* we expect that there is a relationship between the component $\rho^*_{00}(r)$ and the gravitational potential $V(r)$ in the following form [5]

$$\frac{\rho^*_{00}(r)}{\rho^{bare}} \cong \frac{2V(r)}{c^2} \tag{11}$$

where: $V(r)=\frac{GM}{r}$, G is the gravitational constant, M is the mass and r is the distance. After substituting (11) to (10), (on the assumption that $\rho^*_{rr}(r)=0$), we obtain

$$\frac{d^2 r}{dt^2} = -\frac{\partial V(r)}{\partial r} \tag{12}$$

the well-known Newtonian equation of motion in the gravitational potential $V(r)$.

5.2. The rotating body

Let' s consider the slowly rotating body in a static and weak gravitational field. The equation of motion have the form

$$\rho^{bare} \cdot \frac{d^2 x^i}{dt^2} = -\frac{c^2}{2}\frac{\partial \rho^*_{00}(x)}{\partial x^i} + c \cdot \left(\frac{\partial \rho^*_{0k}(x)}{\partial x^j} - \frac{\partial \rho^*_{0j}(x)}{\partial x^k} \right) \frac{dx^i}{dt} \tag{13}$$

The sources of inertia are the following expressions: $\frac{\partial \rho^*_{00}(x)}{\partial x^i}$ and $\frac{\partial \rho^*_{0k}(x)}{\partial x^j} - \frac{\partial \rho^*_{0j}(x)}{\partial x^k}$. These components we can determine from the matrix [6]

$$\rho^*_{\mu\nu}(x) = \rho^{bare} \cdot \begin{pmatrix} -\frac{\omega^2 (x^2+y^2)}{c^2} & \frac{\omega y}{c} & -\frac{\omega x}{c} & 0 \\ \frac{\omega y}{c} & 0 & 0 & 0 \\ -\frac{\omega x}{c} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

where $r^2 = x^2 + y^2$.

Finally, we get well-known equation

$$\frac{d^2 r}{dt^2} = -\omega^2 r - 2\omega \frac{dr}{dt} \tag{14}$$

which includes an real *the centrifugal* and *Coriolis acceleration*.

In the Newtonian approximation the equations of motion (12) and (14) does not depend, *explicitly*, on the mass density.

6. The rotating bucket with water problem

There are two entirely different measurements of the Earth's angular velocity, *astronomical* (from upper culmination to upper culmination of the star) and *dynamic* (by means of Foucault's pendulum experiment), which give *the same results* (in the limit of the experimental errors). In both cases the motion is described with respect to the effective medium and the coincidence of these measurements is the result of massification of the spacetime.

In the famous experiment with *the rotating bucket with water* [7, 8] the motion of water takes place also to relative of the effective medium, therefore the surface of water takes the shape of the parabolic. So, massification of the spacetime explains both these physical phenomena.

7. What is the bare and the effective mass density?

Each theoretical model must correspond with the real of the physical world. We suppose that the bare mass density ρ^{bare} corresponds with *the critical density* $\rho_c = \frac{3H^2}{8\pi G}$ [6], where: H is the Hubble constant. This term is use in the modern cosmology to determine the spatial geometry of the Universe, where ρ_c is the critical density for which the spatial geometry is flat (or Euclidean).

The flat spatial geometry in GR corresponds with the bare medium in our model. The curved spacetime corresponds with the effective medium.

8. Summary

In this paper was applied an alternative attempt to describe gravitational phenomena, using a new idea of the massification of spacetime, which provides the following benefits:

1. During any change in state of motion of the body appears the inertia, which source is the spacetime with the effective mass density.
2. The inertia becomes an intrinsic property of massification of the spacetime.
3. The magnitude of the inertia of any body is determined by massification of the spacetime.
4. Inertial forces, appearing in the non-inertial frames of reference, there are no longer fictitious forces.
5. In the gravitational field clocks and roots indicate the different time and length, than in the absence of the field. This difference results from the change of the effective mass density in a gravitational field [9].

9. Conclusion

The idea of massification of the spacetime, although a very attractive, requires experimental confirmation. Predicted the annual relative change of the fluctuation in the effective mass as resulting from ellipticity of the orbit for the Earth, is equal to 6.6×10^{-10} [9].

GR does not predicts a such fluctuations.

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