

The growth of the Universe. Another approach to cosmology.

Vyacheslav A. Ruchin*

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Abstract

At present, the idea that our space-time is emergent, as a kind of low-energy phase transition into some kind of condensed matter and superconductor, is becoming quite popular. This space-time is formed as a kind of new network, the volume of which, when measured in Planck units, is equal to the number of network nodes. The growth of such a network and the increase in its volume is not due to the stretching of existing cells, but as a result of adding new cells. Considering the growth of such a network in a similar way to the growth of the volume of a new phase, one can naturally explain the redshift and many other problems of modern cosmology.

Keywords: Emergent space-time, Phase transitions, Universe growth, Redshift.

1 Introduction

First, we noticed that already within the framework of standard GR, taking into account the structure of vacuum in the form of a certain crystalline medium can lead to an inflationary scenario and give rise to dark energy and dark matter effects [1]. It was a good try in this direction. However, further work in this direction showed that the consideration of our vacuum as some kind of condensed matter has a deeper foundation than it seemed to us at the beginning, and gives serious impetus to attempts to take a fresh look at our emergent space-time and many physical processes [2–20].

The theory of the Higgs boson with a mass of about 125 GeV and the associated Higgs field, which plays a fundamental role in particle physics, was inspired by ideas from condensed matter physics. This ideas originally had been offered by Philip Anderson [21, 22]. He discussed spontaneous symmetry breaking (SSB) in superconducting condensates and considered it as a general “mechanism” for the processes of emergence and mass generation. Many physicists have urged that the name of this process should be changed to the “Anderson-Higgs” mechanism [23]. Moreover, the ideas of SSB and emergence have become dominant in many classic papers on theoretical physics.

Kenneth Wilson considered our space in the form of a superconductor, the structure of which was represented by a kind of graph. Thus the space was imagined as a lattice made of nodes connected by edges [2, 5, 6].

*E-Mail: vyacheslav.ruchin@gmail.com

This direction, that our emergent spacetime and vacuum are a kind of condensed matter and a superconductor that appear as a phase transition and elementary particles appear as excitations and SSB of this condensate, gets serious development and has become very popular.

«The similarities between the vacuum of space and low-temperature phases of matter are legendary in physics.» - Robert Laughlin [3].

Assuming that physics should be discrete on the Planck scale, the models for the formation of such space-time networks (spin network, causal set, ...) were considered as composed of discrete, indivisible spacetime atoms. Such space-time grows by the accretion of new spacetime atoms [2, 24–28].

In light of this approach, it has also become a topic of great interest in recent cosmology, considering the emergent scenario of cosmic evolution. Particular interest in this was fueled by the fact that it describes a non-singular origin of the Universe unlike the Big-Bang models [29–33, 50]. Such cosmic evolution scenarios based on non-equilibrium thermodynamics and phase transitions are also becoming increasingly popular.

2 Model of the emergence and growth of the Universe

Let's continue moving in this direction. Lee Smolin [2]: "The big bang is then not the origin of all that exists, but only a kind of phase transition by which a new region of space and time was created, in a phase different than the one from which it came". Dynamics the growth of such a new region of space and time resulting from a phase transition is determined by the accretion of new spacetime atoms. In principle, these emergent space-time ideas are well known and actively developed, as noted in the introduction. The only new thing I want to suggest for thinking about is to try to consider the "incredible". Let us consider the mechanism of accretion of new spacetime atoms not in the spirit of the now popular **mathematical** model 'transitive percolation' proposed by Rideout and Sorkin [26], but in the spirit of the growth of new phases commonly observed in nature. Thus, the growth of the structure and the attachment of new cells to it occurs at the boundary of the formed new phase.

Such an emergent space-time like a condensed network, which has the properties of a superconductor, can be represented, as noted by Kenneth Wilson [5, 6], by a kind of graph and as a lattice made of nodes (spacetime atoms) connected by edges. Accordingly, we can consider the growth and increase in the size of such a superconductor lattice formed as a result of a phase transition, and the effects arising from this.

3 Redshift

Accordingly to Quantum Fields on a Lattice [35], we consider a discretized space-time lattice which extends over $l = l_1 = l_2 = l_3$ in every spatial direction and length $l_4 = t$ in the temporal direction ($\hbar = 1$).

$$\Lambda = \{x = (x, x_4) \in dZ^4 \mid 0 \leq x_\mu \leq d(l_\mu - 1)\},$$

where d - is lattice spacing.

The propagation of waves on such a lattice as well as for a scalar field must be subject to periodic boundary conditions in the form:

$$\phi(x + d\hat{\mu}l_\mu) = \phi(x)$$

where $\hat{\mu}$ is the unit vector in the μ -direction.

The Fourier transform of a function $f(x)$ is defined by

$$\tilde{f}(x) \equiv \sum_x d^4 e^{-ipx} f(x)$$

that should be periodic in momentum-space.

The allowed lattice momenta are given by

$$p_\mu = \frac{2\pi}{dl_\mu} n_\mu, n_\mu = 0, 1, \dots, l_\mu - 1.$$

The wavelength propagating in the lattice space will be

$$\lambda_i = \frac{L_i}{n_i}, \quad (1)$$

where $L_i = dl_i$ is the lattice spatial size ($i=1,2,3$).

It should be taken into account that the formula (1) is valid only for weak fields and long-wavelength oscillations that satisfy the superposition principle.

Consequently, the spectrum of waves in the lattice will change depending on its size. In quantum field theory we get the same result of a change in the wavelength inside a potential well depending on its size.

Note: This is a well-known effect. If, for example, a vibrating string on a guitar, which is a one-dimensional chain of interconnected atoms, is clamped in different places, then the sound it emits will also change.

Let us take a simplified case of an isotropic spherical lattice. Then the propagation of waves in any direction can be considered as vibrations of a one-dimensional chain of atoms along the diameter of this ball, which coincides with the direction of these waves.

From the above formulas, it is easy to see if the radius of such a lattice will change $R(t) = a(t)R_0$, then the length of the wave traveling in it will also automatically change. Denote t_0 – observation time and t_1 – wave emission time ($t_0 > t_1$). Respectively, λ_0 – observed wavelength, λ_1 – emitted wave, a_0 – scale factor $a(t_0)$, $a_1 = a(t_1)$. Then the spectral shift will be

$$Z_c = \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a_0}{a_1} - 1.$$

We can expand $a(t)$ into a series $a(t_1) = a(t_0)[1 + (t_1 - t_0)H_0 + \dots]$, where H_0 is a constant related to the rate of change in the lattice size. In a linear approximation the resulting redshift can be expressed in terms of the time difference between emission and absorption or in terms of the distance between the emission and absorption places $l = c(t_0 - t_1)$, where c - wave propagation speed.

$$Z_c \approx (t_0 - t_1)H_0, \quad (2)$$

$$cZ_c \approx lH_0, \quad (3)$$

Therefore, if we follow this popular model and we said “A”, that our emergent space-time appears due to a phase transition, then we must also say “B”:

“The growth of this new phase leads to a redshift within it”.

Thus, if we consider our emergent space-time as a kind of phase transition by which a new region of space-time was created and grows similarly to the usual growth of a new phase and the growth of a crystal, then we will get the same formulas for the redshift as in standard cosmology. Only the mechanism of such expansion of the Universe "slightly" differs from those accepted in general relativity. Instead of Big-Bang and inflation, there will be a new phase nucleation and growth by adding new elements to it.

Note: If the shape of the emergent space of our Universe is not spherical and $L_i \neq L_j$, then, as it can be seen from (1), this should lead to anisotropy and a difference in the spectra observed in these directions. Although the redshift itself, if the growth of space occurs proportionally in all directions, will remain isotropic.

Such a new lattice structure of condensed matter (we can call it a world crystal), growing as a result of a phase transition that forms the growing space-time of our Universe, will be similar to the quantum vacuum in quantum field theory. And the observed matter, the particles of which are born from this vacuum as excitations leading to spontaneous symmetry breaking, will be similar to the defects of this world crystal.

Herewith, the growing world crystal (spin-network, quantum vacuum, superconductor) of our Universe and the defects born in it (observed matter) will have a uniform density, which is generally typical for crystals.

4 Conclusions

There is the well-known statement of Robert Laughlin [3]: "There are legends in physics about the similarity between the cosmic vacuum and the low-temperature phases of matter."

And by simply following this popular trend that our emergent space-time has the properties of condensed matter and arises and grows as a result of a phase transition, similar to the growth of a crystal, we **automatically** come to explain the cosmological redshift and some other problems.

We note that such a model naturally solves some problems of cosmology:

1. **It explains the redshift**, which, as we have seen, will automatically appear in this case according to Hubble's law.

2. **It describes a non-singular origin of the Universe.**

3. **It very naturally removes the worst problem of the cosmological constant Λ** , which was associated with the cosmological vacuum energy density and the expansion rate of the Universe [2, 36]. In our model, the expansion (growth) of the Universe is explained by the standard mechanisms of phase transitions. The density of the vacuum lattice (the world crystal) may well correspond to those huge values predicted by quantum field theory, and this absolutely does not prevent us from obtaining the growth rate of the Universe in accordance with the observed data.

4. It proposes a natural candidate for the role of dark energy.

The growth rate of a crystal or a new phase strongly depends on the properties of the external medium and the interaction with it. In our model, the environment external to our Universe will be naturally dark for us and will play the role of dark energy, which affects the expansion of our space-time.

5. Multiverse.

Our model also leads to the increasingly popular idea of the Multiverse [37, 38]. Since the space-time of our Universe arises and grows as a result of a certain phase transition, an external environment must exist outside our Universe. It is obvious that, under these conditions of the phase transition, other nuclei (other Universes) of the new phase can also arise and grow in this external medium. In our model, however, they will have a similar structure and properties.

6. The consequences of this model are in many respects similar to the results in standard cosmology.

It should also be noted that the cosmological scenario resulting from such a model is in many respects similar to the established inflationary scenarios and Hot Big Bang theories in general relativity. The redshift in this model is also explained by the expansion (growth) of the Universe. Since the space-time of our Universe is considered as a kind of spin-network (multilayered superconductor, crystal lattice), this network has some binding energy. At high temperatures above this binding energy, this network (our space-time) will melt and cannot exist. Our space-time network emerges only at a temperature below the critical as a low-temperature phase transition. As Lee Smolin [2] describes it: "... what is usually called the big bang is, in fundamental terms, the big freeze. What caused our world to exist was probably not so much an explosion as an event that caused a region of the Universe to cool drastically and freeze." Thus, in this model, our Universe also goes through a transition from hot to cool drastically and freeze, with all the known consequences that follow from this.

Thus it turns out, if we follow the ideas expressed here, then today's cosmology, built on general relativity, can be preserved in many aspects, but in many respects fundamentally reworked.

5 Appendix

A. New phase growth rate

As already noted, there are many models for the growth of a new phase that try to take into account the properties of the medium and the conditions under which the phase transition occurs.

According to the rather widespread Wilson–Frenkel model of the phase transition kinetics of liquid freezing, the growth rate of a new solid phase V under the condition of low overcooling ΔT is determined by the formula [39]:

$$V = A\nu \exp\left(-\frac{E}{kT}\right) \exp\left(-\frac{\Delta S}{k}\right) \left\{1 - \exp\left[-\frac{\Delta H}{k}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]\right\} \simeq \beta^T \Delta T, \quad (4)$$

$$\beta^T \simeq A\nu \frac{\Delta S}{kT} \exp\left(-\frac{\Delta S}{k}\right) \exp\left(-\frac{E}{kT}\right)$$

β^T is called the kinetic coefficient of crystallization, A is a constant that depends on the properties of the crystal, ν is the frequency of thermal vibrations of atoms in a crystal and in a liquid (both frequencies are assumed to be equal). To change the configuration, atoms have to overcome the energy barrier E , and this probability is expressed by $\exp(-\frac{E}{kT})$ at a temperature T . The entropy difference between the liquid and solid phases $\Delta S = S_L - S_S$. The latent heat L generated at the melting point corresponds to the enthalpy difference between the liquid and solid phases as $L = H_L - H_S = \Delta H$.

Let us emphasize the main conclusion from equation (4), which is also confirmed in other models: **The growth rate of the new phase will accelerate with increasing supercooling ΔT .**

Even in laboratories, it is impossible to create ideal crystals growth. Obviously, due to various stochastic processes, the magnitude of supercooling will also fluctuate, and with it the growth rate of the new phase.

Probably, it is precisely such fluctuations that we have in our observable Universe.

"The new finding suggests that **the Universe has slowed down and speeded up, not just once, but 7 times in the last 13.8 billion years**, on average emulating dark matter in the process" [40, 41].

On our Earth we also had already several phases of cooling and warming.

B. Experiments to test the increase in distances between galaxies

Time dilation [the stretching of time by a factor of $(1+z)$] is a fundamental property of an expanding Universe. However, as stated in the study [42], over 800 quasars on time-scales from 50 d to 28 yr are monitored: "**The results . . . provide strong evidence that the effects of time dilation are not seen in quasar light curves.** This clearly runs against expectations based on a conventional cosmological viewpoint, and so in this section we examine ways in which the results may be understood. ... it turns out to be surprisingly hard to formulate a conclusive test for time dilation."

In a study by Foley et al. (2005 [43]) on supernova light curves, it was evaluated as confirmation of the time dilation. However, these conclusions has been challenged in the Crawford's paper (2009 [44]).

Other different indirect tests are also mentioned to verify whether the Universe is expanding or static [45]. However, these experiments also did not provide convincing evidence that the distance between galaxies is increasing. "Nobody has ever directly observed a galaxy distance being increased with time. The motion of the putative expansion of the Universe is so slow that it is unmeasurable."

C. Emergent space-time and Einstein's equations

Having said A, we must also say B.

1. By asserting that our universe is born and grows as a result of a phase transition due to the addition of new space-time atoms, we automatically come to the conclusion that the physics of this process is also described by the equations of phase transitions, and not by Einstein's equations.

This is fundamentally different from the purely mathematical model of standard cosmology, in which our universe seemed to always exist (just in a superdense state) and then suddenly began to expand.

2. An increase in the size of our emergent space-time as the growth of a new phase automatically leads to a redshift, which will be basic, relating to our entire universe. In addition, other effects leading to redshift can be taken into account. They will influence local redshift deviations from the general background.

3. Einstein's equations

$$R_{\mu\nu} - g_{\mu\nu}R/2 + \Lambda g_{\mu\nu} = \varkappa T_{\mu\nu}.$$

do not describe the emergence of a new space-time. They describe only how matter (including material vacuum) deforms **an already existing space-time**, which already has a certain metric.

Gravity does not create our emergent space-time. Gravity only deforms it (stretches, compresses, bends), already existing.

Here we return to Sakharov's 1967 idea of "induced gravity" [46], which currently enjoying a significant resurgence, that gravity is not "fundamental", but emerges in the already existing vacuum structure due to the influence of particles arising in it.

This also includes the work of H. Kleinert, who proposed considering our space-time as a world crystal, and gravity as the influence of defects arising in it on it [47, 48], G. t' Hooft's article "Crystalline Gravity" [49] and a large stream of articles on induced and emergent gravity (for example: [50–53]).

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