Self-Variation Theory

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***Abstract*.** In this article we present the principles and main consequences of Self-Variation Theory. The Theory is based on three principles, the principle of Self-Variation, principle of conservation of energy-momentum and the definition of the rest mass of a fundamental particle. The main conclusions of the Theory are the following; it predicts an internal structure of the particles, predict and justifies the particle interactions, predicts and justifies the cosmological data and proves that Self-Variation is related to quantum phenomena.

Keywords: Electromagnetism, Gravity, Particle Interactions, Origin of Universe, Evolution of Universe, Structure of Matter, Quantum Phenomena.

Contents:

1. Introduction

A principle absent from the current theories of physics that is introduced by the Self-Variation Theory is the principle of self-variation. It is a simple, though obviously a somewhat unexpected principle. With the term "self-variation principle" we mean an exactly determined increase of the rest mass of fundamental particles and generally of the "self-variating charge" Q.

Taking into account the energy-momentum conservation principle, the self-variation of the rest mass of the material particle can only take place with the simultaneous emission of energy-momentum into the surrounding spacetime of the particle. The combination of the principle of self-variation with the conservation of energy-momentum has as a consequence the presence of energy-momentum in the surrounding spacetime of the material particle. The introduction of the principle of the rest mass self-variation was made with the expectation that this energy-momentum in spacetime could provide a cause for the interaction of material particles. In retrospect, this expectation was confirmed. Taking into account the existence of the gravitational interaction, we introduce the self-variation of the rest mass. Similarly, due to the existence of the electromagnetic interaction we introduce the principle of self-variation of the electric charge. Generally each interaction results from a "self-variating charge" Q.

The Self-Variation Theory is based on three principles, which we present in section 2.

1. The principle of Self-Variation.
2. The principle of conservation of energy-momentum.
3. A definition of the rest mass of fundamental particles.

The main conclusions of the Theory are the following.
1. It predicts structure the fundamental particles.
2. It predicts the emergence of particle interactions in a unified way.
3. It predicts and justifies the cosmological data.
4. It shows that quantum phenomena are implicit in the Self-Variation Theory.

2. Axiomatic foundation of Self-Variation Theory

In a N -dimensional Riemannian spacetime (see, [1]) the Self-variation Theory is based on three principles, the principle of Self-Variation, the principle of conservation of energy-momentum and the definition of the rest mass of fundamental particles.

A. The self-variation principle

With the term "self-variation principle" we mean an exactly determined increase of the rest mass of material particles. Moreover the self-variation principle generally applies to all kind of charges of the fundamental particles. Direct consequence of the principle of self-variation is that energy, momentum, angular momentum and charge (if the particle is charged) are distributed in the surrounding spacetime. For example, to compensate for the increase, in absolute value, of the negative electric charge of the electron, the particle emits a corresponding positive electric charge into the surrounding spacetime. As a consequence of this emission the total electric charge is conserved. Similarly, the increase of the rest mass of the material particle involves the "emission" of negative energy as well as momentum in the spacetime surrounding the material particle (spacetime energy-momentum) P .

We generally denote the rest mass or charge of particle with Q . The principle of self-variation quantitatively describes the interaction of the 'self-variation charges'. Let

$Q = Q\left(x^0, x^1, x^2, ..., x^{N-1}\right) = Q\left(x^k\right)$, $N \in \mathbb{N}^* = \{1, 2, 3, ...\}$

be the self-variating charge and let P be the energy-momentum the particle emit in spacetime as a consequence of the self-variation of the charge Q. The self-variation principle asserts that valid

$$\frac{\partial Q}{\partial x^k} = \frac{b}{\hbar} P_k Q \tag{2.1}$$

$k = 0, 1, 2, ..., N-1$

in every system of reference

$\mathrm{O}\left(x^0, x^1, x^2, ..., x^{N-1}\right) = \mathrm{O}\left(x^k\right)$

where $\hbar = \dfrac{h}{2\pi}$ is the reduced Planck constant and $b \in \mathbb{C}$, $b \neq 0$ is a constant. $P_0 = \dfrac{iE}{c}$ denotes the energy and $x^0 = ict$ the time measured by an observer, where c is vacuum velocity of light and i is the imaginary unit, $i^2 = -1$. If $Q = m_0$ equation (1) becomes

$$\frac{\partial m_0}{\partial x^k} = \frac{b}{\hbar} P_k m_0 . \tag{2.2}$$

The principle of Self-variation quantitatively describes the interaction of material particles with the spacetime energy-momentum. For the formulation of the equations the following symbolism is used,

W is the energy of the particle,

$\mathbf{J}$ is the momentum of the particle,

m_0 is the rest mass of the particle,

E is the energy of the spacetime energy-momentum related to the particle,

$\mathbf{P}$ is the momentum of the spacetime energy-momentum related to the particle,

E_0 is the rest energy of the spacetime energy-momentum related to the particle. We define the N-vectors,

$$X = \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ . \\ . \\ . \\ x^{N-1} \end{bmatrix} = \begin{bmatrix} ict \\ x^1 \\ x^2 \\ . \\ . \\ . \\ x^{N-1} \end{bmatrix}, \tag{2.3}$$

$$J = \begin{bmatrix} J_0 \\ J_1 \\ J_2 \\ . \\ . \\ . \\ J_{N-1} \end{bmatrix} = \begin{bmatrix} \frac{iW}{c} \\ J_1 \\ J_2 \\ . \\ . \\ . \\ J_{N-1} \end{bmatrix}, \tag{2.4}$$

$$P = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ . \\ . \\ . \\ P_{N-1} \end{bmatrix} = \begin{bmatrix} \frac{iE}{c} \\ P_1 \\ P_2 \\ . \\ . \\ . \\ P_{N-1} \end{bmatrix}, \tag{2.5}$$

$$C=\begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ . \\ . \\ . \\ C_{N-1} \end{bmatrix}, \tag{2.6}$$

where, $C = J + P$.

In equation (2.1), the momentum J of the particle is due to the charge Q. The momentum P arises as a consequence of the Self-Variation of the charge Q. The physical quantities Q, J, P are determined at the same point X of spacetime.

B. The principle of conservation of energy-momentum

The material particle and the spacetime energy-momentum with which the material particle interacts comprise a dynamic system, which we call “generalized particle”. We consider the covariant (see, [2, 3]) momentum of the particle J, the momentum of spacetime P and the total momentum C of generalized particle,

$$C_n = J_n + P_n,\ n = 0,1,2,...,N-1. \tag{2.7}$$

As a consequence of Equation (2.1), the N-vectors C, J and P are covariant. Equation (2.7) expresses the energy-momentum conservation of the generalized particle in a N-dimensional spacetime.

C. The rest mass of the material particles

As invariant physical quantities, the rest masses corresponding to the N-vectors J, P, C are given by the following equations,

$$J_n J^n = m_0^2 c^2, \tag{2.8}$$

$$P_n P^n = \frac{E_0^2}{c^2}, \tag{2.9}$$

$$C_n C^n = M_0^2 c^2. \tag{2.10}$$

For the contravariant N-vectors we have $C^n = g^{nk} C_k$, $P^n = g^{nk} P_k$, $J^n = g^{nk} J_k$ where g^{ij} is the metric tensor. The N-vector C is constant, therefore rest mass M_0 is also constant. In Equations (2.8), (2.9), (2.10) we follow Einstein's summation convention for terms where an index appears twice.

The goal of Self-Variation Theory is to find the functions $J = J(X)$, $m_0 = m_0(X)$, $P = P(X)$ and $E_0 = E_0(X)$. The differential equations resulting from the axiomatic foundation of the Theory give specific solutions for these functions. These solutions have a common feature. The material particle has internal structure, even if we assume it to be a point. In the context of the Self-Variation Theory, the generalized particle replaces the concept of the material particle.

Concluding the section we present three direct consequences of the principles of the Theory. The first of these is given by the following equations,

$$\frac{\partial P_k}{\partial x^a} = \frac{\partial P_a}{\partial x^k}, \tag{2.11}$$

$$\frac{\partial J_k}{\partial x^a} = \frac{\partial J_a}{\partial x^k}. \tag{2.12}$$

Indeed, from Equation (2.1) we get

$$\left(\frac{\partial Q}{\partial x^k}\right)_{;a} = \frac{b}{\hbar}\left(P_k Q\right)_{;a}$$

where with $;a$ we denote the covariant derivative with respect to x^a. Then we get,

$$\frac{\partial^2 Q}{\partial x^a \partial x^k} - \Gamma^l_{ka}\frac{\partial Q}{\partial x^l} = \frac{b^2}{\hbar^2}P_k P_a Q + \frac{b}{\hbar}Q\left(\frac{\partial P_k}{\partial x^a} - \Gamma^l_{ka}P_l\right)$$

and equivalently we get,

$$\frac{\partial^2 Q}{\partial x^a \partial x^k} - \Gamma^l_{ka}\frac{\partial Q}{\partial x^l} = \frac{b^2}{\hbar^2}P_k P_a Q + \frac{b}{\hbar}Q\frac{\partial P_k}{\partial x^a} - \frac{b}{\hbar}Q\Gamma^l_{ka}P_l$$

and with equation (2.1) we get,

$$\frac{\partial^2 Q}{\partial x^a \partial x^k} - \frac{b}{\hbar}Q\Gamma^l_{ka}P_l = \frac{b^2}{\hbar^2}P_k P_a Q + \frac{b}{\hbar}Q\frac{\partial P_k}{\partial x^a} - \frac{b}{\hbar}Q\Gamma^l_{ka}P_l$$

and finally we obtain,

$$\frac{\partial^2 Q}{\partial x^a \partial x^k} - \frac{b^2}{\hbar^2}P_k P_a Q = \frac{b}{\hbar}Q\frac{\partial P_k}{\partial x^a}.$$

Similarly, from the equation

$$\frac{\partial Q}{\partial x^\alpha} = \frac{b}{\hbar}P_\alpha Q$$

we get,

$$\frac{\partial^2 Q}{\partial x^k \partial x^\alpha} - \frac{b^2}{\hbar^2}P_\alpha P_k Q = \frac{b}{\hbar}Q\frac{\partial P_\alpha}{\partial x^k}.$$

Therefore we have

$$Q\frac{\partial P_k}{\partial x^a} = Q\frac{\partial P_a}{\partial x^k}$$

and taking into consideration that $Q \neq 0$ we get Equation (2.11). From Equations (2.11) and (2.7) we get Equation (2.12). In the proof process we used the symbols of Christoffel,

$$\Gamma^k_{an} = \frac{1}{2}g^{k\nu}\left(\frac{\partial g_{a\nu}}{\partial x^n} + \frac{\partial g_{\nu n}}{\partial x^a} - \frac{\partial g_{an}}{\partial x^\nu}\right). \ \square$$

If $E_0 \neq 0$, the rest mass $\frac{E_0}{c^2}$ is Self-Varying. Therefore, for each solution $J = J(X)$, $m_0 = m_0(X)$, $P = P(X)$ and $E_0 = E_0(X)$ that we get from the differential equations of the Theory, the following symmetry criterion applies.

Symmetry criterion

If $E_0 \neq 0$ them one of the following equations holds,

$$\frac{\partial E_0}{\partial x^k} = \frac{b}{\hbar} J_k E_0 \ \mathit{or} \ \frac{\partial E_0}{\partial x^k} = -\frac{b}{\hbar} J_k E_0 . \tag{2.13}$$

We can also give a second interpretation to the symmetry criterion. Equation (2.2) could be

$$\frac{\partial m_0}{\partial x^k} = \frac{b}{\hbar} P_k m_0 \ \text{or} \ \frac{\partial m_0}{\partial x^k} = -\frac{b}{\hbar} P_k m_0 .$$

Choosing the equation

$$\frac{\partial m_0}{\partial x^k} = \frac{b}{\hbar} P_k m_0 ,$$

for the resting energy E_0 we expect the equation

$$\frac{\partial E_0}{\partial x^k} = -\frac{b}{\hbar} J_k E_0 \tag{2.14}$$

to hold, If $E_0 \neq 0$.

The relative position of N-vectors J and P in spacetime can be given by the following equations,

$$P_0 = \Phi_{00} J_0 + \Phi_{01} J_1 + \Phi_{02} J_2 + \ldots + \Phi_{0(N-1)} J_{N-1}$$
$$P_1 = \Phi_{10} J_0 + \Phi_{11} J_1 + \Phi_{12} J_2 + \ldots + \Phi_{1(N-1)} J_{N-1}$$
$$P_2 = \Phi_{20} J_0 + \Phi_{21} J_1 + \Phi_{22} J_2 + \ldots + \Phi_{2(N-1)} J_{N-1}$$

. (2.15)

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$$P_{(N-1)} = \Phi_{(N-1)0} J_0 + \Phi_{(N-1)1} J_1 + \Phi_{(N-1)2} J_2 + \ldots + \Phi_{(N-1)(N-1)} J_{N-1}$$

where $\Phi_{ij} = \Phi_{ij}(X)$. Denoting T the $N \times N$ matrix $\{\Phi_{ij}\}$, Equation (2.13) is written in the form

$$P = TJ \ . \tag{2.16}$$

From Equations (2.15) and (2.7) we get,

$$(T + I) J = C \tag{2.17}$$

where I is the $N \times N$ unit matrix. Equations (2.16) and (2.17) are valid in any frame of reference.

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3. Self-Variation in the spacetime of Special Relativity

In this section we study the generalized particle in the flat 4-dimensional spacetime of Special Relativity (Minkowski spacetime), (see, [1-4]). This study is fundamental, since it highlights the basic consequences of the Self-Variation of material particles. Moreover, this study is a model for the study of the generalized particle in curved spacetime.

In the flat 4-dimensional spacetime of Special Relativity Equations (2.3) - (2.6) and (2.8) - (2.10) take the form,

$$X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} ict \\ x \\ y \\ z \end{bmatrix}, \tag{3.1}$$

$$J = \begin{bmatrix} J_0 \\ J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} \frac{iW}{c} \\ J_x \\ J_y \\ J_z \end{bmatrix}, \tag{3.2}$$

$$P = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} \frac{iE}{c} \\ P_x \\ P_y \\ P_z \end{bmatrix}, \tag{3.3}$$

$$C = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}, \tag{3.4}$$

$$J_0^2 + J_1^2 + J_2^2 + J_3^2 + m_0{}^2 c^2 = 0, \tag{3.5}$$

$$P_0^2 + P_1^2 + P_2^2 + P_3^2 + \frac{E_0^2}{c^2} = 0, \tag{3.6}$$

$$c_0^2 + c_1^2 + c_2^2 + c_3^2 + M_0^2 c^2 = 0 \tag{3.7}$$

respectively.

We consider an inertial frame of reference $\mathrm{O}'(x_0', x_1', x_2', x_3')$ moving with velocity $(u, 0, 0)$ with respect to another inertial frame of reference $\mathrm{O}(x_0, x_1, x_2, x_3)$, with their origins O' and O coinciding at $x_0' = x_0$. With this symbolism the Lorentz-Einstein transformations have the following form,

$$\begin{aligned} x_0' &= \gamma\left(x_0 - i\frac{u}{c}x_1\right) & \frac{\partial}{\partial x_0'} &= \gamma\left(\frac{\partial}{\partial x_0} - i\frac{u}{c}\frac{\partial}{\partial x_1}\right) \\ x_1' &= \gamma\left(x_1 + i\frac{u}{c}x_0\right) & \frac{\partial}{\partial x_1'} &= \gamma\left(\frac{\partial}{\partial x_1} + i\frac{u}{c}\frac{\partial}{\partial x_0}\right) \\ x_2' &= x_2 & \frac{\partial}{\partial x_2'} &= \frac{\partial}{\partial x_2} \\ x_3' &= x_3 & \frac{\partial}{\partial x_3'} &= \frac{\partial}{\partial x_3} \end{aligned} \tag{3.8}$$

$$\begin{aligned} J_0' &= \gamma\left(J_0 - i\frac{u}{c}J_1\right) & P_0' &= \gamma\left(P_0 - i\frac{u}{c}P_1\right) \\ J_1' &= \gamma\left(J_1 + i\frac{u}{c}J_0\right) & P_1' &= \gamma\left(P_1 + i\frac{u}{c}P_0\right), \\ J_2' &= J_2 & P_2' &= P_2 \\ J_3' &= J_3 & P_3' &= P_3 \end{aligned} \tag{3.9}$$

where $\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$.

From these transformations and Equation (2.15) we get the following equations,

$$\begin{aligned} &\Phi_{ij} = -\Phi_{ji}, i \neq j \\ &\Phi_{00} = \Phi_{11} = \Phi_{22} = \Phi_{33} = \Phi \end{aligned} \tag{3.10}$$

and transformations,

$$\Phi' = \Phi \tag{3.11}$$

$$\begin{aligned} \Phi_{01}' &= \Phi_{01} \\ \Phi_{02}' &= \gamma\left(\Phi_{02} + i\frac{u}{c}\Phi_{21}\right), \\ \Phi_{03}' &= \gamma\left(\Phi_{03} - i\frac{u}{c}\Phi_{13}\right) \end{aligned} \tag{3.12}$$

$$\begin{aligned} \Phi_{32}' &= \Phi_{32} \\ \Phi_{13}' &= \gamma\left(\Phi_{13} + i\frac{u}{c}\Phi_{03}\right). \\ \Phi_{21}' &= \gamma\left(\Phi_{21} - i\frac{u}{c}\Phi_{02}\right) \end{aligned} \tag{3.13}$$

It follows from our study that as we move from one frame of reference to another through Lorentz-Einstein transformations, we get equations that apply to the same frame of reference. They are Equations (3.10). Also, the vectors $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$,

$$\boldsymbol{\alpha} = \begin{pmatrix} ic\Phi_{01} \\ ic\Phi_{02} \\ ic\Phi_{03} \end{pmatrix},$$

$$\boldsymbol{\beta} = \begin{pmatrix} \Phi_{32} \\ \Phi_{13} \\ \Phi_{21} \end{pmatrix}$$

are transformed like the electromagnetic field. The vector $\boldsymbol{\alpha}$ corresponds to the electric field and the vector $\boldsymbol{\beta}$ to the magnetic one.

From Equations (2.15) and (3.10) we get,

$$\begin{aligned} P_0 &= \Phi J_0 + \Phi_{01} J_1 + \Phi_{02} J_2 + \Phi_{03} J_3 \\ P_1 &= -\Phi_{01} J_0 + \Phi J_1 - \Phi_{21} J_2 + \Phi_{13} J_3 \\ P_2 &= -\Phi_{02} J_0 + \Phi_{21} J_1 + \Phi J_2 - \Phi_{32} J_3 \\ P_3 &= -\Phi_{03} J_0 - \Phi_{13} J_1 + \Phi_{32} J_2 + \Phi J_3 \end{aligned}. \tag{3.14}$$

The determinant $|T|$ of the system of Equations (3.14) is given by the following equation,

$$|T| = \Phi^4 + \Phi^2 \left(\Phi_{01}^2 + \Phi_{02}^2 + \Phi_{03}^2 + \Phi_{32}^2 + \Phi_{13}^2 + \Phi_{21}^2 \right) + \left(\Phi_{01}\Phi_{32} + \Phi_{02}\Phi_{13} + \Phi_{03}\Phi_{21} \right)^2,$$

as obtained after the necessary calculations. If $P \neq 0$, the system of equations (3.14) is non-homogeneous its determinant is non-zero,

$$\Phi^4 + \Phi^2 \left(\Phi_{01}^2 + \Phi_{02}^2 + \Phi_{03}^2 + \Phi_{32}^2 + \Phi_{13}^2 + \Phi_{21}^2 \right) + \left(\Phi_{01}\Phi_{32} + \Phi_{02}\Phi_{13} + \Phi_{03}\Phi_{21} \right)^2 \neq 0. \tag{3.15}$$

From the inequality (3.15) it follows that if $\Phi_{ij} = 0$ for every $i \neq j$, then $\Phi \neq 0$. If $\Phi = 0$ then $\Phi_{01}\Phi_{32} + \Phi_{02}\Phi_{13} + \Phi_{03}\Phi_{21} \neq 0$. One of the conclusions derived from the study we did is given by the following Internal Symmetry Theorem.

Internal Symmetry Theorem

I. *If* $P \neq 0$ *the following applies.*

A. *If* $\Phi = 0$ *them* $\Phi_{01}\Phi_{32} + \Phi_{02}\Phi_{13} + \Phi_{03}\Phi_{21} \neq 0$.

B. *If* $\Phi_{ij} = 0$ *for each* $i \neq j$ *then the following applies.*

1. *The 4-vectors* P *and* J *are parallel,*

$$P = \Phi J. \tag{3.16}$$

2. Exactly one of the following applies,

$$E_0 = m_0 c^2 \text{ and } M_0 = 0 \tag{3.17}$$

or

$$\Phi = K \exp\left[-\frac{b}{\hbar} \left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3 \right) \right], \tag{3.18}$$

$$m_0 = \pm \frac{M_0}{1+\Phi}, \tag{3.19}$$

$$E_0 = \pm \frac{\Phi M_0 c^2}{1+\Phi}, \tag{3.20}$$

$$J_i = \frac{c_i}{1+\Phi}, i = 0,1,2,3, \tag{3.21}$$

$$P_i = \frac{\Phi c_i}{1+\Phi}, i = 0,1,2,3 \tag{3.22}$$

where $K \neq 0$ *is a dimensionless constant.*

II. *If* $P = 0$ *then the following applies.*

1. $E_0 = 0$. (3.23)

2. $\Phi^4 + \Phi^2\left(\Phi_{01}^2 + \Phi_{02}^2 + \Phi_{03}^2 + \Phi_{32}^2 + \Phi_{13}^2 + \Phi_{21}^2\right) + \left(\Phi_{01}\Phi_{32} + \Phi_{02}\Phi_{13} + \Phi_{03}\Phi_{21}\right)^2 = 0$. (3.24)

Proof. A. A has been proved before, following Inequality (3.15).

B. 1. if $\Phi_{ij} = 0$ for each $i \neq j$, Equation (3.16) results from the system of Equations (3.14).

2. From Equation (3.16) we have $P_i = \Phi J_i$ and with Equation (2.7) we get $c_i - J_i = \Phi J_i$ and equivalently we obtain,

$$\left(\Phi + 1\right) J_i = c_i. \tag{3.25}$$

If $\Phi = -1$ we have $c_i = 0$ and $P_i = -J_i$. Then, from Equation (3.7) we obtain $M_0 = 0$ and from Equations (3.5), (3.6) we obtain, $E_0 = m_0 c^2$.

If $\Phi \neq -1$, from Equation (3.25) we get,

$$J_i = \frac{c_i}{1+\Phi}. \tag{3.26}$$

From equations (3.5) and (2.2) we get,

$$2J_0\frac{\partial J_0}{\partial x_k} + 2J_1\frac{\partial J_1}{\partial x_k} + 2J_2\frac{\partial J_2}{\partial x_k} + 2J_3\frac{\partial J_3}{\partial x_k} + 2\frac{b}{\hbar}P_k m_0^2 c^2 = 0$$

and with Equation (3.5) we get,

$$J_0\frac{\partial J_0}{\partial x_k} + J_1\frac{\partial J_1}{\partial x_k} + J_2\frac{\partial J_2}{\partial x_k} + J_3\frac{\partial J_3}{\partial x_k} - \frac{b}{\hbar}P_k\left(J_0^2 + J_1^2 + J_2^2 + J_3^2\right) = 0$$

and with Equation (2.7) we get,

$$J_0\frac{\partial J_0}{\partial x_k} + J_1\frac{\partial J_1}{\partial x_k} + J_2\frac{\partial J_2}{\partial x_k} + J_3\frac{\partial J_3}{\partial x_k} - \frac{b}{\hbar}\left(c_k - J_k\right)\left(J_0^2 + J_1^2 + J_2^2 + J_3^2\right) = 0$$

and with Equation (3.26) we get,

$$\frac{c_0}{1+\Phi}\frac{\partial}{\partial x_k}\left(\frac{c_0}{1+\Phi}\right) + \frac{c_1}{1+\Phi}\frac{\partial}{\partial x_k}\left(\frac{c_1}{1+\Phi}\right) + \frac{c_2}{1+\Phi}\frac{\partial}{\partial x_k}\left(\frac{c_2}{1+\Phi}\right) + \frac{c_3}{1+\Phi}\frac{\partial}{\partial x_k}\left(\frac{c_3}{1+\Phi}\right)$$

$$-\frac{b}{\hbar}\left(c_k - \frac{c_k}{1+\Phi}\right)\left(\frac{c_0^2}{\left(1+\Phi\right)^2} + \frac{c_1^2}{\left(1+\Phi\right)^2} + \frac{c_2^2}{\left(1+\Phi\right)^2} + \frac{c_3^2}{\left(1+\Phi\right)^2}\right) = 0$$

and after the calculations we get,

$$\frac{\partial \Phi}{\partial x_k} = -\frac{b c_k}{\hbar}\Phi. \tag{3.27}$$

From Equation (3.27) we obtain,

$$\Phi = K\exp\left[-\frac{b}{\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3\right)\right]$$

where K is a dimensionless constant physical quantity.

From Equations (3.16) and (3.26) we obtain,

$$P_i = \frac{\Phi c_i}{1+\Phi}.$$

From this Equation and (3.6) we obtain,

$$E_0 = \pm \frac{\Phi M_0 c^2}{1+\Phi}.$$

Similarly, from Equations (3.26) and (3.5) we obtain,

$$m_0 = \pm \frac{M_0 c^2}{1+\Phi}.$$

The proof is completed by applying the symmetry criterion. For Equations (3.17) we have,

$$\frac{\partial E_0}{\partial x_k} = \frac{\partial m_0 c^2}{\partial x_k}$$

and with Equation (2.2) we get,

$$\frac{\partial E_0}{\partial x_k} = \frac{b}{\hbar} P_k m_0 c^2$$

and with Equation (3.17) we get,

$$\frac{\partial E_0}{\partial x_k} = \frac{b}{\hbar} P_k E_0$$

and considering that it is $P_k = -J_k$ we obtain,

$$\frac{\partial E_0}{\partial x_k} = -\frac{b}{\hbar} J_k E_0.$$

From Equations (3.19) and (3.20) we get,

$$E_0^2 = \Phi^2 m_0^2 c^4. \qquad (3.28)$$

Then we have

$$2E_0 \frac{\partial E_0}{\partial x_k} = 2\Phi \frac{\partial \Phi}{\partial x_k} m_0^2 c^4 + 2\Phi^2 m_0 c^4 \frac{\partial m_0}{\partial x_k}$$

and with the Equations $\frac{\partial \Phi}{\partial x_k} = -\frac{bc_k}{\hbar}\Phi$ and (2.2) we get,

$$E_0 \frac{\partial E_0}{\partial x_k} = -\Phi \frac{bc_k}{\hbar} \Phi m_0^2 c^4 + \Phi^2 \frac{bP_k}{\hbar} m_0^2 c^4$$

and with the Equation (3.28) we get,

$$E_0 \frac{\partial E_0}{\partial x_k} = -\frac{bc_k}{\hbar} E_0^2 + \frac{bP_k}{\hbar} E_0^2$$

and considering that it is $E_0 \neq 0$ we get,

$$\frac{\partial E_0}{\partial x_k} = -\frac{bc_k}{\hbar} E_0 + \frac{bP_k}{\hbar} E_0$$

and equivalently we get,

$$\frac{\partial E_0}{\partial x_k} = \frac{b}{\hbar}\left(P_k - c_k\right) E_0$$

and with the Equation (2.7) we obtain,

$$\frac{\partial E_0}{\partial x_k} = -\frac{b}{\hbar} J_k E_0 .$$

II. 1. It follows from Equation (3.6). 2. If $P = 0$ the system of Equations (3.14) is homogeneous so the determinant of is equal to zero. □

The Internal Symmetry Theorem is generally valid for any self-variating charge, since in equation (2.1), the momentum J of the particle is due to the charge Q.

Equations (3.17) predict a generalized particle with zero total rest mass, $M_0 = 0$. In addition, the Equation $E_0 = m_0 c^2$ applies. The Internal Symmetry Theorem gives no other information about this particle.

For the generalized particle of Equations (3.18) - (3.22), the Internal Symmetry Theorem gives a remarkable set of information. Equations (3.19) and (3.20) give the distribution of the total rest mass M_0 in m_0 and $\frac{E_0}{c^2}$. Similarly, Equations (3.21) and (3.22) give the distribution of the total momentum c_k along the x_k axis. That is, we have energy-momentum and rest-mass distribution in space-time. This distribution is determined by the function Φ. If $b = i$ in Equation (3.18) the distribution is periodic. In general, if the constant b is not a real number, $b \in \mathbb{C} - \mathbb{R}$ the distribution has wave characteristics. If it is a real number, $b \in \mathbb{R}$ the distribution is non-periodic.

From Equation (3.21) we have

$$\frac{\partial J_i}{\partial x_k} = -\frac{c_i}{\left(1+\Phi\right)^2}\frac{\partial \Phi}{\partial x_k}$$

and with Equation (3.27) we get,

$$\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar}\frac{c_i c_k}{\left(1+\Phi\right)^2}\Phi$$

and with Equations (3.21), (3.22) we obtain,

$$\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i . \tag{3.29}$$

From Equations (3.29) and (2.7) we obtain,

$$\frac{\partial P_i}{\partial x_k} = -\frac{b}{\hbar} P_k J_i . \tag{3.30}$$

From equations (3.29) and (3.30) it follows that the Internal Symmetry Theorem gives the rates of change of the 4-vectors J and P.

The function Φ also depends on the 4-vector C. If $c_1 = c_2 = c_3 = 0$ we have

$$\Phi = K \exp\left(-\frac{b}{\hbar} c_0 x_0\right).$$

Then, from Equation (3.7) we get $c_0 = \pm M_0 c$ and

$$\Phi = K \exp\left(-\frac{b}{\hbar} c_0 x_0\right) = K \exp\left(\pm\frac{b M_0 c^2}{\hbar} t\right).$$

Then, from Equation (3.19) we obtain,

$$m_0 = \pm\frac{M_0}{1 + K \exp\left(\pm\frac{b M_0 c^2}{\hbar} t\right)} . \tag{3.31}$$

Equation (3.31) gives the rest mass m_0 as a function of time in an inertial frame of reference in which is $c_1 = c_2 = c_3 = 0$. We can easily prove the corresponding equation for the charge q,

$$q = \pm \frac{Q}{1 + K \exp\left(\pm \frac{bM_0' c^2}{\hbar} t \right)}, \tag{3.32}$$

where Q in this Equation is a constant (and q is the self-varying charge). The rest mass M_0' is due to the energy-momentum that the particle has due to the charge q. Equations (3.31) and (3.32) give the increase in rest mass and electric charge, as required by the Self-Variation principle, of a particle that is stationary ($c_1 = c_2 = c_3 = 0$) with respect to an observer in the flat spacetime.

In II of Theorem we can give the following interpretation. If $P = 0$ the Self-Variation is negated. However, if we assume that this has a chance of happening for a time interval δt, II of the theorem gives us the consequences. In this time interval the rest mass m_0 is constant, while Equations (3.23) and (3.24) apply. If in addition $\Phi = 0$, from Equation (3.24) we get $\Phi_{01}\Phi_{32} + \Phi_{02}\Phi_{13} + \Phi_{03}\Phi_{21} = 0$. Therefore vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are perpendicular. In addition, Equation (3.23) applies, $E_0 = 0$. This situation corresponds to an 'electromagnetic wave'. Thus we conclude that the rest mass m_0 may be constant for a period of time, but this period of time is always associated with an 'electromagnetic wave'.

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4. Electromagnetic interaction

In this section we present the potentials that are compatible with the Self-Variation principle and replace the Liénard-Wiechert potentials (see, [1], [2]). We consider an electric point charge q moving arbitrarily in an inertial frame of reference $\mathrm{O}(t, x, y, z)$. We assume that the electromagnetic field propagates with speed $\boldsymbol{\upsilon}$, $\|\boldsymbol{\upsilon}\| = c$ where c is the speed of light in vacuum. As a consequence of Self-change, at time t, when q is at point P, it acts at point A with the value it had at point E, at the decelerating time w. We use the following symbolism, $\overrightarrow{EA} = \mathbf{r}$,

$\|\mathbf{r}\| = r$, $\vec{OE} = \mathbf{r}_p(w)$, $\vec{OP} = \mathbf{r}_p(t)$, $E\left(w, x_p(w), y_p(w), z_p(w)\right)$, $P\left(t, x_p(t), y_p(t), z_p(t)\right)$, $A(t, x, y, z)$,

where $O(0,0,0)$. The index p in the coordinates x_p, y_p, z_p indicates the position of the point particle carrying the charge q, at the corresponding moment in time w or t. At point E we denote $\mathbf{u}(w) = \mathbf{u}$ the velocity and $\mathbf{a}(w) = \mathbf{a}$ the acceleration of q (see [4], Fig. 1). With this symbolism we have,

$$\mathbf{r} = \begin{pmatrix} x - x_p(w) \\ y - y_p(w) \\ z - z_p(w) \end{pmatrix}, \tag{4.1}$$

$$r = \|\mathbf{r}\| = \left(\left(x - x_p(w)\right)^2 + \left(y - y_p(w)\right)^2 + \left(z - z_p(w)\right)^2\right)^{\frac{1}{2}}, \tag{4.2}$$

$$w = t - \frac{r}{c}, \tag{4.3}$$

$$\boldsymbol{\upsilon} = c\frac{\mathbf{r}}{r} = \frac{c}{r}\begin{pmatrix} x - x_p(w) \\ y - y_p(w) \\ z - z_p(w) \end{pmatrix} = \begin{pmatrix} \upsilon_x \\ \upsilon_y \\ \upsilon_z \end{pmatrix} = \begin{pmatrix} \upsilon_1 \\ \upsilon_2 \\ \upsilon_3 \end{pmatrix}. \tag{4.4}$$

The velocity $\mathbf{u} = \mathbf{u}(w)$ of the q at point E is,

$$\mathbf{u} = \begin{pmatrix} \dfrac{dx_p(w)}{dw} \\ \dfrac{dx_p(w)}{dw} \\ \dfrac{dx_p(w)}{dw} \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}. \tag{4.5}$$

Auxiliary equations

We prove the following list of equations which we will use next.

From Equation (4.2) we have

$$\frac{\partial r}{\partial t} = \frac{1}{2r}\left(2\left(x - x_p(w)\right)\left(-\frac{dx_p(w)}{dw}\frac{\partial w}{\partial t}\right) + 2\left(y - y_p(w)\right)\left(-\frac{dy_p(w)}{dw}\frac{\partial w}{\partial t}\right) + 2\left(z - z_p(w)\right)\left(-\frac{dz_p(w)}{dw}\frac{\partial w}{\partial t}\right)\right)$$

and with Equations (4.4) and (4.5) we get

$$\frac{\partial r}{\partial t} = -\frac{1}{r}(\mathbf{r} \cdot \mathbf{u})\frac{\partial w}{\partial t}$$

and with Equation (4.4) we get

$$\frac{\partial r}{\partial t} = -\left(\frac{\mathbf{u} \cdot \boldsymbol{\upsilon}}{c}\right)\frac{\partial w}{\partial t}$$

and with Equation (4.3) we get

$$\frac{\partial r}{\partial t} = -\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c}\right)\left(1-\frac{\partial r}{\partial t}\right)$$

and equivalently we obtain,

$$\frac{\partial r}{\partial t} = -\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}. \tag{4.6}$$

From Equations (4.3) and (4.6) we obtain,

$$\frac{\partial w}{\partial t} = \frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}. \tag{4.7}$$

Starting again from Equation (4.2) we obtain,

$$\nabla r = \begin{pmatrix} \frac{\partial r}{\partial x} \\ \frac{\partial r}{\partial y} \\ \frac{\partial r}{\partial z} \end{pmatrix} = \frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{\boldsymbol{\upsilon}}{c}. \tag{4.8}$$

From Equations (4.3) and (4.8) we obtain,

$$\nabla w = -\frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{\boldsymbol{\upsilon}}{c^2}. \tag{4.9}$$

From Equation (4.1) we have

$$\frac{\partial \mathbf{r}}{\partial t} = \begin{pmatrix} -\frac{dx_p(w)}{dw}\frac{\partial w}{\partial t} \\ -\frac{dy_p(w)}{dw}\frac{\partial w}{\partial t} \\ -\frac{dz_p(w)}{dw}\frac{\partial w}{\partial t} \end{pmatrix} = \frac{\partial w}{\partial t}\mathbf{u}$$

and with Equation (4.7) we obtain,

$$\frac{\partial \mathbf{r}}{\partial t} = -\frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\mathbf{u}. \tag{4.10}$$

From Equation (4.4) we have

$$\frac{\partial \boldsymbol{\upsilon}}{\partial t} = -\frac{c}{r^2}\frac{\partial r}{\partial t} + \frac{c}{r}\frac{\partial \mathbf{r}}{\partial t}$$

and with Equations (4.4), (4.6) and (4.10) we obtain,

$$\frac{\partial \boldsymbol{\upsilon}}{\partial t} = \frac{c}{r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\left(\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\boldsymbol{\upsilon}-\mathbf{u}\right). \tag{4.11}$$

From Equation (4.4) we have

$$\upsilon_x = \frac{c}{r}\left(x - x_p\left(w\right)\right)$$

and differentiating with respect to x we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{c}{r^2}\frac{\partial r}{\partial x}\left(x - x_p\left(w\right)\right) + \frac{c}{r}\left(1 - \frac{\partial x_p\left(w\right)}{\partial x}\right)$$

and equivalently we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{c}{r^2}\frac{\partial r}{\partial x}\left(x - x_p\left(w\right)\right) + \frac{c}{r}\left(1 - \frac{dx_p\left(w\right)}{dw}\frac{\partial w}{\partial x}\right)$$

and with Equation (4.5) we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{c}{r^2}\frac{\partial r}{\partial x}\left(x - x_p\left(w\right)\right) + \frac{c}{r}\left(1 - u_x\frac{\partial w}{\partial x}\right)$$

and with Equation (4.4) we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{1}{r}\frac{\partial r}{\partial x}\upsilon_x + \frac{c}{r}\left(1 - u_x\frac{\partial w}{\partial x}\right)$$

and with Equations (4.8) and (4.9) we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{\upsilon_x^2}{cr}\frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} + \frac{c}{r}\left(1 + \frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{u_x\upsilon_x}{c^2}\right)$$

and equivalently we obtain,

$$\frac{\partial \upsilon_x}{\partial x} = \frac{c}{r} + \frac{\upsilon_x\left(u_x - \upsilon_x\right)}{cr\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}.$$

Working similarly, we finally obtain,

$$\frac{\partial \upsilon_i}{\partial x_j} = \begin{cases} \dfrac{c}{r} + \dfrac{\upsilon_i\left(u_i - \upsilon_i\right)}{cr\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)},\ if\ \ i = j \\[2ex] \dfrac{\upsilon_j\left(u_i - \upsilon_i\right)}{cr\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)},\ if\ \ i \neq j \end{cases}, \tag{4.12}$$

where $i, j = 1,2,3$ and $\left(x_1, x_2, x_3\right) = \left(x, y, z\right)$.

Now we have,

$$\nabla\cdot\boldsymbol{\upsilon} = \frac{\partial \upsilon_x}{\partial x} + \frac{\partial \upsilon_y}{\partial y} + \frac{\partial \upsilon_z}{\partial z}$$

and with Equation (4.12) we get,

$$\nabla\cdot\boldsymbol{\upsilon} = \frac{3c}{r} + \frac{\upsilon_x\left(u_x - \upsilon_x\right) + \upsilon_y\left(u_y - \upsilon_y\right) + \upsilon_z\left(u_z - \upsilon_z\right)}{cr\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}$$

and equivalently we get,

$$\nabla \cdot \boldsymbol{\upsilon} = \frac{3c}{r} + \frac{\upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2 - u_x\upsilon_x - u_y\upsilon_y - u_z\upsilon_z}{cr\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}$$

and taking into consideration that $\upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2 = c^2$ we get,

$$\nabla \cdot \boldsymbol{\upsilon} = \frac{3c}{r} + \frac{c^2 - \mathbf{u}\cdot\boldsymbol{\upsilon}}{cr\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}$$

and equivalently we obtain,

$$\nabla \cdot \boldsymbol{\upsilon} = \frac{2c}{r}. \tag{4.13}$$

Working similarly we obtain,

$$\nabla \times \boldsymbol{\upsilon} = curl\boldsymbol{\upsilon} = \frac{1}{cr\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\left(\boldsymbol{\upsilon}\times\mathbf{u}\right). \tag{4.14}$$

If a physical quantity f is defined at the point E, $f = f(w)$ then we have,

$$\frac{\partial f(w)}{\partial t} = \frac{df(w)}{dw}\frac{\partial w}{\partial t}$$

and with Equation (4.7) we obtain,

$$\frac{\partial f(w)}{\partial t} = \frac{df(w)}{dw}\frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}. \tag{4.15}$$

Similarly, from Equation (4.9) we obtain,

$$\nabla f(w) = -\frac{df(w)}{dw}\frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{\boldsymbol{\upsilon}}{c^2}. \tag{4.16}$$

From Equations (4.15) and (4.16) we obtain,

$$\nabla f(w) = -\frac{\partial f(w)}{\partial t}\frac{\boldsymbol{\upsilon}}{c^2}. \tag{4.17}$$

As a consequence of Self-Variation, at time t the electric charge acts at point A with the value it has at point E. Therefore, $q = q(w)$ and from Equations (4.15), (4.16) and (4.17) For $f = q$ we obtain,

$$\frac{\partial q}{\partial t} = \frac{dq}{dw}\frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}, \tag{4.18}$$

$$\nabla q = -\frac{dq}{dw}\frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{\boldsymbol{\upsilon}}{c^2}, \tag{4.19}$$

$$\nabla q = -\frac{\partial q}{\partial t}\frac{\boldsymbol{\upsilon}}{c^2}. \tag{4.20}$$

We now consider the acceleration vector $\mathbf{a} = \mathbf{a}(w)$ of q at the moment w located at point E,

$$\mathbf{a} = \mathbf{a}(w) = \frac{d\mathbf{u}(w)}{dw}. \tag{4.21}$$

Applying equations (4.15) and (4.16) for the velocity components $\mathbf{u}$ we obtain,

$$\frac{\partial \mathbf{u}}{\partial t} = \frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\mathbf{a}, \tag{4.22}$$

$$\frac{\partial u_i}{\partial x_j} = -\frac{\upsilon_j a_i}{c^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}, \tag{4.23}$$

where $i, j = 1,2,3$ and $(x_1, x_2, x_3) = (x, y, z)$. Applying equations (4.15) and (4.16) for the velocity components $\mathbf{a}$ we obtain,

$$\frac{\partial \mathbf{a}}{\partial t} = \frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\mathbf{b}, \tag{4.24}$$

$$\frac{\partial a_i}{\partial x_j} = -\frac{\upsilon_j b_i}{c^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}, \tag{4.25}$$

where $\mathbf{b} = \mathbf{b}(w) = \frac{d\mathbf{a}(w)}{dw}$.

Using the previous Equations we obtain the following equations,

$$\frac{\partial(\mathbf{u}\cdot\boldsymbol{\upsilon})}{\partial t} = \frac{\boldsymbol{\upsilon}\cdot\mathbf{a}}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} + \frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})^2 - c^2u^2}{cr\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}, \tag{4.26}$$

$$\nabla(\mathbf{u}\cdot\boldsymbol{\upsilon}) = \frac{c}{r}\mathbf{u} + \frac{u^2 - \mathbf{u}\cdot\boldsymbol{\upsilon}}{r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c} - \frac{\boldsymbol{\upsilon}\cdot\mathbf{a}}{c\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c}, \tag{4.27}$$

$$\frac{\partial(\boldsymbol{\upsilon}\cdot\mathbf{a})}{\partial t} = \frac{\boldsymbol{\upsilon}\cdot\mathbf{b}}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} + \frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})(\boldsymbol{\upsilon}\cdot\mathbf{a}) - c^2(\mathbf{u}\cdot\mathbf{a})}{cr\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}, \tag{4.28}$$

$$\nabla(\boldsymbol{\upsilon}\cdot\mathbf{a}) = \frac{c}{r}\mathbf{a} - \frac{\boldsymbol{\upsilon}\cdot\mathbf{b}}{c^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\boldsymbol{\upsilon} + \frac{\mathbf{u}\cdot\mathbf{a} - \boldsymbol{\upsilon}\cdot\mathbf{a}}{cr\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\boldsymbol{\upsilon}, \tag{4.29}$$

after the necessary calculations.

Liénard-Wiechert potentials

With the notation we follow, the Liénard-Wiechert scalar -vector potential pair $(V_{LW}, \mathbf{A}_{LW})$ is given by the equations,

$$V_{LW} = \frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}, \tag{4.30}$$

$$\mathbf{A}_{LW} = V_{LW}\frac{\mathbf{u}}{c^2} = \frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\frac{\mathbf{u}}{c^2}. \tag{4.31}$$

The electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ at point $A(x, y, z)$ are given by the pair $(V, \mathbf{A})$ of the scalar potential V and the vector potential $\mathbf{A}$ respectively, through equations

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \tag{4.32}$$

$$\mathbf{B} = \nabla \times \mathbf{A}. \tag{4.33}$$

Through Equations (4.30), (4.31) and (4.32), (4.33) the Liénard-Wiechert potentials give the following equations for the electromagnetic field at point A,

$$\mathbf{E} = \frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right) + \frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right], \tag{4.34}$$

$$\mathbf{B} = \frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\mathbf{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right) + \frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\mathbf{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right)-\frac{\boldsymbol{\upsilon}}{c}\times\mathbf{a}\right]. \tag{4.35}$$

The first terms in the second members of Equations (4.34), (4.35) give the electromagnetic field accompanying the electric charge in its movement, and the second terms the electromagnetic radiation.

Self-Variation potentials

As a consequence of Self-Variation, the electromagnetic potential splits into two pairs of potentials. One pair,

$$\begin{aligned} V_u &= \frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2} \\ \mathbf{A}_u &= V_u\frac{\boldsymbol{\upsilon}}{c^2} = \frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2} \end{aligned} \tag{4.36}$$

gives the electromagnetic field that accompanies the electric charge in its motion,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)$$

$$\mathbf{B}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\mathbf{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right) \qquad (4.37)$$

The other pair,

$$V_a=\frac{(\boldsymbol{\upsilon}\cdot\mathbf{a})q}{4\pi\varepsilon_0 c^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}$$

$$\mathbf{A}_a=V_a\frac{\boldsymbol{\upsilon}}{c^2}=\frac{(\boldsymbol{\upsilon}\cdot\mathbf{a})q}{4\pi\varepsilon_0 c^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2}, \qquad (4.38)$$

gives the electromagnetic radiation,

$$\mathbf{E}=\frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]$$

$$\mathbf{B}=\frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\mathbf{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right)-\frac{\boldsymbol{\upsilon}}{c}\times\mathbf{a}\right]. \qquad (4.39)$$

From (4.37) and (4.39) we get Equations (4.34). The Liénard-Wiechert and Self-Variation potentials give the same equations for the electromagnetic field strength.

From the potentials (4.36) we prove the first of Equations (4.37). Similarly, the proof of the second is done, as well as the proof of Equations (4.39) from the potentials (4.38).

Proof. From Equation (4.32) and (4.36) we have,

$$\mathbf{E}=-\nabla\left(\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right)-\frac{\partial}{\partial t}\left(\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2}\right)$$

and equivalently we get,

$$\mathbf{E}=-\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\nabla q-q\nabla\left(\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right)-\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial q}{\partial t}$$

$$-q\frac{\partial}{\partial t}\left(\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2}\right) \tag{4.40}$$

and with Equation (4.20) we get,

$$\mathbf{E}=-q\nabla\left(\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right)-q\frac{\partial}{\partial t}\left(\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2}\right) \tag{4.41}$$

and equivalently we get,

$$\mathbf{E}=\frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\nabla\left(\frac{u^2}{c^2}\right)-\left(1-\frac{u^2}{c^2}\right)q\nabla\left(\frac{1}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right)$$

$$+\frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\partial\left(\frac{u^2}{c^2}\right)}{\partial t}\frac{\boldsymbol{\upsilon}}{c^2}-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c^2}\right) \qquad . \tag{4.42}$$

$$-\left(1-\frac{u^2}{c^2}\right)q\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial}{\partial t}\left(\frac{1}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right)$$

From Equation (4.17) for $f\left(w\right)=u^2\left(w\right)$ we get,

$$\nabla\left(\frac{u^2}{c^2}\right)=-\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial}{\partial t}\left(\frac{u^2}{c^2}\right). \tag{4.43}$$

From Equations (4.42) and (4.43) we get,

$$\mathbf{E}=-\left(1-\frac{u^2}{c^2}\right)q\nabla\left(\frac{1}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right)-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c^2}\right)$$

$$-\left(1-\frac{u^2}{c^2}\right)q\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial}{\partial t}\left(\frac{1}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right)$$

and equivalently we get,

$$\mathbf{E}=-\left(1-\frac{u^2}{c^2}\right)q\left(-\frac{1}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\nabla r+\frac{2}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\nabla\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)\right)$$

$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c^2}\right)$$

$$-\left(1-\frac{u^2}{c^2}\right)q\frac{\boldsymbol{\upsilon}}{c^2}\left(-\frac{1}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\partial r}{\partial t}+\frac{2}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\partial}{\partial t}\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)\right)$$

and equivalently we obtain,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left(\nabla r+\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial r}{\partial t}\right)$$

$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c^2}\right) \qquad . \tag{4.44}$$

$$-\frac{2\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\nabla\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)+\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial}{\partial t}\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)\right)$$

From Equations (4.8) and (4.10) we get,

$$\nabla r+\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial r}{\partial t}=\frac{\boldsymbol{\upsilon}}{c}\,. \tag{4.45}$$

From Equations (4.26) and (4.27) we get,

$$\nabla\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)+\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial}{\partial t}\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)=\frac{1}{r}\left(\frac{\mathbf{u}}{c}-\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\frac{\boldsymbol{\upsilon}}{c}\right). \tag{4.46}$$

From Equations (4.44) and (4.34), (4.45), (4.11), (4.4) we get,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c}$$

$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)$$

$$-\frac{2\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\mathbf{u}}{c}-\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\frac{\boldsymbol{\upsilon}}{c}\right)$$

and equivalently we get,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c}$$

$$+\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}(1-2)\frac{\mathbf{u}}{c}$$

$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}-2\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\right)\frac{\boldsymbol{\upsilon}}{c}$$

and equivalently we get,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c}+\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\frac{\boldsymbol{\upsilon}}{c}$$

$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\mathbf{u}}{c}$$

and equivalently we get,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(1-\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}+\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\right)\frac{\boldsymbol{\upsilon}}{c}$$

$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\mathbf{u}}{c}$$

and equivalently we get,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\boldsymbol{\upsilon}}{c}-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\mathbf{u}}{c}$$

and equivalently we obtain,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right).\ \square$$

In the proof we followed, the transition from Equation (4.40) to (4.41) was made as a consequence of Equation (4.20). This Equation expresses the Self-Variation of the electric charge q. If we assume that the charge q does not self-variate, from the potentials (4.36) we directly obtain Equation (4.41). The Self-Variation potentials give the same electromagnetic field whether we consider the electric charge to vary according to the Self-Variation principle or to be constant.

Applying Maxwell's Equations for the electromagnetic field of Equations (4.34), (4.35) it follows that at point A there is an electric charge, as a consequence of Self-Variation, with density ρ and current density $\mathbf{j}$,

$$\rho = -\frac{dq}{dw}\frac{1-\frac{u^2}{c^2}}{4\pi c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}$$

$$. \tag{4.47}$$

$$\mathbf{j} = \rho\boldsymbol{\upsilon} = -\frac{dq}{dw}\frac{1-\frac{u^2}{c^2}}{4\pi c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\boldsymbol{\upsilon}$$

As a consequence of Self-Variation, in the surrounding spacetime of q there is an electric charge of opposite sign ($\frac{dq}{dw} > 0$), as follows from Equations (4.47).

We prove the first of Equations (4.47). Similarly, the proof of the second Equation is made.

Proof. From Maxwell's first law we have,

$$\rho = \varepsilon_0 \nabla\cdot\mathbf{E}\,. \tag{4.48}$$

We write equation (4.34) in the form,

$$\mathbf{E} = q\left(\frac{1-\frac{u^2}{c^2}}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)+\frac{1}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]\right). \tag{4.49}$$

If we ignore Self-Variation and consider q constant, at point A there is no electric charge. Thus from Equations (4.48) and (34.49) we get,

$$\nabla\cdot\left(\frac{1-\frac{u^2}{c^2}}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)+\frac{1}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]\right)=0\,. \tag{4.50}$$

From Equations (4.48) and (4.49) we get,

$$\rho = \varepsilon_0\nabla q\cdot\left(\frac{1-\frac{u^2}{c^2}}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)+\frac{1}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]\right)$$

$$+\varepsilon_0 q\nabla\cdot\left(\frac{1-\frac{u^2}{c^2}}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)+\frac{1}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]\right)$$

and with Equation (4.50) we get,

$$\rho=\nabla q\cdot\left(\frac{1-\dfrac{u^2}{c^2}}{4\pi r^2\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)+\frac{1}{4\pi c^2 r\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\dfrac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]\right)$$

and equivalently we get,

$$\rho=\frac{1-\dfrac{u^2}{c^2}}{4\pi r^2\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\nabla q\cdot\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)+\frac{1}{4\pi c^2 r\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\nabla q\cdot\left[\frac{\left(\dfrac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]$$

and with Equation (4.19) we get,

$$\rho=-\frac{1-\dfrac{u^2}{c^2}}{4\pi r^2\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{dq}{dw}\frac{1}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{\boldsymbol{\upsilon}}{c^2}\cdot\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)$$

$$-\frac{1}{4\pi c^2 r\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{dq}{dw}\frac{1}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{\boldsymbol{\upsilon}}{c^2}\cdot\left[\frac{\left(\dfrac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]$$

and equivalently we get,

$$\rho=-\frac{dq}{cdw}\frac{1-\dfrac{u^2}{c^2}}{4\pi r^2\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^4}\frac{\boldsymbol{\upsilon}}{c}\cdot\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)$$

$$-\frac{dq}{cdw}\frac{1}{4\pi c^2 r\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\boldsymbol{\upsilon}}{c}\cdot\left[\frac{\left(\dfrac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]$$

and equivalently we get,

$$\rho=-\frac{dq}{cdw}\frac{1-\dfrac{u^2}{c^2}}{4\pi r^2\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^4}\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)$$

$$-\frac{dq}{cdw}\frac{1}{4\pi c^2 r\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left[\frac{\left(\dfrac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)-\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right]$$

and equivalently we obtain,

$$\rho=-\frac{dq}{cdw}\frac{1-\frac{u^2}{c^2}}{4\pi r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}+0\,.$$

Therefore, the charge density at point A is given by the equation,

$$\rho=-\frac{dq}{cdw}\frac{1-\frac{u^2}{c^2}}{4\pi r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\,.$$

Furthermore, electromagnetic radiation does not contribute to the electric charge of spacetime.

□

We now prove the continuity equation at point A,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j}=0\,. \tag{4.51}$$

Proof. From Equation (4.47) we have,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j}=\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{\upsilon})$$

and equivalently we get,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j}=\frac{\partial\rho}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\rho+\rho\nabla\cdot\boldsymbol{\upsilon}$$

and with Equation (4.13) we get,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j}=\frac{\partial\rho}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\rho+\frac{2c}{r}\rho\,. \tag{4.52}$$

The charge $q=q(w)$ and the velocity $\mathbf{u}=\mathbf{u}(w)$ are defined at point E. Then, from the first of Equations (4.47) we get the density ρ in the form,

$$\rho=-\frac{dq}{cdw}\frac{1-\frac{u^2}{c^2}}{4\pi r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}=\frac{f(w)}{4\pi cr^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\,. \tag{4.53}$$

From Equations (4.52) and (4.53) we get,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j}=\frac{1}{4\pi cr^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\partial f(w)}{\partial t}-\frac{2f(w)}{4\pi cr^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\partial r}{\partial t}+\frac{3f(w)}{4\pi c^3r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^4}\frac{\partial(\mathbf{u}\cdot\boldsymbol{\upsilon})}{\partial t}$$

$$+\boldsymbol{\upsilon}\cdot\left(\frac{1}{4\pi cr^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\nabla f(w)-\frac{2f(w)}{4\pi cr^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\nabla r+\frac{3f(w)}{4\pi c^3r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^4}\nabla(\mathbf{u}\cdot\boldsymbol{\upsilon})\right)$$

$$+\frac{2f(w)}{4\pi r^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}$$

and equivalently we get,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j}=\frac{1}{4\pi cr^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\partial f(w)}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla f(w)\right)$$

$$-\frac{2f(w)}{4\pi cr^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\partial r}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla r\right)+\frac{3f(w)}{4\pi c^3r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^4}\left(\frac{\partial(\mathbf{u}\cdot\boldsymbol{\upsilon})}{\partial t}+\boldsymbol{\upsilon}\nabla(\mathbf{u}\cdot\boldsymbol{\upsilon})\right). \tag{4.54}$$

$$+\frac{2f(w)}{4\pi r^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}$$

From Equations (4.15) and (4.16) we get,

$$\frac{\partial f(w)}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla f(w)=0\,. \tag{4.55}$$

From Equations (4.6) and (4.8) we get,

$$\frac{\partial r}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla r=c\,. \tag{4.56}$$

From Equations (4.26) and (4.27) we get,

$$\frac{\partial(\mathbf{u}\cdot\boldsymbol{\upsilon})}{\partial t}+\boldsymbol{\upsilon}\nabla(\mathbf{u}\cdot\boldsymbol{\upsilon})=0\,. \tag{4.57}$$

From Equations (4.54) and (4.55), (4.56), (4.57) we get,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j}=-\frac{2cf(w)}{4\pi cr^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}+\frac{2f(w)}{4\pi r^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}$$

and equivalently we obtain,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j}=0\,.\ \square$$

The continuity equation expresses the conservation of charge distributed in spacetime. This conservation of charge is equivalently expressed through the equation,

$$q(t)=q(w)+\int_V \rho dV \ . \tag{4.58}$$

Considering the independence of velocity c (Einstein, 1905) from velocity $\mathbf{u}$ at point E, the volume V in Equation (4.58) is a sphere centered at point E and radius r. Equation (4.58) can also be proved independently of the continuity equation, by using the auxiliary Equations (4.6) – (4.29) (see [3], pp. 101-102). From Equation (4.58) it follows that two observers in points E and P, for the same particle (carrying the charge q) measure a value $q(t)$ for their own particle and $q(w)$ the value with which the particle of the other acts in theirs.

To understand the physical content of Equation (4.58), let us assume that the particle at point E is an electron carrying a charge q. In the time interval from w to t, $\delta t=t-w=\frac{r}{c}$, the increase in q is balanced by the charge of spacetime, which is distributed over the sphere with center E and radius r. The charge of spacetime is due to the electromagnetic field that accompanies the electron. If we assume that this field exists in every case, the increase of q is continuous. We now assume that the electron is stationary ($\mathbf{u}=\mathbf{0}$) at point E. The increase of q to over time is given by the equation (3.32). Therefore, the constant rest mass M_0' determines the increase in q over time.

The increase of rest mass to over time is given by equation (3.31). Considering that the electric charge of the electron contributes a small percentage to its total rest mass, we conclude that in Equations (3.31) and (3.32) is, $M_0' << M_0$. If we assume that in the same particle the constant b is the same for rest mass and electric charge, then then we conclude that the charge q of the electron increases at a much lower rate, compared to the rate of increase of its rest mass m_0. This conclusion is confirmed by the cosmological data, as we will see in section 6.

Self-variation potentials are compatible with Lorentz-Einstein transformations and, obviously, with the Self-variation principle. The Liénard-Wiechert potentials were published (1899) six years before the publication of Special Relativity (1905) by Einstein. After the formulation of Special Relativity it was shown that they are compatible with Lorentz-Einstein transformations. From Equations (4.30), (4.31) it is proven that the Liénard-Wiechert potentials are not compatible with the Self-Variation principle. For them to be compatible, the Self-Variation principle should have given the equation

$$\nabla q=-\frac{\partial q}{\partial t}\frac{\mathbf{u}}{c^2}$$

and not (4.20),

$$\nabla q=-\frac{\partial q}{\partial t}\frac{\mathbf{v}}{c^2}.$$

If we denote by L the set of equations that are compatible with the Lorentz-Einstein transformations and by S the set of equations that are compatible with then it is $S\subset L$. Regarding the mathematical formalism of the laws of physics, the Self-Variation Theory imposes additional constraints than those imposed by Special Relativity.

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5. Gravitational interaction

The Self-Variation Theory formulates gravity and electromagnetism with the same equations. These Equations concern the field created by the rest mass / electric charge of a particle. The central equation of the Theory relates three physical quantities, the rest mass or charge of the field source, the relative velocity of the field source to the observer, and the propagation velocity of the field relative to the observer. These velocities are directly related to the potential and intensity of the field measured by an observer. The first calculations give consistency of the Theory at the distance scales that we have observational data. Theory predicts increased stellar velocities on the outskirts of galaxies. It also predicts increased velocities of galaxies on the outskirts of galaxy clusters. In the context of the Theory, the equations we present in this section apply to all interactions, not just gravity and electromagnetism. Further investigation of the equations will yield the complete, precise prediction of the Theory.

Through a series of mathematical calculations described in detail in section 4, the Self-Variations principle necessarily involves a modification of the electromagnetic potential. For comparison the classical electromagnetic Liénard–Wiechert potentials are,

$$V_{LW} = \frac{q}{4\pi\varepsilon_0 r\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}$$

$$\mathbf{A}_{LW} = V\frac{\mathbf{u}}{c^2},$$

whereas the corresponding Self-Variation potentials are,

$$V = \frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2} + \frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})q}{4\pi\varepsilon_0 c^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2},$$

$$\mathbf{A} = V\frac{\boldsymbol{\upsilon}}{c^2}.$$

The difference lies in the potential of Equations (4.38), which are not present in Equations (4.31).

Gravitational potential

The Self-Variation potential for the gravitational interaction is derived from the above Equations of the electromagnetic Self-Variation potential by substituting the charge q, with the rest mass M, of the source of the gravitational field hence, $\frac{q}{4\pi\varepsilon_0} \to -GM$, where G, is the constant of gravity and by substituting the acceleration $\boldsymbol{\alpha}$ of the particle in the electromagnetic field, with the intensity $\mathbf{g}$ of the gravitational field, hence, $\boldsymbol{\alpha} \to \mathbf{g}$. Also notice that now $\boldsymbol{\upsilon}$, represents the speed of propagation of the gravitational field, hence we must substitute the speed of light in vacuum c, with the speed of propagation of the gravitational field υ, hence, $c \to \upsilon$. These substitutions lead to the corresponding gravitational potentials of the Self-Variation,

$$V = -\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} - \frac{GM}{\upsilon^3}\frac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}, \tag{5.1}$$

$$\mathbf{A} = V\frac{\boldsymbol{\upsilon}}{\upsilon^2}.$$

where $\mathbf{u}$, is the velocity of the rest mass M relative to the observer, and r, is the distance from the rest mass M. Deriving the gravitational potentials in this way, implies that there is a gravitational analog to the magnetic field $\mathbf{B}$, and has units s^{-1} (see Equations (4.30) – (4.33)). Notice that in the limit case where the speed of propagation of the gravitational field approaches infinity, $\upsilon \to \infty$, we get the limit potential

$$V = -\frac{GM}{r},$$

which is no other than the one of classical mechanics which assumed instant action of gravity at distance r.

Like the corresponding Equation for electromagnetism, Equation (5.1) refers to the gravitational field created by the rest mass of a particle. By taking into account the distribution of particles in spacetime we get the gravitational field on a macroscopic scale.

Potential, propagation speed and intensity of the gravitational field caused by a single rest mass M

The differential equations we get from Equation (5.1) depend on the direction of the vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$. In this section we study in detail one of these cases. Considering that the vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have opposite directions, $\boldsymbol{\upsilon} = \upsilon \frac{\mathbf{r}}{r}$ and $\mathbf{g} = -g \frac{\mathbf{r}}{r}$, $\boldsymbol{\upsilon} \cdot \mathbf{g} = -\upsilon g$, where $\upsilon = \|\boldsymbol{\upsilon}\|$ and $g = \|\mathbf{g}\|$. Then

$$\frac{d\boldsymbol{\upsilon}}{dt} = \frac{d\upsilon}{dt}\frac{\mathbf{r}}{r} + \upsilon \frac{d}{dt}\left(\frac{\mathbf{r}}{r}\right) = \mathbf{g} = -g\frac{\mathbf{r}}{r}. \tag{5.2}$$

Then from Equation (5.1) we have

$$V = -\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} + \frac{GM}{\upsilon^2}\frac{g}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}. \tag{5.3}$$

The gravitational field intensity $\mathbf{g}(r)$ is given by

$$\mathbf{g}(r) = -\nabla V(r) = -\frac{dV}{dr}\frac{\mathbf{r}}{r}. \tag{5.4}$$

In Equation (5.2) (see, section 4) we have that, $dr = -cdw$. However using the symbols of the current section this Equation is written as, $dr = -\upsilon dt$. From Equation (5.2) we get $\frac{d\upsilon}{dt} = -g$. Combining these Equations we get

$$\upsilon\frac{d\upsilon}{dr} = g = \frac{dV}{dr}, \tag{5.5}$$

where with t, we have denoted the time of the observer. From Equation (5.5) we have,

$$\upsilon^2 = \sigma + 2V \tag{5.6}$$

where σ is a constant. In this section we study the case $\sigma \neq 0$.

From Equations (5.3) and (5.5) we get,

$$V = -\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} + \frac{1}{\upsilon^2}\frac{GM}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{dV}{dr}. \tag{5.7}$$

Then, from Equations (5.6) and (5.7) we obtain the differential equation for the potential,

$$V = -\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} + \frac{1}{\sigma+2V}\frac{GM}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{dV}{dr} \tag{5.8}$$

and the differential equation for the speed of propagation of the gravitational field,

$$\upsilon^2 - \sigma = -\frac{2GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} + \frac{1}{\upsilon^2}\frac{GM}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{d\upsilon^2}{dr}. \tag{5.9}$$

Equation (5.9) relates three physical quantities, the rest mass M of the field source, the velocity $\mathbf{u}$ of the field source relative to the observer, and the speed of propagation $\boldsymbol{\upsilon}$ of the field relative to the observer. The fact that this equation relates only these three physical quantities makes it fundamental to the gravitational interaction. For the observer, the properties of spacetime depend on the rest mass M .

In the electromagnetic interaction the Self-Variation Theory predicts two independent pairs of potentials. One gives the electromagnetic field of the moving electric charge and depends on the velocity $\mathbf{u}$ of the charge (see Equations (4.36)),

$$V_u = \frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}$$

$$\mathbf{A}_u = V_u \frac{\boldsymbol{\upsilon}}{c^2}.$$

The other gives the electromagnetic radiation emitted by the electric charge and depends on the acceleration $\mathbf{a}$ of the charge (see Equations (4.38)),

$$V_\alpha = \frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})q}{4\pi\varepsilon_0 c^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}$$

$$\mathbf{A}_a = V_a \frac{\boldsymbol{\upsilon}}{c^2}.$$

As a consequence of the substitution $\mathbf{a}\to\mathbf{g}$, this separation cannot be made in the gravitational interaction. The intensity of the gravitational field, the acceleration of gravity $\mathbf{g}$ is related to both the

$$-\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}$$

term and the

$$-\frac{GM}{\upsilon^3}\frac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}$$

term in the second part of Equation (5.1). In addition to Equation (45.4), the potential and the gravitational field strength are also related to each other through Equation (5.1).

Gravitational interaction of two bodies

We study the case where a body of rest mass m moves in the gravitational field of a stationary body of rest mass $M >> m$.

In polar coordinates (r,θ), the orbits $r=r(\theta)$ of the body of rest mass m is given by the solution of the system of equations,

$$L = mr^2\frac{d\theta}{dt} = constant \tag{5.10}$$

$$\ddot{r} - r\left(\dot{\theta}\right)^2 = -g(r) \tag{5.11}$$

and

$mV(r) + K = E = constant$. (5.12)

In these Equations t is the time of the observer, L and K the angular momentum and the kinetic energy of the body of rest mass m, $\ddot{r} = \frac{d^2 r}{dt^2}$, $\dot{\theta} = \frac{d\theta}{dt}$ and E the mechanical energy of the system of the two bodies.

From the solution of the differential Equation (5.8) we get the potential $V(r)$. Then, from Equation (5.5) we get the intensity $g(r)$ of the field. Alternatively, from the solution of the differential Equation (5.9) we obtain the speed of propagation of the field $\upsilon(r)$. Then, from Equation (5.6) we get the potential $V(r)$. The velocity $u = u(r)$ is obtained from Equation (5.12).

The solutions given by differential equations (5.8) and (5.9) depend on the inner product $\mathbf{u} \cdot \frac{\boldsymbol{\upsilon}}{\upsilon^2}$.

If $\mathbf{u} \cdot \frac{\boldsymbol{\upsilon}}{\upsilon^2} = 0$ we get the simplest possible solutions.

Circular orbits

The inner product $\mathbf{u} \cdot \boldsymbol{\upsilon}$ in the differential Equations (5.8) and (5.9) has the consequence that complex calculations are required for their solution. These calculations are simplified in the case of circular orbits. The case of circular orbits is a first approach to the conclusions drawn from the Equations we present in this section.

For the body of rest mass m we have $\frac{d\boldsymbol{\upsilon}}{dt} = \frac{d\mathbf{u}}{dt} = \mathbf{g}$. In the case of circular orbit it is

$$\mathbf{u} \cdot \mathbf{g} = \mathbf{0} \tag{5.13}$$

and taking into consideration that the vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have opposite directions we have

$$\mathbf{u} \cdot \boldsymbol{\upsilon} = \mathbf{0} . \tag{5.14}$$

Now we have

$$\frac{du^2}{dt} = \frac{d\mathbf{u}^2}{dt} = 2\mathbf{u}\frac{d\mathbf{u}}{dt} = 2\mathbf{u} \cdot \mathbf{g}$$

and with Equation (5.13) we get

$$\frac{du^2}{dt} = 0 , \tag{5.15}$$

where $u = \|\mathbf{u}\|$. Therefore the velocity u is constant in the circular orbit.

From Equations (5.9) and (5.14) we have

$$\upsilon^2 - \sigma = -\frac{2GM}{r}\left(1 - \frac{u^2}{\upsilon^2}\right) + \frac{GM}{\upsilon^2}\frac{d\upsilon^2}{dr} . \tag{5.16}$$

Let

$$x = \frac{\sigma}{GM} r \tag{5.17}$$

and let

$$f(x) = \frac{\upsilon^2(x)}{c^2} . \tag{5.18}$$

From Equations (5.16), (5.17) and (5.18) we get the following differential equation,

$$\frac{df}{dx}-\frac{c^2}{\sigma}f^2+f-\frac{2}{x}\left(f-\frac{u^2}{c^2}\right)=0. \tag{5.19}$$

In Equation (5.19) it is

$$0\le\frac{u^2}{c^2}<1. \tag{5.20}$$

The solution we get from the differential equation (5.19) depends on the value of the quotients $\frac{u^2}{c^2}$ and $\frac{c^2}{\sigma}$. Solving differential equation (5.19) we get the speed $\upsilon=\upsilon(x)$ from Equation (5.18).

From Equations (5.6) and (5.18) we get the field potential as a function of x,

$$V(x)=-\frac{\sigma}{2}+\frac{c^2}{2}f(x). \tag{5.21}$$

From equations (5.21) and Transformation (5.17) we get the field potential as a function of r.

From Equation (5.4) we have

$$\mathbf{g}(r)=-\frac{dV}{dr}\frac{\mathbf{r}}{r}=-\frac{dV}{dx}\frac{dx}{dr}\frac{\mathbf{r}}{r}$$

and with the Transformation (5.17) we have

$$\mathbf{g}(x)=-\frac{\sigma}{GM}\frac{dV}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (5.6) we have

$$\mathbf{g}(x)=-\frac{\sigma}{2GM}\frac{d\upsilon^2}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (5.18) we have

$$\mathbf{g}(x)=-\frac{\sigma c^2}{2GM}\frac{df}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (5.19) we obtain the intensity $\mathbf{g}$ of the field as a function of x,

$$\mathbf{g}(x)=-\frac{\sigma c^2}{2GM}\left(\frac{c^2}{\sigma}f^2(x)-f(x)+\frac{2}{x}\left(f(x)-\frac{u^2}{c^2}\right)\right)\frac{\mathbf{r}}{r}. \tag{5.22}$$

The functions $\upsilon=\upsilon(r)$, $V=V(r)$ and $\mathbf{g}=\mathbf{g}(r)$ are obtained from Equations (5.18), (5.21) and (5.22) through the Transformation (5.17).

As a consequence of Equation (5.19) the velocity u and the gravitational field are mutually dependent. This is a clear conclusion of the Theory, which has its origin in Equation (5.1). The solution of the differential Equation (5.19) gives pairs $(f(x),u)$. Included in these solutions are velocities u that correspond to the increased velocities of stars in the outskirts of galaxies and the increased velocities of galaxies in the outskirts of galaxy clusters.

Potential, propagation speed, and intensity of the field induced by a stationary rest mass relative to an observer

If the observer and the source of the field M are stationary between them, $\mathbf{u}=\mathbf{0}$, then from Equations (5.8) and (5.9) we have

$$V=-\frac{GM}{r}+\frac{GM}{\sigma+2V}\frac{dV}{dr} \tag{5.23}$$

and

$$\upsilon^2 - \sigma = -\frac{2GM}{r} + \frac{GM}{\upsilon^2}\frac{d\upsilon^2}{dr}. \tag{5.24}$$

From Equation (5.24), again applying Transformations (5.17) and (5.18) we get,

$$\frac{df}{dx} - \frac{c^2}{\sigma} f^2 + f - \frac{2f}{x} = 0. \tag{5.25}$$

Solving (5.25) for f, we have

$$f(x) = \frac{\sigma}{c^2}\frac{x^2}{ke^x + x^2 + 2x + 2} \tag{5.26}$$

where k, is the integration constant. Then from (5.18), (5.26) we have

$$\upsilon^2(x) = \sigma\frac{x^2}{ke^x + x^2 + 2x + 2}. \tag{5.27}$$

Finally applying the Transformation (5.17) to (5.27) we get the speed of propagation of the gravitational field, as derived from the Self-Variation gravitational potential, with respect to r,

$$\upsilon^2(r) = \sigma\frac{a^2r^2}{ke^{ax} + a^2r^2 + 2ar + 2}, \tag{5.28}$$

where $a = \dfrac{\sigma}{GM}$. (5.29)

From Equations (5.6) and (5.27) we obtain the gravitational potential with respect to x,

$$V(x) = -\sigma\frac{ke^x + 2x + 2}{2\left(ke^x + x^2 + 2x + 2\right)}. \tag{5.30}$$

Then from Equations (5.30) and Transformation (5.17) we obtain the gravitational potential with respect to r,

$$V(r) = -\sigma\frac{ke^{ar} + 2ar + 2}{2\left(ke^{ar} + a^2r^2 + 2ar + 2\right)}. \tag{5.31}$$

The gravitational field intensity $\mathbf{g}$ is calculated as follows. From Equation (5.4) and Transformation (5.17) we get

$$\mathbf{g}(x) = -\frac{\sigma}{GM}\frac{dV(x)}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (5.6) we get,

$$\mathbf{g}(x) = -\frac{\sigma}{2GM}\frac{d\upsilon^2(x)}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (5.18) we get,

$$\mathbf{g}(x) = -\frac{\sigma c^2}{2GM}\frac{df(x)}{dx}\frac{\mathbf{r}}{r}$$

and with Equations (5.25) we get,

$$\mathbf{g}(x) = -\frac{\sigma c^2}{2GM}\left(\frac{c^2}{\sigma} f^2(x) - f(x) + \frac{2}{x} f(x)\right)\frac{\mathbf{r}}{r}$$

and with (5.26) we obtain,

$$\mathbf{g}(x) = -\frac{\sigma c^2}{2GM}\left(\frac{c^2}{\sigma} f^2(x) - f(x) + \frac{2}{x} f(x)\right)\frac{\mathbf{r}}{r} = -\frac{\sigma^2}{2GM}\frac{2kxe^x - kx^2e^x + 2x^2 + 4x}{\left(ke^x + x^2 + 2x + 2\right)^2}\frac{\mathbf{r}}{r}\,. \qquad (5.32)$$

The function $f = f(x)$ is given by Equation (5.26). From Transformation (5.17) and Equations (5.26) and (5.32) we get the field intensity $\mathbf{g} = \mathbf{g}(r)$ as a function of r.

We made the substitution $\frac{q}{4\pi\varepsilon_0} \to -GM$ (and not $\frac{q}{4\pi\varepsilon_0} \to GM$) in order for the gravitational interaction to be attractive. However, this is not achieved. From Equation (5.32) it follows that there are values of k and x for which gravity is repulsive. Consequently, the general case of the gravitational interaction is obtained by substituting $\frac{q}{4\pi\varepsilon_0} \to \pm GM$. Equation (5.1) is common to electromagnetism and gravity. Through the transformations

$\frac{q}{4\pi\varepsilon_0} \leftrightarrow \pm GM$, $\mathbf{a} \leftrightarrow \mathbf{g}$ and $c \leftrightarrow \upsilon$

we pass from one interaction to another.

For $\mathbf{u} = \mathbf{0}$ and $\frac{q}{4\pi\varepsilon_0} \to +GM$ the equivalents of Equations (5.24), (5.25) and (5.26) are,

$$\upsilon^2 - \sigma = \frac{2GM}{r} - \frac{GM}{\upsilon^2}\frac{d\upsilon^2}{dr},$$

$$\frac{df}{dx} + \frac{c^2}{\sigma} f^2 - f - \frac{2f}{x} = 0,$$

$$f(x) = \frac{\sigma}{c^2}\frac{x^2}{ke^{-x} + x^2 - 2x + 2}.$$

Then we get,

$$\upsilon^2(x) = \sigma\frac{x^2}{ke^{-x} + x^2 - 2x + 2}, \qquad (5.33)$$

$$V(x) = -\sigma\frac{ke^{-x} - 2x + 2}{2\left(ke^{-x} + x^2 - 2x + 2\right)} \qquad (5.34)$$

and

$$\mathbf{g}(x) = -\frac{\sigma^2}{2GM}\frac{-2kxe^{-x} - kx^2e^{-x} + 2x^2 - 4x}{\left(ke^{-x} + x^2 - 2x + 2\right)^2}\frac{\mathbf{r}}{r}\,. \qquad (5.35)$$

By comparing the triads of Equations ((5.27), (5.30), (5.32)) and ((5.33), (5.34), (5.35)) the similarities and differences of the $\frac{q}{4\pi\varepsilon_0} \to -GM$ and $\frac{q}{4\pi\varepsilon_0} \to +GM$ substitutions emerge.

As a consequence of the equality $-\frac{GM}{x} = \frac{GM}{-x}$, the triads of Equations ((5.27), (45.30), (5.32)) and ((5.33), (5.34), (5.35)) are related through the $x \leftrightarrow -x$ transformation. Through this transformation one triad arises from the other. From Equation (5.1) it follows that for the two substitutions, this symmetry also exists in the case $\mathbf{u} \neq \mathbf{0}$. Therefore, for each solution $\upsilon(x)$,

$V(x)$, $g(x)$ of the Equations of the field we also get its "complementary" solution through the transformation $x \to -x$.

The case $\mathbf{u}=\mathbf{0}$ is the simplest. In addition, it gives exact solutions of υ, V and $\mathbf{g}$. Therefore, the initial investigation of the equations of this section can be done for all possible substitutions with $\mathbf{u}=\mathbf{0}$.

As a consequence of the equivalence

$$\mathbf{u}\cdot\boldsymbol{\upsilon}=0 \Leftrightarrow \begin{cases} \mathbf{u}=\mathbf{0} \\ or \\ \mathbf{u}\perp\boldsymbol{\upsilon} \end{cases},$$

the cases $\mathbf{u}=\mathbf{0}$ and $\mathbf{u}\perp\boldsymbol{\upsilon}$ (circular orbits) are given by Equations (5.19) and (5.18), (5.21), (5.22),

$$\frac{df}{dx}-\frac{c^2}{\sigma}f^2+f-\frac{2}{x}\left(f-\frac{u^2}{c^2}\right)=0,$$

$$\upsilon^2(x)=c^2 f(x),$$

$$V(x)=-\frac{\sigma}{2}+\frac{c^2}{2}f(x),$$

$$\mathbf{g}(x)=-\frac{\sigma c^2}{2GM}\left(\frac{c^2}{\sigma}f^2(x)-f(x)+\frac{2}{x}\left(f(x)-\frac{u^2}{c^2}\right)\right)\frac{\mathbf{r}}{r}.$$

Then, through the transformation $x \to -x$ we obtain the complementary solution.

Comparison of complementary solutions

We now make a first comparison of the complementary solutions.

From Equation (5.32) we have that gravity is repulsive ($g(x)<0$) for the values of k and x for which,

$$2kx-kx^2+2x^2+4x<0.$$

From Equations (5.27), (5.30) and (5.32) we get the limit values,

$\upsilon(0)=0$ and $\lim_{x\to\infty}\upsilon(x)=0$,

$V(0)=-\frac{\sigma}{2}$ and $\lim_{x\to\infty}V(x)=-\frac{\sigma}{2}$,

$g(0)=0$ and $\lim_{x\to\infty}g(x)=0$.

From Equation (5.35) we have that gravity is repulsive ($g(x)<0$) for the values of k and x for which,

$$-2kx-kx^2+2x^2-4x<0.$$

From Equations (5.33), (5.34) and (5.35) we get the limit values,

$\upsilon(0)=0$ and $\lim_{x\to\infty}\upsilon(x)=\sqrt{\sigma}$ ($\sigma>0$),

$V(0)=-\frac{\sigma}{2}$ and $\lim_{x\to\infty}V(x)=-\frac{\sigma}{2}$,

$g(0)=0$ and $\lim_{x\to\infty}g(x)=0$.

These limit values do not depend on the rest mass M.

The general equations for the case $\boldsymbol{\upsilon} = \upsilon \frac{\mathbf{r}}{r}$ and $\mathbf{g} = -g\frac{\mathbf{r}}{r}$

From Equation (5.9) and transformation (5.17) we get the following equation,

$$\left(\frac{\upsilon^2}{\sigma}-1\right)\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2 x = -2+2\frac{u^2}{\upsilon^2}+\frac{1}{\upsilon^2}\frac{xd\upsilon^2}{dx}. \tag{5.36}$$

From Equation (5.36) and the transformation $x \to -x$ we get the complementary equation

$$-\left(\frac{\upsilon^2}{\sigma}-1\right)\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2 x = -2+2\frac{u^2}{\upsilon^2}+\frac{1}{\upsilon^2}\frac{xd\upsilon^2}{dx}. \tag{5.37}$$

For the case we studied ($\boldsymbol{\upsilon} = \upsilon \frac{\mathbf{r}}{r}$ and $\mathbf{g} = -g\frac{\mathbf{r}}{r}$), (5.36) and (5.37) are the general Equations. Corresponding equations are obtained for all possible combinations in the directions of vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$. The resulting solutions of these differential equations depend on the inner product $\mathbf{u}\cdot\frac{\boldsymbol{\upsilon}}{\upsilon^2}$.

In the differential equations (5.16) and (5.24) the unknown function is the υ^2. Thus we made the transformation (5.18). In the differential equations (5.36) and (5.37) the unknown function is the υ. Thus we make the transformation

$$\upsilon(x) = c\Phi(x) \tag{5.38}$$

and we get,

$$\left(\frac{c^2}{\sigma}\Phi^2-1\right)\left(1-\frac{u\cos\theta}{c}\frac{1}{\Phi}\right)^2 x = -2+2\frac{u^2}{c^2}\frac{1}{\Phi^2}+\frac{2}{\Phi}\frac{xd\Phi}{dx}, \tag{5.39}$$

$$-\left(\frac{c^2}{\sigma}\Phi^2-1\right)\left(1-\frac{u\cos\theta}{c}\frac{1}{\Phi}\right)^2 x = -2+2\frac{u^2}{c^2}\frac{1}{\Phi^2}+\frac{2}{\Phi}\frac{xd\Phi}{dx}, \tag{5.40}$$

where θ is the angle of the vectors $\mathbf{u}$ and $\boldsymbol{\upsilon}$.

After finding the function $\Phi(x)$, the velocity $\upsilon(x)$ is given by equation (5.38). The field potential $V(x)$ is given by Equation (5.6). The field strength is given by equation (5.4) written in the form $\mathbf{g}(x) = -\frac{\sigma}{GM}\frac{dV}{dx}\frac{\mathbf{r}}{r}$.

The differential Equations (5.38), (5.39) give triads of solutions $(\Phi(x),u,\theta)$. Included in these solutions are velocities u that correspond to the increased velocities of stars in the outskirts of galaxies and the increased velocities of galaxies in the outskirts of galaxy clusters. The theoretical prediction coincides with the observational data. For given velocities u, the solutions given by Equations (5.39) and (5.40) depend on the quotient $\frac{c^2}{\sigma}$. For a given quotient $\frac{c^2}{\sigma}$, Equations (5.39) and (5.40) give solutions for specific velocities u. There are pairs $\left(\frac{c^2}{\sigma},u\right)$ for which equations (5.39) and (5.40) give realistic solutions. A criterion for whether a solution is realistic or not arises from the values that the speed $\upsilon(x)$ takes, as given by equation (5.38). As the propagation speed of the field, it is not necessarily less than c. However, if we assume that the carrier of gravity is a particle, then we have $\upsilon(x) \leq c$. In any case, we can draw conclusions about the field from the possible values of its propagation speed.

In a first approach, the value of the quotient $\frac{c^2}{\sigma}$ can be estimated from the already known observational data (see, [1] – [3] and [6] – [15]). For such a measurement, at the macrocosmic scale the distribution of particles in spacetime must be taken into account and not only their total rest mass. A more accurate prediction of the Theory for the velocities of stars in the outskirts of galaxies and the velocities of galaxies in the outskirts of galaxy clusters can be made using appropriate mathematical models on the respective distance scales. For the functions f and Φ we have $f(x) = \Phi^2(x)$, as obtained from Equations (5.18) and (5.38).

The potentials V, $\mathbf{A}$

The complete investigation of the Theory of Gravitation is done through Equation (5.1) and the pair of potentials $(V, \mathbf{A})$, where

$$\mathbf{A} = V\frac{\boldsymbol{\upsilon}}{\upsilon}.$$

These potentials relate the gravitational field to spacetime, through the equations,

$$\mathbf{g} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t},$$

$$\mathbf{B} = \nabla \times \mathbf{A},$$

where $\mathbf{B}$ is a gravitational proportional to the magnetic field and has units s^{-1} (see Equations (4.30) – (4.33)).

As a consequence of Self-Variation, at time t, the rest mass / electric charge located at point P acts on point A with the value it had at another point E (see [5], Fig. 1). The intensity of the gravitational field given by the potentials $(V, \mathbf{A})$ is the same whether we consider the rest mass M to be constant or consider it to vary according to the Principle of Self-Variation (see, section 4). This property of the potentials of the Self-Variation Theory allows us to solve the differential Equations of the gravitational field by considering the rest mass M as constant.

We have presented the Equations for Gravity as predicted by the Self-Variation Theory. The substitution $\boldsymbol{\alpha} \to \mathbf{g}$, by which we get the Gravitational potential of Self-Variation from the corresponding Electromagnetic potential, is an idea belonging to Einstein. Without this substitution the Gravitational field of Self-Variation cannot arise. The Equations we have presented include Einstein's proposal for the equivalence of acceleration and gravity.

Through the substitutions $\frac{q}{4\pi\varepsilon_0} \leftrightarrow \pm GM$, $\mathbf{a} \leftrightarrow \mathbf{g}$, $c \leftrightarrow \upsilon$ Equation (5.1) is common to gravity and electromagnetism. By doing all the combinations we get the possible equations of the gravitational interaction. A common characteristic of the resulting cases is that gravity is attractive or repulsive as the distance from the rest mass changes. The correlation of attraction / repulsion with the distance from the rest mass is a direct consequence of the Equations of this section.

If in Equation (5.1) we replace the rest mass with the "Self-Variating Charge Q" we get the "Unified Self-Variation Interaction". In gravitational interaction Self-Variating charge is the rest mass, $Q=M$. In electromagnetic interaction the Self-Variating charge is the electric charge, $Q=q$. An issue for investigation is the possible physical quantities Q.

As a consequence of the Self-Variation of rest masses, spacetime contains negative energy (see, Equations (4.47) for electric charge). As distance scales increase spacetime contains a large number of particles distributed over a negative energy background. This background of negative energy arises from the Self-Variation of the remaining masses of the material particles of the universe. At the macroscopic scale the gravitational interaction depends on the distribution of particles and negative energy. On the cosmological scale this combination makes the universe flat (see, [4]). A mathematical model for the Theory's predictions at the macrocosmic and cosmological scales is necessary.

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6. The cosmological data as a consequence of the Self-Variation of the material particles

As a consequence of Self-Variation, in our cosmological-scale observations, the rest mass and electric charge (generally the self-varying charge) of a particle have a smaller value than the corresponding values of the same particle in the laboratory, on earth. This fact has consequences for all physical phenomena occurring in distant astronomical objects, which depend on rest mass and electrical charge. These consequences are recorded in the cosmological data.

The reduced values of rest mass in the past time result in the weakening of gravity, compared to its strength on earth and nearby galaxies. This attenuation is extremely large at cosmological-scale distances. The equations given by the Self-Variation Theory predict that gravity cannot cause the universe to collapse or expand. The consequences of gravity are limited to other distance scales, much smaller than the cosmological one.

The Standard Cosmological Model is based on General Relativity. However, it has repeatedly had to introduce additional assumptions in order to bring the Model into agreement with the observational data. From the hypothesis of Dark Matter (see, [21]) and inflation (see [5]), to the more recent hypothesis of Dark Energy (see, [13], [18]). The Standard Cosmological Model is inconsistent with recent measurements from the early twenty-first century to the present (see [1], [3], [4], [10], [15]). There is no hypothesis that could bring the Standard Cosmological Model into agreement with the two measured values for the Hubble constant (see, [16], [17]).

The Internal Symmetry Theorem justifies the so far known cosmological data, in a flat and static universe. The increased velocities of stars on the outskirts of galaxies, and the increased velocities of galaxies on the outskirts of galaxy clusters are justified by the conclusions of section 5 on gravity.

Rest mass and electric charge on the cosmological scale

In a flat and static universe, from Equation (3.31) for $b, K \in \mathbb{R}$, $b > 0$ we get the following equation for the increase in rest mass to over time t,

$$m_0 = m_0(t) = \frac{M_0}{1 + K\exp\left(\frac{bM_0c^2}{\hbar}t\right)}. \tag{6.1}$$

From equations (3.19) and (3.20), requiring

$$m_0 E_0 = \frac{\Phi M_0^2 c^2}{(1+\Phi)^2} < 0$$

we get $\Phi < 0$ and equivalently we get,

$$K < 0. \tag{6.2}$$

In Equation (6.1) the constant K is negative.

We consider an astronomical object at distance r from Earth. The emission of the electromagnetic spectrum of the far-distant astronomical object we observe "now" on Earth has taken place before a time interval $\delta t = \frac{r}{c}$. From equation (6.1) we have that the rest mass $m_0(r)$ on the distant astronomical object at the moment of emission was,

$$m_0(r) = \frac{M_0}{1 + K\exp\left(\frac{bM_0c^2}{\hbar}\left(t - \frac{r}{c}\right)\right)}. \tag{6.3}$$

From Equations (6.1) and (6.3) we obtain,

$$m_0(r) = m_0 \frac{1 + K\exp\left(\frac{bM_0c^2}{\hbar}t\right)}{1 + K\exp\left(\frac{bM_0c^2}{\hbar}t\right)\exp\left(-\frac{bM_0c}{\hbar}r\right)}. \tag{6.4}$$

Different particles have different rest mass M_0. Furthermore, in different particles the Self-Variation can evolve in a different way, which can be expressed by a different value of the constant b. Thus in Equation (6.4) we denote,

$$\frac{bM_0c^2}{\hbar} = \frac{b_k M_0 c^2}{\hbar} = k_p > 0. \tag{6.5}$$

With the index 'p' we denote the particle to which the constant k_p refers. With this symbolism, Equation (6.4) is written in the form,

$$m_0(r) = m_0 \frac{1 + Ke^{k_p t}}{1 + Ke^{k_p t} e^{-k_p \frac{r}{c}}}. \tag{6.6}$$

We now denote by A the time-dependent function,

$$A = A(t) = -\Phi(t) = -Ke^{k_p t} > 0. \tag{6.7}$$

From Equations (6.6) and (6.7) we obtain,

$$m_0(r) = m_0 \frac{1 - A}{1 - Ae^{-k_p \frac{r}{c}}}. \tag{6.8}$$

From Equation (6.7) we obtain,

$$\frac{dA}{dt} = \dot{A} = k_p A > 0\,. \tag{6.9}$$

Similarly, starting from Equation (3.32) we get the following equations,

$$\frac{bM_0'c^2}{\hbar} = \frac{b_k'M_0c^2}{\hbar} = k_p' > 0\,, \tag{6.10}$$

$$q(r) = q\frac{1-B}{1-Be^{-k_p'\frac{r}{c}}}\,, \tag{6.11}$$

$$B = -K'e^{k_p't} > 0\,, \tag{6.12}$$

$$\frac{dB}{dt} = \dot{B} = k_p'B > 0\,. \tag{6.13}$$

The replacement of the function $\Phi(t)$ with the functions $A(t)$ and $B(t)$ was done to preserve the notation of the cosmological scale equations as they are in already published articles (see, [11]).

The redshift of the distant astronomical objects

The fine structure constant a is defined as

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0 c\hbar}\,, \tag{6.14}$$

where q_e is the electric charge of the electron. From Equation (6.11) if $q = q_e$ the charge of electron we obtain,

$$\alpha(r) = \alpha\left(\frac{1-B}{1-Be^{-k_e'\frac{r}{c}}}\right)^2. \tag{6.15}$$

The energy of the electron in the atom is

$$E_n = -\frac{1}{n^2}\frac{Z^2K^2m_eq_e^4}{2\hbar^2}$$

$$n = 1,2,3,\ldots$$

where m_e is the rest mass and q_e is the electric charge of the electron, Z is the atomic number and K is Coulomb's constant (see [2], [19]). The wavelength λ inversely proportional to the photon energy E, $\lambda = \frac{2c\pi\hbar}{E}$ (see, [14]). Therefore, the wave length λ of the linear spectrum is inversely proportional to the factor $m_eq_e^4$. If we denote by λ_0 the wavelength of a photon emitted by an atom "now" on Earth, in the laboratory and by λ the same wavelength of the same atom received "now" on Earth from the far-distant astronomical object, the following relation holds,

$$\frac{\lambda}{\lambda_0} = \frac{m_eq_e^4}{m_e(r)q_e^4(r)}$$

and from equations (6.8) and (6.11) if $q = q_e$ the charge and $m_0 = m_e$ the rest mass of electron we obtain,

$$\frac{\lambda}{\lambda_0} = \frac{1 - Ae^{-k_e\frac{r}{c}}}{1-A}\left(\frac{1-Be^{-k_e'\frac{r}{c}}}{1-B}\right)^4 . \tag{6.16}$$

From equation (6.16) we have for the redshift z,

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1$$

of the astronomical object that

$$z = \frac{1 - Ae^{-k_e\frac{r}{c}}}{1-A}\left(\frac{1-Be^{-k_e'\frac{r}{c}}}{1-B}\right)^4 - 1. \tag{6.17}$$

From Equations (6.15) and (6.17) we get,

$$z = \frac{1 - Ae^{-k_e\frac{r}{c}}}{1-A}\left(\frac{a}{a(r)}\right)^2 - 1. \tag{6.18}$$

Considering that the electric charge of the electron increases at a much slower rate than its rest mass (see comments at the end of section 3 and [8], [20]), from equation (6.18) we get,

$$z = \frac{1 - Ae^{-k_e\frac{r}{c}}}{1-A} - 1$$

and equivalently we obtain,

$$z = \frac{A}{1-A}\left(1 - e^{-k_e\frac{r}{c}}\right). \tag{6.19}$$

For small distances r, from Equation (6.19) we get,

$$z = \frac{A}{1-A}\left(1 - 1 + k_e\frac{r}{c}\right)$$

and equivalently we get,

$$z = \frac{k_e A}{c(1-A)} r$$

and comparing this Equation with Hubble's law $cz = Hr$ (see, [7]) we get,

$$H = H_e = \frac{k_e A}{1-A}, \tag{6.20}$$

where $H = H_e$ is the Hubble constant for the linear spectrum of atoms. If the electromagnetic radiation we measure depends on ′′heavy′′ particles, such as the proton and the neutron, Equation (20) becomes,

$$H_p = \frac{k_p A}{1-A}. \tag{6.21}$$

The Self-Variation Theory predicts the measurement of at least two values of the Hubble constant (see, [16], [17]). On the cosmological scale, the Self-Variation of the electron and the heavy particles correspond to different values of Hubble's constant.

Taking into account that $H > 0$, $A > 0$ and $k_e > 0$, from Equation (6.20) we get,

$A < 1$.

From Equation (6.10) we get,

$$z < \frac{A}{1-A}$$

and taking into account that $A > 0$ we obtain,

$$\frac{z}{1+z} < A < 1. \tag{6.22}$$

From this inequality and considering the possible values of redshift we conclude that,

$$A \to 1^- . \tag{6.23}$$

From equations (6.21) and (6.9) we have,

$$\frac{dH}{dt} = \dot{H} = \frac{H^2}{A}. \tag{6.24}$$

From this Equation and relation (6.23) we conclude that Hubble's constant increases slightly with time.

The rest mass of the electron as a function of redshift

From equations (6.8) and (6.19) we obtain,

$$m_e(z) = \frac{m_e}{1+z}. \tag{6.25}$$

The redshift is measured with great precision. Therefore, Equation (6.25) gives the relation between $m_e(z) = m_e(r)$ and m_e very precisely.

A large set of physical phenomena and mechanisms depend on the rest mass of the electron. Therefore it is important to know precisely its value in distant astronomical objects. This accuracy is given by Equation (6.25).

The diminished energies of distant astronomical objects

From Equation (6.25) we get,

$$E(z) = \frac{E}{1+z} \tag{6.26}$$

for the energy $E = m_e c^2$ of the electron.

Taking into account the two measured values of the Hubble constant (see [16], [17]) we conclude that equation (6.26) is also approximately valid for heavy particles. This equation predicts that the energy resulting from hydrogen fusion and nuclear reactions is reduced in distant astronomical objects.

The Thomson and Klein-Nishina scattering coefficients as a function of redshift of the distant astronomical objects

The laboratory value of the Thomson scattering coefficient is given by equation,

$$\sigma_T = \frac{8\pi}{3}\frac{q_e^2}{m_e^2 c^4}, \tag{6.27}$$

where m_e the rest mass and q_e the electric charge of the electron. Thus we have,

$$\frac{\sigma_T(z)}{\sigma_T} = \left(\frac{m_e}{m_e(z)}\right)^2 \left(\frac{\alpha}{\alpha(z)}\right)^2$$

and taking into account the very slow rate of change of the fine structure constant ($\alpha(z) \approx \alpha$) we get,

$$\frac{\sigma_T(z)}{\sigma_T} = \left(\frac{m_e}{m_e(z)}\right)^2$$

and with Equation (6.25) we obtain,

$$\frac{\sigma_T(z)}{\sigma_T} = (1+z)^2. \tag{6.28}$$

The Thomson coefficient concerns the scattering of photons with low energy E . For photons with high energy E the photon scattering is determined from the Klein-Nishina coefficient,

$$\sigma = \frac{3}{8}\sigma_T \frac{m_0 c^2}{E}\left(\ln\left(\frac{2E}{m_0 c^2}\right) + \frac{1}{2}\right) \tag{6.29}$$

in the laboratory and,

$$\sigma(z) = \frac{3}{8}\sigma_T(z) \frac{m_0(z) c^2}{E(z)}\left(\ln\left(\frac{2E(z)}{m_0(z) c^2}\right) + \frac{1}{2}\right) \tag{6.30}$$

in an astronomical object with redshift z . From Equations $E(z) = m(z)c^2$ and (6.26) we get,

$$\frac{m_0(z)}{E(z)} = \frac{m_0}{E}$$

and Equation (6.30) becomes,

$$\sigma(z) = \frac{3}{8}\sigma_T(z) \frac{m_0 c^2}{E}\left(\ln\left(\frac{2E}{m_0 c^2}\right) + \frac{1}{2}\right)$$

and with Equation (6.29) we get,

$$\frac{\sigma(z)}{\sigma} = \frac{\sigma_T(z)}{\sigma_T}$$

and with Equation (6.28) we obtain,

$$\frac{\sigma(z)}{\sigma} = \frac{\sigma_T(z)}{\sigma_T} = (1+z)^2. \tag{6.31}$$

From Equation (6.31) we conclude that the Thomson and Klein-Nishina scattering coefficients increase with redshift and in the same way.

From Equation (6.19) we obtain,

$$\lim_{r\to\infty} z = \frac{A}{1-A}. \tag{6.32}$$

Then from Equations (6.31) and (6.32) we get,

$$\frac{\sigma(r\to\infty)}{\sigma} = \frac{\sigma_T(r\to\infty)}{\sigma_T} = \frac{1}{(1-A)^2}. \tag{6.33}$$

Considering the limit (6.23) and Equation (6.33) we conclude that the Thomson and Klein-Nishina scattering coefficients had enormous values in the very early universe. In its initial phase the universe was totally opaque. From this initial phase stems the Cosmic Microwave Background Radiation (see, [9], [12]) we observe today.

The ionization and excitation energies of atoms as a function of redshift of the distant astronomical objects

The ionization energy as well as the excitation energy of atoms X_n is proportional to the factor $m_e q_e^4$, where m_e is the rest mass and q_e the electric charge of the electron. Therefore we have,

$$\frac{X_n(r)}{X_n} = \frac{m_e(r)}{m_e}\left(\frac{q_e(r)}{q_e}\right)^4$$

and considering that the electric charge of the electron increases at a much slower rate than its rest mass we get,

$$\frac{X_n(r)}{X_n} = \frac{m_e(r)}{m_e}$$

and with equation (6.25) we have,

$$\frac{X_n(r)}{X_n} = \frac{X_n(z)}{X_n} = \frac{1}{1+z}$$

and equivalently we obtain,

$$X_n(r) = X_n(z) = \frac{X_n}{1+z}. \tag{6.34}$$

From Equation (6.34) we conclude that the ionization and excitation energies of atoms decrease with increasing redshift. This fact has consequences on the degree of ionization of atoms in the distant astronomical objects.

The number of excited atoms in a gas in a state of thermodynamic equilibrium is given by Boltzmann's equation,

$$\frac{N_n}{N_1} = \frac{g_n}{g_1}\exp\left(-\frac{X_n}{KT}\right), \tag{6.35}$$

where N_n is the number of atoms at energy level n, X_n the excitation energy from the the 1st to the *n*th energy level, $K = 1\cdot 38\times 10^{-23} JK^{-1}$ Boltzmann's constant, T the temperature in degrees Kelvin, and g_n the multiplicity of level n, i.e. the number of levels into which level n is split apart inside a magnetic field.

From Equations (6.34) and (6.35) we obtain,

$$\frac{N_n}{N_1}=\frac{g_n}{g_1}\exp\left(-\frac{X_n}{KT(1+z)}\right). \tag{6.36}$$

For the hydrogen atom for $n=2$, $X_2=10.5eV=16.4\times10^{-10}J$, $g_1=2$, $g_2=8$ and at the surface of the Sun where $T\approx6000K$ equation (6.36) implies that just one in 10^8 atoms is at state $n=2$. Correspondingly from equation (6.36) and for $z=1$ wee have $z=1$, for $z=2$ we have $\frac{N_2}{N_1}=5.8\times10^{-3}$, and for $z=5$ we have $\frac{N_2}{N_1}=0.15$.

From Equations (6.34) and (36) we conclude that in the past, the universe went through an ionization phase of possibly long duration.

The position-momentum uncertainty as a function of redshift of the distant astronomical objects

Combining equations (3.21) and (6.7) we have,

$$J_i=\frac{c_i}{1-A(t)}$$

in the laboratory, and

$$J_i=\frac{c_i}{1-A\left(t-\frac{r}{c}\right)}=J_i=\frac{c_i}{1-A(t)\exp\left(-k_p\frac{r}{c}\right)}$$

for an astronomical object at distance r, and combining these two equations with equation (6.8) we get,

$$\frac{J_i(r)}{J_i}=\frac{m_0(r)}{m_0}$$

and with equation (6.25) we obtain,

$$\frac{J_i(r)}{J_i}=\frac{1}{1+z}. \tag{6.37}$$

From the position-momentum uncertainty, for the axis x_i we have,

$$J_i\Delta x_i\sim\hbar$$

in the laboratory, and

$$J_i(z)\Delta x_i(z)\sim\hbar$$

for the astronomical object, and combining these two relations we get,

$$J_i(z)\Delta x_i(z)=J_i\Delta x_i$$

and with equation (6.37) we have,

$$\Delta x_i(z)=(1+z)\Delta x_i. \tag{6.38}$$

From equation (6.38) we conclude that the uncertainty (see, [6]) $\Delta x_i(z)$ of position of a material particle increases with the redshift. Moreover, as the universe evolved towards the state we observe today, the uncertainty of position of material particles was decreasing.

From equations (6.38) and (6.32) we have,

$$\Delta x_i(r\to\infty)=\frac{\Delta x_i}{1-A}. \tag{6.39}$$

Considering the limit (6.23) and Equation (6.39) we conclude that in the very early universe there existed great uncertainty of position of material particles. The same conclusions arise for the Bohr radius,

$$R_{Bohr}(z) = (1+z) R_{Bohr}, \tag{6.40}$$

$$R_{Bohr}(r \to \infty) = \frac{R_{Bohr}}{1-A}. \tag{6.41}$$

On the type Ia supernovae

The production of energy in the universe is mainly through hydrogen fusion and nuclear reactions. Therefore, the energy produced in the past at distant astronomical objects was smaller than the corresponding energy produced today in our galaxy (see, Equation (6.26)). Furthermore, the Self-variation of the electron's rest mass played a defining role in the energy produced in the past at distant cosmological objects. This is due to the fact that the fundamental astrophysical parameters depend on the rest mass of the electron, which depends on the redshift.

A characteristic example concerns type Ia supernovae. The value of the rest mass of the electron, given as a function of the redshift z from Equation (6.25), plays a defining role at all phases of evolution of a star which ends up exploding as a type Ia supernova. As a consequence of equations (6.25) and (6.38) the intrinsic luminosity of supernovae of type Ia supernovae depends on redshift. The dependence of brightness on redshift is recorded at the seemingly long distances of type Ia supernovae (see, [13], [18]).

The evolution of the universe. Vacuum state

From equation (6.34) it follows that as the universe evolved to the state we observe today the ionization energy increased. This prediction is generally valid for any kind of negative dynamical energies which bind together material particles to produce more complex particles.

From equation (6.25) we have,

$$\Delta m_0(z) c^2 = \frac{\Delta m_0 c^2}{1+z}$$

for the energy $\Delta m_0 c^2$, the mass deficiency, which ties together the particles which constitute the nuclei of the elements. According to this Equation the energy $\Delta m_0 c^2$, like the ionization energies, increased as the universe evolved towards its present state.

From Equations (6.25) and (6.32) we have,

$$m_0(r \to \infty) = m_0(1-A) \neq 0. \tag{6.42}$$

Considering the limit (6.23) and Equation (6.42) we conclude that, as the universe tends toward its initial state, the rest masses of material particles tend to zero,

$$m_0(r \to \infty) = m_0(1-A) \to 0. \tag{6.43}$$

With the notation we follow, from Equation (3.20) we have,

$$E_0(r \to \infty) = 0. \tag{6.44}$$

According to the relations (6.43) and (6.44) the initial state of the universe slightly differed from vacuum. Considering the conservation of energy-momentum, we conclude that the total mass /

energy of the universe asymptotically tend to zero or its is zero. We called this initial state of the universe the 'Vacuum State'.

As a consequence of the Vacuum State, the gravitational interaction cannot play the role attributed to it by the Standard Cosmological Model. Gravity cannot cause either the collapse or the expansion of the universe.

The gravitational interaction strengthens with the passage of time, as the rest masses of material particles increase. From one point and on this is in position to accumulate matter within “small” regions of space. The role of gravity is limited to the creation of the large structures of the universe.

A comparison of the cosmological predictions of Self-variation Theory versus the Standard Cosmological Model, based on the cosmological data

In this subsection we compare the predictions of the Standard Cosmological Model (SCM) and the Self-Variation Theory (SVT), based the 14 main cosmological data.

1. Origin of the universe

SCM, Big Bang.
SVT, Vacuum State.

2. Redshift

SCM, it is a direct consequence of the expansion of the Universe.
SVT, it is a direct consequence of the self-variation of rest mass of the electron. In the past (large redshift) the electron transition energies inside the atom were much smaller.

3. Cosmic Microwave Background Radiation

SCM, it is a remnant of the Big Bang.
SVT, it is a consequence of the enormous values of the Thomson and Klein-Nishina scattering coefficients in the distant past.

4. Increased luminosity distances of type Ia

SCM, is forced to do the Dark Energy case.
SVT, it is a direct consequence of the self-variation of the fundamental parameters of astrophysics (mass of electron, ionization energy and degree of ionization of atoms, Thomson and Klein-Nishina scattering coefficients, Bohr radius, production of energy via hydrogen fusion and nuclear reactions). In the distant past these parameters had different values.

5. Flatness of the Universe

SCM, an attempt is made to justify it with the Inflation hypothesis.

SVT, the total energy content of the Universe is predicted to be zero, therefore the Universe on the grand scale is and was always flat.

6. Nucleosynthesis of the chemical elements

SCM, the prediction agrees with observations, for a particular decrease rate of the temperature versus the expansion rate of the universe that is adopted.
SVT, further investigation of the Internal Symmetry Theorem in curved spacetime is required.

7. Ionization of atoms in the early universe

SCM, it is predicted as a consequence of the high temperatures after the Big Bang.
SVT, it is a direct consequence of the dependence of the ionization energy from redshift. In the past (large redshift) the ionization energy was much smaller.

8. Distribution of matter on the cosmological scale.

SCM, it inconsistent with recent measurements.
SVT, it is consistent with the recent measurements.

9. Variation of the fine structure constant

SCM, is not predicted.
SVT, it is a direct consequence of the Self-variation of the electric charge.

10. The Horizon problem

SCM, an attempt is made to explain it with the hypothesis of inflation.
SVT, the Self-variation model predicts that the position-momentum uncertainty is huge in the early universe, tending to infinity in the distant past. Therefore everything was connected in the early universe.

11. The larger than expected velocities of astronomical objects at the outskirts of large structures in the universe

SCM, is forced to do the Dark Matter case.
SVT, it is predicted (see, section 5).

12. Absence of magnetic monopoles in the universe

SCM, magnetic monopoles have never been observed, hence the problem for SCM, since it predicts their existence as a consequence of the Big Bang.
SVT, the detailed study of electromagnetism in section 4 rules out the existence of magnetic monopoles.

13. Olbers' paradox

SCM, it is justified by the expansion of the universe.
SVT, it is a consequence of Equation (6.26).

14. The two measured values of Hubble's constant

SCM, the two measured values of the Hubble constant are incompatible with the SCM.
SVT, on the cosmological scale, the Self-Variation of the electron and the heavy particles correspond to different values of Hubble's constant.

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7. Self-Variation and quantum phenomena. The structure of matter

In the context of the Self-Variation Theory, the structure of matter is determined by the relative position of the N -vectors J and P in curved N -dimensional spacetime. In flat 4-dimensional spacetime this structure is given by the Internal Symmetry Theorem. The Theorem predicts a distribution of energy-momentum and charge of every material particle in spacetime. It also gives the rate of change of the 4-vectors J and P, predicting a dynamical system that we called a generalized particle. A central role in the generalized particle is played by the function Φ (see, Equation (3.18)). It justifies the cosmological data, in a flat static universe, while if $b \in \mathbb{C} - \mathbb{R}$ it predicts wave behavior of the generalized particle, i.e. matter. Therefore, the Intrinsic Symmetry Theorem is related to fundamental physical phenomena and mechanisms, such as the structure and wave behavior of matter, quantum phenomena and interactions. Hence, the investigation of the Internal Symmetry Theorem in the curved N -dimensional spacetime is the main goal of the Self-Variation Theory.

The Internal Symmetry Theorem in curved N-dimensional spacetime

We follow the proof procedure of the Internal Symmetry Theorem in flat spacetime. From Equation (2.8) we get,

$$\left(J_n J^n\right)_{;k} = \left(m_0^2 c^2\right)_{;k}$$

and equivalently we get,

$$\left(J_n J^n\right)_{;k} = \frac{\partial}{\partial x_k}\left(m_0^2 c^2\right)$$

and with Equation (2.2) we obtain,

$$\left(J_n J^n\right)_{;k} = \frac{b}{\hbar} P_k m_0^2 c^2 , \qquad (7.1)$$

where with $;k$ we denote the covariant derivative with respect to x^k.

Equation (2.7) applies to the N -vectors J and P,

$$C_n = J_n + P_n ,\ n = 0,1,2,...,N-1. \qquad (7.2)$$

The relative position of the N-vectors J and P in spacetime is given by Equation (2.15),

$$\begin{aligned}
P_0 &= \Phi_{00}J_0 + \Phi_{01}J_1 + \Phi_{02}J_2 + ... + \Phi_{0(N-1)}J_{N-1} \\
P_1 &= \Phi_{10}J_0 + \Phi_{11}J_1 + \Phi_{12}J_2 + ... + \Phi_{1(N-1)}J_{N-1} \\
P_2 &= \Phi_{20}J_0 + \Phi_{21}J_1 + \Phi_{22}J_2 + ... + \Phi_{2(N-1)}J_{N-1} \\
&. \qquad\qquad\qquad\qquad\qquad\qquad . \qquad (7.3) \\
&. \\
&. \\
P_{(N-1)} &= \Phi_{(N-1)0}J_0 + \Phi_{(N-1)1}J_1 + \Phi_{(N-1)2}J_2 + ... + \Phi_{(N-1)(N-1)}J_{N-1}
\end{aligned}$$

The Internal Symmetry theorem is given by the solution of the system of Equations (7.1), (7.2) and (7.3). As a consequence of the covariant derivative with respect to x^k in the first part of

Equation (7.1), the investigation of the Theorem requires complex mathematical calculations. An exception is 4-dimensional flat spacetime, as we saw in section 3.

Self-Variation propagates as a 'disturbance' in space-time. The transmission of this disturbance in spacetime is given by the functions Φ_{ij} , after solving the system of Equations (7.1), (7.2) and (7.3). These functions give the rate of change of J , P , therefore they are always related to an interaction. As a consequence of Equation (7.3), the Interior Symmetry Theorem is always related to a set of matrices. The propagation of Self-Variation in spacetime is equivalent to wave behavior of matter.

Physicists have come into contact many times with the system of Equations (7.1), (7.2), (7.3). Bohr's work (see, [1]), corresponds to a standing wave for which, however, we do not know the wave function. Schrödinger understands the wave behavior of matter (see, [4]), defines the homonymous operators and applies them to a dynamical system, the hydrogen atom. Heisenberg introduces matrices to quantum mechanics and formulates the uncertainty principle (see [3]). Dirac introduces non-commutative matrices in his work on quantum mechanics (see, [2]), but does not know Equations (7.2) and (7.1), (7.3) and the cause of quantum phenomena. Dirac studied a dynamical system, the generalized particle of the electron. The generalized particle is always associated with an interaction. In Dirac's work, this interaction is expressed through potential, which plays a central role in his work. This was followed by the work of many distinguished physicists who brought quantum mechanics to the level we know it today.

The contribution of the Self-Variation Theory to theoretical physics is summarized in the Theorem of Internal Symmetry, i.e. the solution of the system of Equations (7.1), (7.2), (7.3), and the requirement that the mathematical form of physical laws must be compatible with the of Self-Variation principle. This requirement places additional restrictions, than those set by the Theory of Relativity on the mathematical form of physical laws. The Intrinsic Symmetry Theorem relates to the observations we make on all distance scales, from the microcosm to the cosmological scale. In section 3 we presented the simplest solution of the system of Equations (7.1), (7.2) (7.3), in four-dimensional flat spacetime. In section 6 we saw the predictions of the Theorem on the cosmological scale. In sections 4 and 5, we took into account that the potential is not uniquely defined and required it to be compatible with the Self-Variation principle. Thus we obtained the potential of the electromagnetic field, as given by the pairs of Equations (4.36) and (4.38).

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8. Conclusion

The Self-Variation Theory provides a mathematically consistent paradigm for nature. The origin, evolution and current form of the universe are consistent with the theoretical prediction. At all distance scales, from the microcosm to observations billions of light years away, the Theory is remarkably consistent with experimental and observational data.