Dark Matter versus Dark Energy in Clusters using the Dark Matter by Gravitation theory –V3

Author Manuel Abarca Hernandez   email   mabarcaher1@gmail.com

1. ABSTRACT

This paper develops the Dark Matter by Gravitation theory, DMbG theory hereafter, in clusters of galaxies.
 I have been developing this theory in galaxies since 2013 and I have published more than 20 papers most of them using rotation curves of galaxies, especially the ones belonging to M31 and Milky Way. So to understand this paper is compulsory to consult the paper *A DM theory by gravitation for galaxies and clusters-V2.* Vixra:2312.0002 where it is fully developed the DMbG theory.

An important results got by the DMbG theory is that total mass associated to a galactic halo up to a specific radius depend on the square root of radius without limit. Apparently, this result would be absurd because of divergence of the total mass. However it is the Dark energy the responsible to counterbalance the DM. In this paper the DMbG theory finds impressing theoretical results that in the last chapter has been checked with recent data published for the Virgo cluster.

As at cluster scale the DE is not negligible, in this work it is defined, the total mass as baryonic matter plus DM and the gravitating mass as the addition of the total mass plus the negative mass associated to dark energy.

In clusters it is defined the zero gravity radius (R_{ZG} hereafter) as the radius needed by the dark energy to counter balanced the total mass (baryonic and DM). It have been found, as universal formulas, that the ratio $R_{ZG}/R_{VIR} \approx 7.3$ and its Total mass associated at R_{ZG} is $\approx 2.7 \cdot M_{VIR}$ This works postulates that the factor 2.7 may equilibrate the strong imbalance between the local Universe mater density versus the Global matter density. Currently this fact is a big conundrum in cosmology.

Also it has been found that the zero velocity radius, R_{ZV} hereafter, i.e. the cluster border because of the Hubble flow, is $\approx 0.8 \cdot R_{ZG}$ and its gravitating mass is $\approx M_{VIR}$

By derivation of gravitating mass function it is calculated that a half of R_{ZG} this function reaches its maximum whose value is $\approx 1.5 \cdot M_{VIR}$

Some important theoretical calculus have been checked using recent data for Coma and Virgo clusters. Namely, it is calculated $R_{ZG} = 20$ Mpc and its total mass for Coma and $R_{ZV} = 10.3$ Mpc and its gravitating mass for Virgo.

The theoretical finding in this work are organised around a dozen of notable formulas although there are four

especially remarkable: $M_{TOTAL}(<R_{ZG}) = \left(\dfrac{100}{\Omega_{DE}}\right)^{1/5} \cdot M_{VIR} \approx 2.7 \cdot M_{VIR}$ The total mass (baryonic + DM) associated to a cluster is 2.7 times the virial mass, as a universal law.

The ratio $\dfrac{R_{ZG}}{R_{VIR}} = \left(\dfrac{100}{\Omega_{DE}}\right)^{2/5} \approx 7.277$ The ratio of zero gravity radius versus virial radius is 7.3 as universal law.

The maximum of gravitating mass is reached at $\approx 0.49 \cdot R_{ZG}$ and $M_G^{MAXIMUM}(<0.49 \cdot R_{ZG}) \approx 1.57 \cdot M_{VIR}$

The zero velocity radius is reached $\approx 0.8\ R_{ZG}$ and its gravitating mass is $M_G(<R_{ZV}) \approx M_{VIR}$. Notice that the gravitating mass is decreasing from its maximum at $0.5 \cdot R_{ZG}$ up to R_{ZG} where is zero.

TABLE OF CONTENTS

2. INTRODUCTION

The basis of this paper are developed in [1] Abarca, M. 2023, so it is highly recommended to read it to understand the meaning of this paper. The dark matter by gravitation theory, DMbG theory hereafter, is an original theory developed since 2013 through more than 20 papers, although in [1] Abarca, M. 2023 is published the best version as physical as mathematically. Therefore is not possible to understand this paper if reader have not at least a general knowledge about the DMbG theory.

In fact in [1] Abarca, M. 2023, the chapters 16 and 17 are dedicated to cluster where it is introduced the formula for R_{ZG} and some others to extend the theory from galaxies to clusters. The newness of the important results got in this paper are due to the possibility to approximate the virial radius to R_{200} and the virial mass to M_{200} , the chapter 3 is dedicated to explore this approximation using recent data published for some important clusters such as Virgo, Coma or some others.

At cluster scale Dark Energy, DE, is not negligible and it is possible to associate to DE a negative density of mass. In this work it is crucial to distinguish between total mass, which is baryonic matter plus DM, and the gravitating mass which is the addition of total mass plus a negative quantity that represents the DE.

Reader knows the virial radius and virial mass concepts at cluster scale, but perhaps does not know the zero gravity radius concept, which is introduced and developed in chapter 5. The R_{ZG} is related to distances between clusters, because each cluster has a sphere with radius R_{ZG} where the gravitational field associated to the cluster dominates. For example if virial radius of Virgo is 1,7 Mpc then Virgo R_{ZG} is 12.4 Mpc, so it is not possible to have another cluster of galaxies in dynamic equilibrium inside this sphere.

3. VIRIAL MASS AND VIRIAL RADIUS IN CLUSTER OF GALAXIES

As reader knows it is a good estimation about virial radius and virial mass for cluster of galaxies to consider Rvir = R_{200} and Mvir= M_{200}. Where R_{200} is the radius of a sphere whose mean density is 200 times bigger than the critic

density of Universe $\rho_C = \dfrac{3H^2}{8\pi G}$ and M_{200} will be the total mass enclosed by the radius R_{200}.

In this epigraph will be shown some data published by prestigious researchers that confirm this approximation.

It is right to get the following relation between both concepts $R_{VIR}^3 \approx R_{200}^3 = \dfrac{G \cdot M_{200}}{100 \cdot H^2}$ or $M_{VIR} \approx M_{200} = \dfrac{100 H^2 R_{200}^3}{G}$

The checking process will begin with the bigger cluster in the Local Universe. The graph below comes from [7] Seong –A Oh.2023.At the foot notes, they inform that virial radius is 2.8 Mpc, so using the above formula it is got Mvirial = 2.5E15 Msun, that match with mass published.
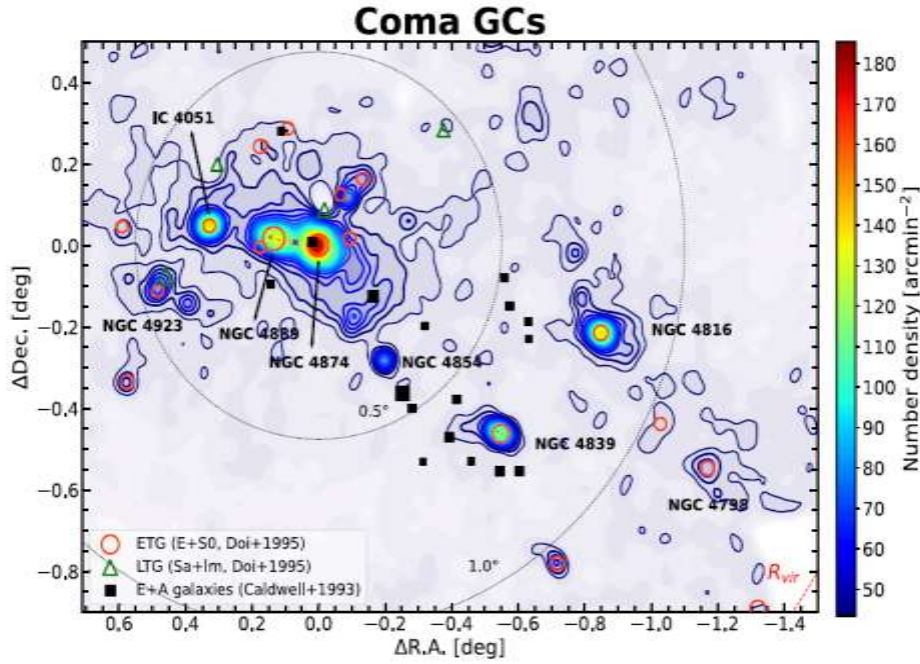
Figure 3. Spatial number density contour map of GCs in the Coma field including NGC 4839 and NGC 4816 (see S. Oh et al. 2023, in preparation, for details). Dotted line circles represent $R = 0°.5$, $1°.0$, and $R_{vir}(=2.8$ Mpc) from NGC 4874 at the Coma center. Red circles and green triangles mark early-type galaxy members, and late-type galaxy members (Doi et al. 1995). Black boxes mark E+A galaxies (Caldwell et al. 1993). The contour levels denote $2\sigma_{bg}$ and higher with an interval of $1\sigma_{bg}$ where σ_{bg} denotes the background fluctuation. The contour maps were smoothed using a Gaussian filter with $\sigma_G = 1'$. The color bar represents the GC number density.

Coma Cluster parameters according [7] Seong –A Oh.et al. 2023		Data from Dynamic method M_{VIR} = 2.7E15 Msun and R_{VIR} = 2.8 Mpc

Parameter	Main Cluster
Heliocentric galaxy velocity, v_h	7167 km s^{-1}
Heliocentric group velocity, v_h	6853 km s^{-1}
Velocity dispersion, σ_v	1082 km s^{-1}
Virial mass (dynamics),[b] M_{vir}	$2.7 \times 10^{15} M_\odot$
Weak-lensing mass,[c] M_{WL}	$1.2 \times 10^{15} M_\odot$

Using in the approximation formula 2.8 Mpc as R_{200} is got M_{200} =2.5E15 Msun whose relative difference versus M_{VIR} is 7.4%

The author [7] Seong, shows that the formula to calculate Virial mass is M_{VIR}= 1.5E6.h^{-1}·σ_V^3 Msun where σ_V =1082 km/s So the formula gives the mass in Msun units on condition that velocity dispersion σ_V units are km/s and h = 0.7

3.1 CHECKING THE VIRIAL MASS APROXIMATION ON A SAMPLE OF CLUSTERS AND GROUP OF G.

Data [4] R.Ragusa et al.2022				
Group of galaxies G. Or Clusters C.	Virial Radius	Virial Mass	Mass calculated	Relative diff for M
Name	Mpc	10^{13} Msun	1E13Msun	%
Antlia C.	1,28	26,3	2,39E+01	-9,21E+00
NGC596/584 G.	0,5	1,55	1,42E+00	-8,18E+00
NGC 3268 G.	0,9	8,99	8,30E+00	-7,67E+00
NGC 4365Virgo SubG.	0,32	0,4	3,73E-01	-6,73E+00
NGC 4636 Virgo SubG.	0,63	3,02	2,85E+00	-5,73E+00
NGC 4697Virgo Sub G.	1,29	26,9	2,44E+01	-9,14E+00
NGC 5846 G.	1,1	16,6	1,52E+01	-8,71E+00
NGC 6868 G.	0,6	2,69	2,46E+00	-8,57E+00

Data beside in green have been taken from [4] R.Ragusa et al. 2022 and using the formula $M_{200} = \frac{100H^2 R_{200}^3}{G}$ it is calculated its mass associated for each radius. The yellow column shows the relative difference for masses, always under 10 %. The mass calculated are lower than mass published. With these examples it is shown that the consideration of R_{200} and M_{200} as virial radius and virial mass is an acceptable approximation for a wide range of celestial bodies, group of galaxies or clusters.

As the Virgo cluster is the nearest between the big clusters it is crucial to check the approximation for virial mass and radius with its data.

According [12] Karachentsev I.D. et al. 2014.In page 5 it is shown that Rvir = 1.8 Mpc and Mvir = 7E14 Msun. And using the approximate formula Mvir= M_{200} it is got Mvir = 6.64E14 Msun which is quite close to published value.

According [15] Olga Kashibadze, I. Karachentsev 2020, see pag 9, Rg=Rvir= 1.7 Mpc and Mvir= (6.3±0.9)E14 Msun. Using formula for M_{200} for 1.7 Mpc it is got 5.59E14 Msun which match with mass published.

In table below are summarized the results for the two most prominent cluster of galaxies.

Cluster of galaxies	Virial Radius	Virial mass	Calculated M_{200}	Mass Relative diff.
	Mpc	x 10^{14} Msun	x 10^{14} Msun	%
Virgo [13]Kashibadze 2020	1.7	6.3±0.9	5.59	11
Coma [7] Seong-A. 2023	2.8	27	25	7.4

In conclusion R_{200} and M_{200} are a very good estimation for Virial radius and Virial mass for galaxies, group of galaxies and cluster of galaxies, when they are in dynamical equilibrium, as it is well known by the astrophysicist researchers.

4. VIRIAL THEOREM AS A METHOD TO GET THE DIRECT MASS FORMULA IN CLUSTERS

In chapter 9, epigraph 9.8 of paper [1]Abarca,M.2023 was demonstrated that direct formula $M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$ is the most suitable formula to calculate the total mass (baryonic and DM) depending on radius in the galactic halo region. The units of parameter a^2 are $m^{5/2}/s^2$

This chapter is based on the 16 chapter in [1]Abarca,M.2023, where the Direct mass formula is extended to clusters.

4.1 PARAMETER a^2 FORMULA DEPENDING ON VIRIAL RADIUS AND VIRIAL MASS

Due to the fact that the Direct mass formula has one parameter only, is enough to know the mass associated to a specific radius to be able to calculate parameter a^2. That is the situation when it is known the virial mass and the virial radius for a cluster of galaxies.

This formula is only a way to estimate parameter a^2 because outside the virial radius always there will be a fraction of the galaxies belonging to cluster. Anyway, this method may estimate a lower bound of parameter a^2 associated to clusters.The Virial theorem states that $M_{VIRIAL}(<r) = M_{DYNAMICAL}(<r) = \dfrac{V^2 \cdot r}{G}$ is a formula right for a cluster of galaxy on condition that velocity and radius are calculated for galaxies in dynamical equilibrium.

If it is considered that the virial radius is the border of halo cluster where galaxies are in dynamical equilibrium and at the same time is negligible the amount of Baryonic matter outside the sphere with such radius, then it is possible to apply the formula of $M_{DIRECT}(<R_{VIRIAL})$. Then by equation of both formulas will be possible to clear up a^2.

$$M_{VIRIAL}(<R_{VIRIAL}) = \frac{a^2 \cdot \sqrt{R_{VIRIAL}}}{G} \quad \text{so} \quad a^2 = \frac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$$, this formula will be called parameter a^2(M_{VIR}, R_{VIR})

because depend on both measures.

4.2 PARAMETER a^2 FORMULA DEPENDING ON VIRIAL MASS ONLY

In chapter 3 was got this formula $R_{VIR}^3 = \frac{G \cdot M_{VIR}}{100 \cdot H^2}$ as a good approximation between virial mass and virial radius . So

using that formula and by substitution of virial radius in $a^2 = \frac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$ it is right to get parameter a^2 depending on

M_{VIR} only $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$. This formula will be called parameter a^2 (M_{VIR}) as depend on M_{VIR}
only.

With the virial data for some important clusters such as Virgo or Coma cluster will be calculated its parameter **a^2** with

the formula $a^2 = \frac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$ and with the formula $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$.

The last formula is an approximation of the previous formula as it is supposed that $R_{VIR} = R_{200}$. Below are calculated
both formulas and fortunately its relative difference is negligible.

Cluster	Virial Radius	Virial mass	Parameter a^2 (M_{VIR} , R_{VIR})	Parameter a^2 (M_{VIR})	Relativ diff.
	Mpc	$\cdot 10^{14}$ Msun	I.S. units $m^{5/2}/s^2$	I.S. units $m^{5/2}/s^2$	%
Virgo	1.7	6.3±0.9	3.6527E23	3.581E23	2
Coma	2.8	27	1.2198E24	1.2042E24	1.3

Green data come from [13] Olga Kashibadze.2020 and yellow data come from [7] Seong –A Oh,2023

Notice how close are both results for parameter a^2 especially when relative differences for masses are 11% and 7.4%

4.3 CHEKING M_{200} FORMULA WITH M_{200} PUBLISHED ABOUT MILKY WAY

In epigraph 3 was shown that $M_{200} = \frac{100 H^2 R_{200}^3}{G}$ or conversely $R_{200}^3 = \frac{G \cdot M_{200}}{100 \cdot H^2}$ and in epigraph 4 was shown the direct

mass $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$, which gives the total mass into the halo region. This formula is an important

consequence derived from DMbG theory. Using the value of M_{200} and R_{200} into the direct mass formula it is got

$M_{200}(<R_{200}) = \frac{a^2 \cdot \sqrt{R_{200}}}{G}$ and extracting the square root to R_{200} using the formula $R_{200}^3 = \frac{G \cdot M_{200}}{100 \cdot H^2}$ it is got an expression

without R_{200} parameter, where it is right to clear up the value for the mass, $M_{200} = \frac{a^{12/5} \cdot (10 \cdot H)^{-6/15}}{G}$.

This formula gives a new possibility to check the DMbG using the following procedure:

In epigraph 11.3 of [16] Abarca, M. 2023. was got parameter **a_{M-W}** = $3.9 \cdot 10^{10}$ m$^{5/4}$/s , for the MW galactic halo, being
this coefficient the optimal value to fit the rotation curve in halo region according the curve v= a·r $^{-1/4}$. Using the
central hypothesis of the theory was got the direct mass, which is a formula with radius unbounded as according
DMbG theory the DM is linked to gravitational field.

So comparing the value of M_{200} got by that formula with the value published as a result of sophisticated astrophysical
survey of Milky Way, it is possible to check the rightness of calculus and consequently the rightness of the DMbG
theory in general.

Firstly it will be calculated the value got by the formula $M_{200} = \frac{a^{12/5} \cdot (10 \cdot H)^{-6/15}}{G}$. As it depends on parameter H, it is needed to state his value, for example, H= 70 km/s/Mpc, and $\mathbf{a_{M\text{-}W}} = 3.9 \cdot 10^{10}$ m$^{5/4}$/s , so it is right to get M$_{200}$ = 8.976$\cdot 10^{11}$ $M_{\ominus}$ and using $R_{200}^3 = \frac{G \cdot M_{VIR}}{100 \cdot H^2}$ it is right to get R$_{200}$ = 199 kpc.

There are published a lot of studies about DM in Milky Way, but it has been select two recent works whose authors are prestigious astrophysicist. Namely the paper sources for the comparison are: [17] Jeff Shen , G. M. Eadie et al. and [18] E.V. Karukes, M.Benito et al.

M$_{200}$ COMPARISON WITH RESULT GOT BY JEFF SHEN TEAM`S

The clipping text below belong to the abstract of the paper [17] Jeff Shen.2022

Abstract

The mass of the Milky Way is a critical quantity that, despite decades of research, remains uncertain within a factor of two. Until recently, most studies have used dynamical tracers in the inner regions of the halo, relying on extrapolations to estimate the mass of the Milky Way. In this paper, we extend the hierarchical Bayesian model applied in Eadie & Juri to study the mass distribution of the Milky Way halo; the new model allows for the use of all available 6D phase-space measurements. We use kinematic data of halo stars out to 142 kpc, obtained from the H3 survey and Gaia EDR3, to infer the mass of the Galaxy. Inference is carried out with the No-U-Turn sampler, a fast and scalable extension of Hamiltonian Monte Carlo. We report a median mass enclosed within 100 kpc of $M(<100 \text{ kpc}) = 0.69^{+0.05}_{-0.04} \times 10^{12} M_{\odot}$ (68% Bayesian credible interval), or a virial mass of $M_{200} = M(<216.2^{+7.5}_{-7.5} \text{ kpc}) = 1.08^{+0.12}_{-0.11} \times 10^{12} M_{\odot}$, in good agreement with other recent estimates. We analyze our results using posterior predictive checks and find limitations in the model's ability to describe the data. In particular, we find sensitivity with respect to substructure in the halo, which limits the precision of our mass estimates to $\sim$15%.

Beside is remarked two important data: M$_{200}$ and R$_{200}$ $M_{200} = M(<216.2^{+7.5}_{-7.5} \text{ kpc}) = 1.08^{+0.12}_{-0.11} \times 10^{12} M_{\odot}$ as R$_{200}$ = 216.2 kpc and M$_{200}$ = 1.08 $\cdot 10^{12}$ $M_{\ominus}$ then it is possible to clear up the value for H because $R_{200}^3 = \frac{G \cdot M_{VIR}}{100 \cdot H^2}$ Namely this value is H= 67.817 km/s/ Mpc. As it is known the value of H is lightly different depending on technique used to measure it.

Now it will be calculated this value $M_{200} = \frac{a^{12/5} \cdot (10 \cdot H)^{-6/15}}{G}$ using H= 67.817 km/s/ Mpc. As $\mathbf{a_{M\text{-}W}} = 3.9 \cdot 10^{10}$ then M$_{200}$ = 9.09$\cdot 10^{11}$ $M_{\ominus}$ and R$_{200}$ = 204.1 kpc

The value for M$_{200}$ is only 6% lower regarding the lower mark got by Jeff Shen. Obviously both results are very close, because a relative difference of 6% regarding M$_{200}$ is totally negligible. From these results it is possible to state two main conclusions: the first one is that the Jeff Shen team´s has done a magnificent work, and the second one is that the Direct mass formula, using $\mathbf{a_{M\text{-}W}} = 3.9 \cdot 10^{10}$ is an excellent formula to measure the total mass (DM plus baryonic matter) in Milky Way and consequently this result is a new demonstration about the rightness of DMbG theory.

M$_{200}$ COMPARISON WITH RESULT GOT BY EKATERINA KARUKES TEAM`S

The clipped texts below belong to the paper [18] E.V. Karukes. 2020, placed in pages 10 and 11.

After marginalizing over all other parameters and model averaging over baryonic morphologies, we obtain the following determination of the Milky Way halo's dark matter virial mass $\log_{10} M_{200}^{DM}/M_{\odot} = 11.92^{+0.06}_{-0.05}$ or on a linear scale:

$$M_{200}^{DM} = 8.3^{+1.2}_{-0.9} \times 10^{11} M_{\odot}, \qquad (3.1)$$

sity, HPD, interval, i.e., the shortest interval containing 68% of posterior probability). Our estimate of the total mass of the Milky Way –the sum of baryons and dark matter– within the virial radius, is $\log_{10} M_{\text{tot}}/M_{\odot} = 11.95^{+0.04}_{-0.04}$ or on a linear scale:

$$M_{\text{tot}} = 8.9^{+1.0}_{-0.8} \times 10^{11} \, M_{\odot}. \tag{3.2}$$

As the reader may see, this time the agreement is perfect between the result of Karukes team`s $M_{200} = 8.9 \cdot 10^{11} \, M_{\odot}$ and the calculus got by the DMbG theory, using the Direct mass formula $M_{200} = 9.09 \cdot 10^{11} \, M_{\odot}$. Newly two conclusion arisen from these results: The first one is that the Karukes team`s has done a magnificent work and the second one is that the Direct mass formula, using $\mathbf{a_{M\text{-}W}} = 3.9 \cdot 10^{10}$ is an excellent formula to measure the total mass (DM plus baryonic matter) in Milky Way and consequently this result is a new demonstration about the rightness of DMbG theory.

4.4 CHEKING M$_{200}$ FORMULA WITH M$_{200}$ PUBLISHED ABOUT M31

The clipped text below is placed in the abstract of paper [19] Xiangwei Zhang.2024.

determined rotation curve, we have constructed a mass distribution model for M31. Our measurement of the M31 virial mass is $M_{\text{vir}} = 1.14^{+0.51}_{-0.35} \times 10^{12} M_{\odot}$ within $r_{\text{vir}} = 220 \pm 25$ kpc.

In page 9 is mentioned that it has been considered H = 70 km/s/Mpc and as virial mass the value M$_{200}$. Now it will be calculated M$_{200}$ using parameter **a** in the framework of DMbG theory.

In epigraph 9.1 of [16] Abarca, M. 2023. was got parameter $a_{M31} = 4.727513 \cdot 10^{10} \, m^{5/4} / s$ and using H= 70 km/s/Mpc it is right to get the value for $M_{200} = \dfrac{a^{12/5} \cdot (10 \cdot H)^{-6/15}}{G}$ being M$_{200}$ = 1.42·10$^{12} M_{\odot}$ and R$_{200}$ = 232.14 kpc.

So reader can check that the value calculated match fully with the data interval marked in the clipped text.
One more time the DMbG theory has overcome this test, this time using M31, the second best well known galactic rotation curve.

5. DARK MATTER IS COUNTER BALANCED BY DARK ENERGY AT ZERO GRAVITY RADIUS

This chapter is based on the 17 chapter in [1]Abarca,M.2023

The basic concepts about DE on the current cosmology can be studied in [9] Chernin,A.D.

According [11] Biswajit Deb. Plank satellite data (2018) give a new updated, Hubble constant, H = 67.4± 0.5 km/s/Mpc and a new $\Omega_{DE} = 0.6889 \pm 0.0056$. However currently there is a tension regarding Hubble constant as there are published by prestigious researchers others measures for H bigger than 70 Km/s/Mpc. In this paper will be used H= 70 Km/s/Mpc and $\Omega_{DE} = 0.7$ as the fraction of Universal density of DE.

5.1 ZERO GRAVITY RADIUS DEPENDING ON PARAMETER a^2 FORMULA
According [9] Chernin,A.D. in the current cosmologic model ΛCDM, dark energy has an effect equivalent to antigravity i.e. the mass associated to dark energy is negative and the dark energy have a constant density for all the Universe equal to $\varphi_{DE} = \varphi_C \bullet \Omega_{DE} = -6.444 \cdot 10^{-27} kg/m^3$ being $\Omega_{DE} = 0.7$ and $\rho_C = \dfrac{3H^2}{8\pi G} = 9.2\text{E-}27$ kg/m^3 the critic density of the Universe.

As DE density is constant, the total DE mass is proportional to Radius with power 3, whereas DM mass grows with radius power 0.5 so it is right to get a radius where DM is counter balanced by DE.

According [9] Chernin,A.D.The mass associated to DE is $M_{DE}(< R) = -\dfrac{\rho_{DE} 8\pi R^3}{3}$, or equivalently

$M_{DE}(< R) = -\varphi_{DE}\dfrac{8\pi R^3}{3} = \dfrac{-H^2 \cdot \Omega_{DE}}{G} \cdot R^3$. Notice that this author multiply by two the volume of a sphere i.e. he

considers that the effective density of dark energy is two times the $\varphi_{DE} = \varphi_C \bullet \Omega_{DE}$.

[9] Chernin defines gravitating mass $M_G = M_{DE} + M_{TOTAL}$, where M_{TOTAL} is baryonic plus dark matter mass, and defines R_{ZG} , Radius at zero Gravity as the radius where $M_{DE} + M_{TOTAL} = 0$. i.e. when the gravitating mass is zero.

This leads to equation $M_{TOTAL}(< R_{ZG}) = \varphi_{DE}\dfrac{8\pi R_{ZG}^3}{3}$. As Direct mass formula gives the total mass in the framework of

DMbG theory, the previous equation leads to $M_{DIRECT}(< R_{ZG}) = \dfrac{a^2 \cdot \sqrt{R_{ZG}}}{G} = M_{TOTAL}(< R_{ZG}) = \rho_{DE} \cdot \dfrac{8\pi \cdot R_{ZG}^3}{3}$, so it

is possible to clear up rightly $R_{ZG} = \left[\dfrac{3a^2}{8\pi G \rho_{DE}}\right]^{2/5}$ and as $\varphi_{DE} = \dfrac{3 \cdot H^2}{8\pi G}\Omega_{DE}$ then by substitution

$R_{ZG} = \left[\dfrac{a^2}{H^2 \cdot \Omega_{DE}}\right]^{2/5}$ This formula will be called R_{ZG} (parameter a^2).

For example, see [1] Abarca, M. 2023. In epigraph 14.1 was estimated parameter $a^2 = 4.428 \cdot 10^{21}$ (I.S. units) associated to Local Group, adding the four ones associated to MW, M31,LMC and M33, that as it is known they are the most massive galaxies in the Local Group. So using such value, the R_{ZG} for Local Group is 2.19073 Mpc

5.2 ZERO GRAVITY RADIUS FORMULA DEPENDING ON VIRIAL MASS

In previous chapter was got the value for $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$ depending on M_{VIR} as local parameter

only, so by substitution in R_{ZG} formula it is right to get $R_{ZG} = \dfrac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3}\Omega_{DE}^{2/5}}$ where the only local parameter is

M_{VIR} so this formula will be called $R_{ZG}(M_{VIR})$ and $R_{ZG} = K \cdot (M_{VIR})^{1/3}$ where $K = \dfrac{G^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \cdot \Omega_{DE}^{2/5}}$

Below are calculated R_{ZG} by two ways: R_{ZG} (parameter a^2) using Parameter a^2 (M_{VIR}) and R_{ZG} (M_{VIR}). It is remarkable how both calculus are mathematically equivalents as it was expected, i.e. when it is used the Parameter a^2 (M_{VIR}) to calculate R_{ZG} (parameter a^2) then match mathematically with R_{ZG} (M_{VIR}). See in table below how the grey values match perfectly.

Cluster	Virial mass	Parameter a^2 (M_{VIR})	R_{ZG} (parameter a^2)	R_{ZG} (M_{VIR})	Relatif diff.
	$\cdot 10^{14}$ Msun	I.S. units m$^{5/2}$/s^2			%
Virgo	6.3±0.9	3.581E23	12.871	12.871	0
Coma	27	1.2042E24	20.9069	20.9069	0

Green data come from [13] Olga Kashibadze.2020 and yellow data come from [7] Seong –A Oh,2023. With these

important cluster of galaxies, it has been illustrated how the total mass, calculated by $M_{DIRECT}(< r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$, is

counter balanced by dark energy at mega parsecs scale, and precisely this Radius at zero gravity determines the region size where the cluster has gravitational influence.

Data [4] R.Ragusa et al.2022			
Celestial Body	VirialRadius Mpc	Virial Mass 1E13Msun	Zero Grav R Mpc
Antlia cluster	1,28	26,3	9,62E+00
NGC596/584	0,5	1,55	3,74E+00
NGC 3268	0,9	8,99	6,73E+00
NGC 4365	0,32	0,4	2,38E+00
NGC 4636	0,63	3,02	4,68E+00
NGC 4697	1,29	26,9	9,69E+00
NGC 5846	1,1	16,6	8,25E+00
NGC 6868	0,6	2,69	4,50E+00

Beside has been calculated the zero gravity radius for the sample of group of galaxies or cluster published by [4] R.Ragusa et al.2022. See the blue columns.

The column in green shows the R_{ZG} using the formula by the virial mass $R_{ZG} = K \cdot (M_{VIR})^{1/3}$ where K=3.683309948·10^8 (I.S. units).

5.3 TOTAL MASS ASSOCIATED TO A CLUSTER OF GALAXIES

5.3.1 TOTAL MASS ASSOCIATED TO THE SPHERE WITH ZERO GRAVITY RADIUS

In epigraph 5.1 was shown how this equation $M_{TOTAL}(< R_{ZG}) = \rho_{DE} \cdot \frac{8\pi \cdot R_{ZG}^3}{3}$ is used to define R $_{ZG}$. By simplification

it is got $M_{TOTAL}(< R_{ZG}) = \frac{H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = M \cdot R_{ZG}^3$ being M= 5.3984·10^{-26} (I.S.) So that is the total mass formula.

In table below are shown calculus of total mass for Virgo, for Coma cluster and the Local Group of galaxies.

Cluster	Radius ZG Mpc	Parameter a^2 (M$_{VIR}$) m$^{5/4}$/s (I.S.)	M$_{TOTAL}$(<R$_{ZG}$)=M·R$^3_{ZG}$ Msun
Virgo	12.871	3.581E23	1.699945E15
Coma	20.9069	1.2042E24	7.28353E15
Local group	2.19073	4.28E21	8.379931E12

5.3.2 TOTAL MASS AT ZERO GRAVITY RADIUS USING THE VIRIAL MASS

Using rightly the direct mass formula $M_{TOTAL}(< R_{ZG}) = \frac{a^2 \cdot \sqrt{R_{ZG}}}{G}$

As in epigraph 4.2 was got $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$ and in epigraph 5.2 was got $R_{ZG} = \frac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$

by substitution in direct mass $M_{TOTAL}(< R_{ZG}) = \frac{a^2 \cdot \sqrt{R_{ZG}}}{G}$ it is got $M_{TOTAL}(< R_{ZG}) = \frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}} \cdot M_{VIR}$ calling $U = \frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}}$

then $M_{TOTAL}(< R_{ZG}) = U \cdot M_{VIR}$ being U≈ 2.6976.

So may be stated that according Dark matter by gravitation theory, the total mass (baryonic plus DM) enclosed by the sphere with radius R_{ZG} is equivalent to 2.7 times the Virial Mass.

Below are compared the masses for Virgo and Coma clusters using this formula and the previous one by the R_{ZG} to the cubic power. It is clear that both are mathematically equivalents.

Cluster	Virial mass $\cdot 10^{14}$ Msun	$M_{TOTAL}(< R_{ZG})$=U·M$_{VIR}$ Msun	M$_{TOTAL}$(<R$_{ZG}$)=M·R^3$_{ZG}$ Msun	Relatif diff. %
Virgo	6.3±0.9	1.6994E15	1.6994E15	0
Coma	27	7.2835E15	7.2835E15	0

Green data come from [13] Olga Kashibadze.2020 and yellow data come from [7] Seong –A Oh, 2023.

Using the previous calculus, by the direct formula for total mass, may be calculated easily the total mass associated to a radius, which is a fraction of R$_{ZG}$, with the following property: if $(R = f \cdot R_{ZG})$ then

$$M_{TOTAL}(< R) = M_{TOTAL}(< f \cdot R_{ZG}) = \sqrt{f} \cdot U \cdot M_{VIR}$$. because $M_{TOTAL}(< R_{ZG}) = U \cdot M_{VIR}$

For example, if R= 0.5·R$_{ZG}$ then $M_{TOTAL}(< 0.5 \cdot R_{ZG}) = \sqrt{0.5} \cdot U \cdot M_{VIR} \approx 0.707 \cdot U \cdot M_{VIR} \approx 1.9 \cdot M_{VIR}$

A second example using the data for Virgo cluster R$_{ZG}$ = 12.87 and R $_{VIR}$ =1.7 Mpc, so R $_{VIR}$ / R$_{ZG}$ = f = 0.13209 and by the formula $M_{TOTAL}(< R_{VIR}) = M_{TOTAL}(< f \cdot R_{ZG}) = \sqrt{f} \cdot U \cdot M_{VIR} \approx 0.98 M_{VIR}$.

The answer is not exactly Mvir because Rvir =1.7 Mpc is the experimental data. If Rvir is calculated by the formula $R^3_{VIR} \approx R^3_{200} = \frac{G \cdot M_{200}}{100 \cdot H^2}$, using the Mvir as M$_{200}$ then R$_{VIR}$ = 1.7687 Mpc and using this value $M_{TOTAL}(< f \cdot R_{ZG}) = \sqrt{f} \cdot U \cdot M_{VIR} \approx 0.999997 M_{VIR}$

5.3.3 CHEKING THE TOTAL MASS FORMULA INTO THE COMA CLUSTER

According [9] Chernin,A.D. they have estimated R$_{ZG}$ $\approx 20\ Mpc$, and M$_M$ (< R$_{ZG}$) = 6.2·10^{15} Msun. In Chernin´s paper, the M$_M$ represents baryonic matter and DM, which is called total mass in this paper and it is calculated by the direct mass formula.

In epigraph 5.2 was calculated R$_{ZG}$ $\approx 20.9\ Mpc$. which match perfectly with Chernin data. According the formula got in previous epigraph $M_{TOTAL}(< R_{ZG}) = U \cdot M_{VIR}$ In paper [9] Chernin,A.D, the authors do not give virial mass, so it will be considerate virial mass given by [7] Seong –A Oh,2023 M$_{VIR}$ = 2.7·10^{15} Msun so multiplying by factor U $M_{TOTAL}(< R_{ZG}) = U \cdot M_{VIR} = 7.29 \cdot 10^{15}$ Msun and by the formula got in 5.4 it is got that

$M_{TOTAL}(< 20 Mpc) = 7.13 \cdot 10^{15}\ Msun$ whose relative difference versus Chernin value is 15%, being a very good approximation.

5.4 TOTAL DARK ENERGY AT ZERO GRAVITY RADIUS

According [9] Chernin,A.D. $M_{DE}(< R_{ZG}) = -\frac{\rho_{DE} 8\pi R^3_{ZG}}{3} = \frac{-H^2 \cdot \Omega_{DE}}{G} \cdot R^3_{ZG}$ that it is just the opposite value to

$M_{TOTAL}(< R_{ZG}) = \frac{\sqrt[5]{100}}{\Omega^{1/5}_{DE}} \cdot M_{VIR}$ i.e. the total gravitating mass M$_G$(<R$_{ZG}$)=M$_{TOTAL}$ (<R$_{ZG}$)+ M$_{DE}$ (<R$_{ZG}$) enclosed

into the sphere of zero gravity radius is zero, as it was postulated as definition . M$_{TOTAL}$ (<R$_{ZG}$)+ M$_{DE}$ (<R$_{ZG}$) = 0

Therefore $M_{DE}(< R_{ZG}) = -\frac{\sqrt[5]{100}}{\Omega^{1/5}_{DE}} \cdot M_{VIR} = -U \cdot M_{VIR}$ being U≈ 2.6976 Joining both formulas it is got

$M_{DE}(< R_{ZG}) = -\frac{\rho_{DE} 8\pi R^3_{ZG}}{3} = \frac{-H^2 \cdot \Omega_{DE}}{G} \cdot R^3_{ZG} = -U \cdot M_{VIR}$ .This formula may be used to calculate easily the mass

associated to DE if the radius is a fraction of R_{ZG} .i.e. if $(R = f \cdot R_{ZG})$ then $M_{DE}(<R) = -\dfrac{\rho_{DE}8\pi R^3}{3} =$

$$= M_{DE}(< f \cdot R_{ZG}) = \frac{-f^3 \cdot H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = -f^3 \cdot U \cdot M_{VIR}$$

For example using the Virgo data $R_{VIR} = 1.7$ Mpc and $R_{ZG} = 12.871$ then f= $R_{VIR}/R_{ZG} = 0.13208$ and

$M_{DE}(< f \cdot R_{ZG}) = -f^3 \cdot U \cdot M_{VIR} = -0.00621 M_{VIR}$ So at this radius the DE may be considered negligible. However for bigger radius the DE begins to increase and finally at R_{ZG} is able to cancel the total mass (baryonic and DM).

5.5 GRAVITATING MASS FUNCTION

In the epigraph 5.1 was defined the gravitating mass $M_G = M_{DE} + M_{TOTAL}$, where M_{TOTAL} is baryonic plus dark matter mass and M_{DE} is the negative mass associated to DE. As the M_{DE} and M_{TOTAL} depend on the radius, it is got the function of gravitating mass depending on the radius by addition of both types of masses, the M_{DE} as a negative quantity and M_{TOTAL} as a positive quantity.

The best way to calculate the gravitating mass is using the formulas got in epigraph 5.3 and 5.4 where are calculated the both types of masses associated to a radius, which is a fraction of R_{ZG}, i.e. $R = f \cdot R_{ZG}$, these formulas are:

$M_{TOTAL}(< f \cdot R_{ZG}) = \sqrt{f} \cdot U \cdot M_{VIR}$. and $M_{DE}(< f \cdot R_{ZG}) = \dfrac{-f^3 \cdot H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = -f^3 \cdot U \cdot M_{VIR}$ and joining both

formulas it is got $M_G(<R) = M_G(< f \cdot R_{ZG}) = [\sqrt{f} - f^3] \cdot U \cdot M_{VIR}$ where $U \approx 2.6976$. This way the gravitating mass depends on the dimensionless factor f.

DOMINION OF GRAVITATING MASS FUNCTION

Although its mathematical dominion ranges from zero ad infinitum, the real dominion finish at R_{ZG} because at R_{ZG} the gravitating mass is zero so the gravitating mass depending on f finish at 1, and as the gravitating mass is defined by the cluster, its dominion begins at R_{VIR} so the gravitating mass depending on f begins at R_{VIR}/R_{ZG}

In chapter 7 it will be shown that $\dfrac{R_{ZG}}{R_{VIR}} = \left(\dfrac{100}{\Omega_{DE}}\right)^{2/5}$ =7.277 therefore it is concluded that the gravitating mass

depending on $f = R/R_{ZG}$ has as dominion the interval (0.13732, 1)

Gravitating mass function/Virial mass
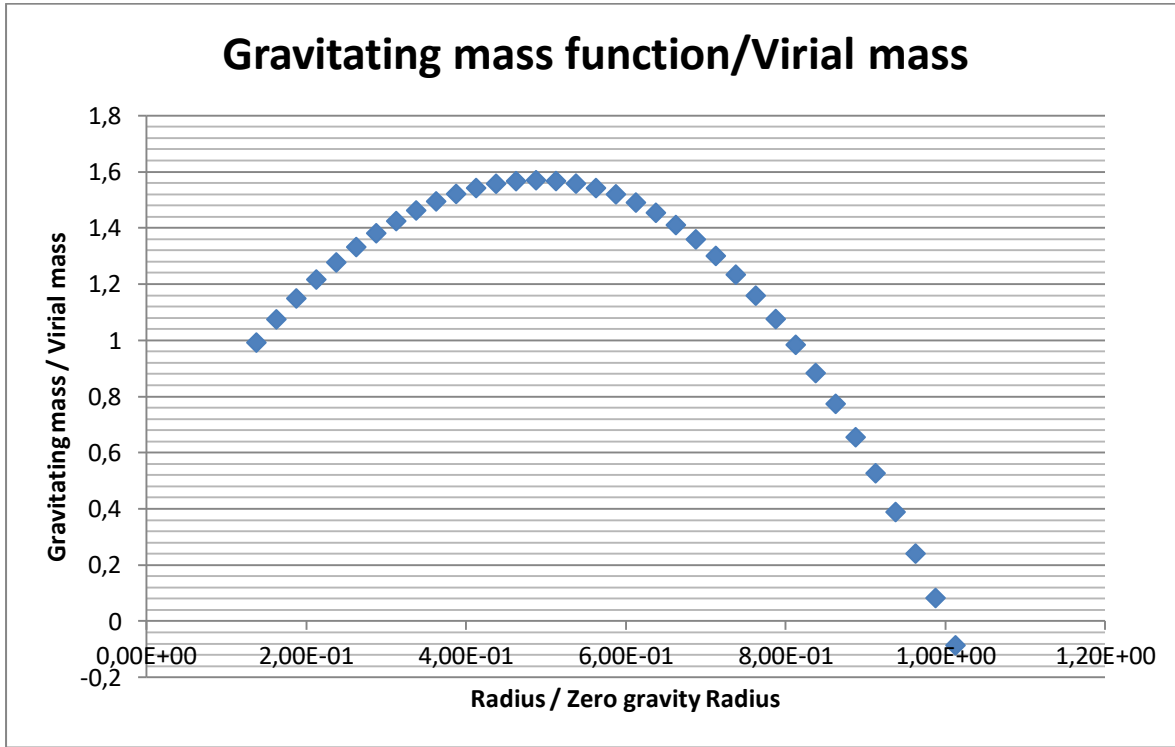
Above is represented the ratio gravitating mass/ virial mass versus the ratio R/R_{ZG} in its dominion and close to 0.5 there is the function maximum that will be calculated below.

5.7 CALCULUS FOR THE MAXIMUM OF GRAVITATING MASS

Given $M_G(<r) = -\varphi_{DE}\dfrac{8\pi R^3}{3} + \dfrac{a^2 \cdot \sqrt{R}}{G}$ It is clear that such function is zero at R=0 and at $R_{ZG} = \left[\dfrac{a^2}{H^2 \cdot \Omega_{DE}}\right]^{2/5}$ (See

epigraph 5.1) therefore there will be a maximum for the gravitating mass function, that it is found easily by

derivation, being $R_M = \dfrac{1}{\sqrt[5]{36}} \cdot R_{ZG} \approx 0.48836 \cdot R_{ZG}$. or $f = \dfrac{1}{\sqrt[5]{36}}$

By substitution of this value $R_M = \dfrac{1}{\sqrt[5]{36}} \cdot R_{ZG}$

$M_G(< R_M) = M_G(< f \cdot R_{ZG}) = \left[\sqrt{f} - f^3\right] \cdot U \cdot M_{VIR}$ where $U \approx 2.6976$ or $M_G^{MAXIMUM}(< R_M) = 1.57 \cdot M_{VIR}$

These findings are simple and impressing. Put in brief may be stated one interesting result, got by the Dark Matter by Gravitation theory:

For any cluster of galaxies at a half of R_{ZG} roughly, it is reached the maximum of gravitating mass which is 1.5 M_{VIR} roughly.

5.7.1 TOTAL MASS AT RADIUS WHERE GRAVITATING MASS HAS A MAXIMUM

In order to compare the gravitating mass with the total mass, now it is calculated the total mass using the direct mass formula with the property shown in epigraph 5.3. if $(R = f \cdot R_{ZG})$ then $M_{TOTAL}(< R) = \sqrt{f} \cdot U \cdot M_{VIR}$.

Now $R_M = \dfrac{1}{\sqrt[5]{36}} \cdot R_{ZG}$ So $M_{TOTAL}(< R_M) = \sqrt{36^{-1/5}} \cdot U \cdot M_{VIR} \approx 1.885 M_{VIR}$. Comparing this result with the

previous one $M_G^{MAXIMUM}(< R_M) \approx 1.571 \cdot M_{VIR}$ may be deduced that the DE enclosed into the sphere with radius R_M

Is $M_{DE} = -0.314\ M_{VIR}$. (1) Now it will be calculated that value with formula got in epigraph 5.5, using as $f = \dfrac{1}{\sqrt[5]{36}}$

so $M_{DE}(< f \cdot R_{ZG}) = -f^3 \cdot U \cdot M_{VIR} = -0.3142 M_{VIR}$ that match perfectly with the result (1).

6. ZERO VELOCITY RADIUS BECAUSE OF THE HUBBLE FLOW

It is defined the zero velocity radius as the distance to the cluster centre, where the escape velocity from gravitation field is equal to Hubble flow velocity. i.e. $V_E = V_{HF}$

From classical dynamic, the escape velocity formula is $V_E^2 = \frac{2GM}{R}$

The velocity of Hubble flow is $V_{HF} = H \cdot R$ or $V_{HF}^2 = H^2 \cdot R^2$ and by equation of $V_{HF}^2 = V_E^2$ it is got 2GM=H²·R³

It is needed to use the gravitating mass function depending on radius $M_G(r) = \frac{-H^2 \Omega_{DE} \cdot R^3}{G} + \frac{a^2 \cdot \sqrt{R}}{G}$ as into the cluster halo the DE is not negligible, so the gravitating mass is a simple way to include the DE into the gravitational phenomenon.

By substitution of the gravitating mass function into 2GM=H²·R³, it is right to clear up the value for the zero velocity radius. $R_{ZV} = \left[\frac{2 \cdot a^2}{H^2 \cdot (1+2\Omega_{DE})}\right]^{2/5}$ In the epigraph 4.2 was got parameter $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$. And

by substitution in the zero velocity radius is got . $R_{ZV} = \left[\frac{2 \cdot (GM_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}}{H^2 \cdot (1+2\Omega_{DE})}\right]^{2/5}$ or

$$R_{ZV} = \frac{2^{2/5} \cdot 10^{2/15} \cdot (G \cdot M_{VIR})^{1/3}}{H^{2/3} \cdot (1+2\Omega_{DE})^{2/5}}$$

Cluster	Virial mass	R_{ZV}	V_{HF}
	· 10¹⁴ Msun	Mpc	Km/s
Virgo	6.3±0.9	10.375	726.25
Coma	27	16.85	1179.65

In the table is shown the R_{ZV} and its Hubble flow velocity associated for the two clusters, using the previous formula.

6.1 ZERO VELOCITY RADIUS VERSUS ZERO GRAVITY RADIUS

The ratio between $R_{ZV} = \frac{2^{2/5} \cdot 10^{2/15} \cdot (G \cdot M_{VIR})^{1/3}}{H^{2/3} \cdot (1+2\Omega_{DE})^{2/5}}$ and $R_{ZG} = \frac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$ is a numerical value independent

from M_{VIR}. $f = \frac{R_{ZV}}{R_{ZG}} = \frac{\Omega_{DE}^{2/5} \cdot 2^{2/5}}{(1+2\Omega_{DE})^{2/5}} \approx 0.806058$

Cluster	Virial mass	R_{ZG} (M_{VIR})	$R_{ZV} = f \cdot R_{ZG}$
	· 10¹⁴ Msun		
Virgo	6.3±0.9	12.871	10.375
Coma	27	20.9069	16.85

Beside it is shown the R_{ZG} and R_{ZV} for these two important cluster of galaxies, using the ratio formula.

The R_{ZV} is the real border of the cluster because beyond this radius, any galaxy would be unable to fall toward the cluster, although between the range of radius $[R_{ZV}, R_{ZG}]$ the gravitational field is dominated by the cluster. However the Hubble flow velocity is strong than escape velocity and consequently any galaxy placed in that interval of radius will be able to run away from the cluster halo.

6.2 GRAVITATING MASS AT ZERO VELOCITY RADIUS

Using a procedure similar used to calculate the maximum of the gravitating mass, see epigraph 5.7, being $R_{ZV} = f \cdot R_{ZG}$ where $f \approx 0.806058$ by substitution at formula $M_G(< R_{ZV}) = M_G(< f \cdot R_{ZG}) = \left[\sqrt{f} - f^3\right] \cdot U \cdot M_{VIR}$ where $U = \frac{100^{1/5}}{\Omega_{DE}^{1/5}} \approx 2.6976$ it is right to get that $M_G(< R_{ZV}) \approx 1.009 \cdot M_{VIR} \approx M_{VIR}$.

Notice that $M_G^{MAXIMUM} < (R_M) \approx 1.5 \cdot M_{VIR}$ where $R_M \approx 0.5 \cdot R_{ZG}$ and $M_G < (R_{ZV}) \approx M_{VIR}$ where $R_{ZV} \approx 0.8 \cdot R_{ZG}$. Remember the epigraph 5.6 where is noted that gravitating function is decreasing in the range $[R_{ZV}, R_{ZG}]$.

7. ZERO GRAVITY RADIUS VERSUS VIRIAL RADIUS

In chapter 3 was got $R_{VIR}^3 = \frac{G \cdot M_{VIR}}{100 \cdot H^2}$ or $R_{VIR} = \left(\frac{G \cdot M_{VIR}}{100 \cdot H^2}\right)^{1/3}$ as a good approximation of R_{VIR} is R_{200}. By other side in previous chapter has been got

$$R_{ZG} = \frac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$$ so it is right to get the ratio $\frac{R_{ZG}}{R_{VIR}} = \left(\frac{100}{\Omega_{DE}}\right)^{2/5} = 7.277$ This is an awesome result, because

this ratio is Universal, it is not depend of virial mass.

Celestial Body	VirialRadius Mpc	Virial Mass 1E13Msun	Zero Grav R Mpc	Ratio R_{ZG}/R_{VIR}
Antlia cluster	1,28	26,3	9,62E+00	7,52E+00
NGC596/584	0,5	1,55	3,74E+00	7,49E+00
NGC 3268	0,9	8,99	6,73E+00	7,47E+00
NGC 4365	0,32	0,4	2,38E+00	7,45E+00
NGC 4636	0,63	3,02	4,68E+00	7,42E+00
NGC 4697	1,29	26,9	9,69E+00	7,51E+00
NGC 5846	1,1	16,6	8,25E+00	7,50E+00
NGC 6868	0,6	2,69	4,50E+00	7,50E+00

Beside is shown the ratio for a sample of clusters.

Columns in blue come from [4] R.Ragusa et al.2022

The second columns shows the virial radius for each celestial body.

Column in green is the R_{ZG} calculated and column in pink is the ratio. It is clear that the ratio R_{ZG}/R_{VIR} got in this sample of celestial bodies match very well with the value got by the theory.

Cluster	VirialRadius Mpc	Zero Grav R Mpc	Ratio R_{ZG}/R_{VIR}
Virgo C.	1.7	12.871	7.57
Coma C.	2.8	20.9069	7.467

The results got for the most prominent clusters are impressive as well.

8. CHEKING THE THEORY IN VIRGO CLUSTER

In the paper of [15] A. Simionescu, N et al.2017, in his abstract is published the M_{200} and R_{200} as a result of his X ray technique $M_{200} = 1.05 \pm 0.02 \times 10^{14} \ M_\odot, \ r_{200} = 974.1 \pm 5.7 \ kpc,$ on the left is the clipped text with data.

However in [13] Olga Kashibadze. 2020 , $R_{VIR} = 1.7$ Mpc $M_{VIR} = 6.3E14$ Msun which are totally different.

I have selected the Simionescu paper to show how two relatives recent papers gives a quite different value for two concepts that are very similar: The values for mass and radius at 200 times the critic density of Universe and the Virial mass and radius. This two different data show how difficult is to get accuracy measures even for a cluster not far away, 16.5 Mpc only. Anyway, it is reasonable to consider more trustable the more recent data.

8.1 TOTAL MASS ASSOCIATED UP TO THE ESTIMATED ZERO VELOCITY RADIUS AT 7.3 Mpc

obtained estimate of the total mass of the cluster. The virial mass of the cluster, being determined independently at the scale of $R_g = 1.7$ Mpc from the internal motions, is nearly the same - $M_{VIR} = (6.3 \pm 0.9) \times 10^{14} M_\odot$. The agreement of

Clipped text from the concluding remarks of [13] O. Kashibadze.2020

The value R_g is close to the virial radius.

So using $M_{200} = \frac{100 H^2 R_{200}^3}{G}$ as virial mass it is possible to estimate $R_{200} = 1.77$ Mpc. Anyway in this pape $R_{VIR} = 1.7$ Mpc

In epigraph 5.2 was got the general formula $R_{ZG} = \frac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$ for zero gravity radius and its value for Virgo cluster $R_{ZG} = 12.9$ Mpc. In chapter 6 was got the ratio $\frac{R_{ZV}}{R_{ZG}} = \frac{\Omega_{DE}^{2/5} \cdot 2^{2/5}}{(1 + 2\Omega_{DE})^{2/5}} \approx 0.8$ that gives $R_{ZV} = 10.3$ Mpc

At the concluding remarks [13] O. Kashibadze, the authors gives the range $R_0 = 7\text{-}7.3$ Mpc for R_{ZV} and a value for the total mass $M_T (<R_0) = (7.4 \pm 0.9) \cdot 10^{14}$ Msun. There is an important discrepancy between $R_0 = 7.3$ Mpc and $R_{ZV} = 10.3$ Mpc. However below it is shown how there is a reasonable matching regarding the total mass $M_T (<R_0)$ published and the gravitating mass calculated in the framework of DMbG theory.

As at R_0 radius the DE is not negligible it is needed to use the gravitating mass formula developed in epigraph

5.7 $M_G (< f \cdot R_{ZG}) = [\sqrt{f} - f^3] \cdot U \cdot M_{VIR}$ where $U \approx 2.6976$ so for $R_0 = 7.3$ Mpc and $R_{ZG} = 12.9$ Mpc f= 0.5659

and $M_G (< R_0) = 1.54 \cdot M_{VIR} = 9.7 \cdot 10^{14}$ Msun that differs 17 % only versus $M_T (<R_0) = (7.4 \pm 0.9) \cdot 10^{14}$ Msun, if it is considered the upper value at the interval of masses.

8.2 TOTAL MASS ASSOCIATED UP TO THE TWICE OF VIRIAL RADIUS

As $R_{VIR} = 1.7$ Mpc its twice value is 3.4 Mpc. As $R_{ZG} = 12.9$ Mpc then f = 0.2635 and $M_G (< 2 \cdot R_{VIR}) = \mathbf{1.335} \cdot M_{VIR} = = 8.4 \cdot 10^{14}$ Msun. This value match perfectly with the interval of masses given below in the clipped text.

As mentioned above, the Planck Collaboration (2016) performed a detailed study of the Virgo cluster through Sunyaev-Zeldovich effect and found the total mass of warm/hot gas to be $(1.4 - 1.6) \times 10^{14} M_\odot$. Assuming the cosmic value for the baryon fraction, $f_b = \Omega_b / \Omega_m = 0.1834$, they found that the total mass of the cluster would be $(7.6 - 8.7) \times 10^{14} M_\odot$ on a scale up to 2 times larger than the virial radius.

Clipped text from page 9 of [13] O. Kashibadze.2020

8.3 SOLVING THE CONUNDRUM: LOCAL DENSITY MATTER VERSUS GLOBAL DENSITY MATTER

As this topic has the highest importance I have clipped the text of a paper published for a team of prestigious astrophysicist.

As it has been noted by different authors (Vennik 1984, Tully 1987, Crook et al. 2007, Makarov & Karachentsev 2011, Karachentsev 2012), the total virial masses of nearby groups and clusters leads to a mean local density of matter of $\Omega_m \simeq 0.08$, that is 1/3 the mean global density $\Omega_m = 0.24 \pm 0.03$ (Spergel et al. 2007). One possible explanation of the disparity between the local and global density estimates may be that the outskirts of groups and clusters contain significant amounts of dark matter beyond their virial radii, beyond what is anticipated from the integrated light of galaxies within the infall domain. If so, to get agreement between local and global values of Ω_m, the total mass of the Virgo cluster (and other clusters) must be 3 times their virial masses. A measure of this missing

Clipped text from introduction of paper [12] Karachentsev I.D.,R. Brent Tully, et al. 2014

In page 3 of that paper, they state that at the nearby clusters the mean local density of matter is $\Omega_m = 0.08$, whereas the global mass density in the Universe is $\Omega_m = 0.24$. The authors suggest that a possible solution for this tension would be that the total mass for cluster haloes must be three times the virial mass. That is justly what is found in this paper studying the DM at cluster scale as universal law in the framework of DMbG theory.

In previous epigraphs has been found that $M_{TOTAL}(< R_{ZG}) = \dfrac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}} \cdot M_{VIR} = U \cdot M_{VIR}$, so the factor U $\approx$ 2.7 increase

the total mass (baryonic and DM) almost three times the virial mass associated to any cluster, this way, the Local matter density would be equal to Global matter density as it is expected at the present cosmology.

This result got through the DMbG theory might be a enormous success, because currently this problem remains unsolved, and conversely this result would be a formidable experimental back to DMbG theory.

9. SUMMARY

Thanks to the approximation of virial radius and virial mass for R_{200} and M_{200} it has been possible to get some impressive general results for clusters in the framework of Dark Matter by Gravitation theory.
This is the summary for the main formulas got in the paper.

In chapter 3 using the formula of Universal critic density it got this relation $M_{200} = \dfrac{100 H^2 R_{200}^3}{G}$ and using the

approximation of virial radius and virial mass by R_{200} and M_{200} it is stated $M_{VIR} \approx M_{200} = \dfrac{100 H^2 R_{200}^3}{G}$ This

approximation has been checked into a sample of clusters.
In 4.2 using the previous relation into the direct mass formula allow to get $a^2 = (G \cdot M_{200})^{5/6} \cdot (10 \cdot H)^{1/3}$ or

conversely $M_{200} = \dfrac{a^{12/5} \cdot (10 \cdot H)^{-6/15}}{G}$

In 4.3 and 4.4 is compared the value M_{200} calculated by this formula $M_{200} = \frac{a^{12/5} \cdot (10 \cdot H)^{-6/15}}{G}$, with the value published by prestigious astrophysicist about M_{200} in MW and M31. This comparison has been a successful test to check the rightness of DMbG theory.

In 5.2 is defined R_{ZG} as the radius where the total mass is counterbalanced by the negative mass associated to DE, and

it is got a formula for Radius of Zero gravitating mass, being $R_{ZG} = \dfrac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$

In 5.3 It is got the formula for total mass $M_{TOTAL}(< R_{ZG}) = \frac{H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = M \cdot R_{ZG}^3$ being M= $5.3984 \cdot 10^{-26}$ (I.S.)

In 5.4 is got the total mass associated to R_{ZG} as $M_{TOTAL}(< R_{ZG}) = \dfrac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}} \cdot M_{VIR} = U \cdot M_{VIR}$. Being U≈ 2.7

This result is a highlight of the DMbG theory because is able to increase the current local $\Omega_m = 0.08$ up to almost the global matter density $\Omega_m = 0.24$

In addition it is checked the R_{ZG} formula in the Coma Cluster, with [9] Chernin,A.D.et al. 2013.They give a value R_{ZG} = 20 Mpc, matching fully with result got by the formula (20.9 Mpc). Also the total mass calculated at this radius is 15% bigger versus the mass they have published, which is a very good approximation.

In 5.5 it is got the mass associated to DE at zero gravitating mass $M_{DE}(< R_{ZG}) = \dfrac{-H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = -U \cdot M_{VIR}$

In 5.6 is represented the ratio gravitating mass/ virial mass versus the ratio radius/R_{ZG}

In 5.7 is got the radius where the gravitating mass has a maximum, being $R_M = \frac{1}{\sqrt[5]{36}} \cdot R_{ZG} \approx 0.48836 \cdot R_{ZG}$

And it is got the gravitating mass associated to R_M, i.e. $M_G^{MAXIMUM}(< R_M) \approx 1.571 \cdot M_{VIR}$

In chapter 6 is defined and calculated the Zero velocity radius, being $R_{ZV} = \left[\frac{2 \cdot (GM_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}}{H^2 \cdot (1 + 2\Omega_{DE})} \right]^{2/5}$ also is

calculated the ratio R= $\frac{R_{ZV}}{R_{ZG}} = \frac{\Omega_{DE}^{2/5} \cdot 2^{2/5}}{(1 + 2\Omega_{DE})^{2/5}} \approx 0.8$ and it is got his gravitating mass $M_G(< R_{ZV}) \approx M_{VIR}$.

In chapter 7 is calculated the ratio for any clusters $\dfrac{R_{ZG}}{R_{VIR}} = \left(\dfrac{100}{\Omega_{DE}} \right)^{2/5} = 7.277$ this ratio is universal as it does not

depend on virial mass associated to a specific cluster.

In chapter 8 are checked some results got by the DMbG theory with some data published recently from Virgo cluster, see [13] O. Kashibadze, 2020.Namely the checked results are:

According the theory R_{ZG} = 12.9 Mpc for Virgo cluster which produces a R_{ZV} = 10.3 Mpc. The published data is R_{ZV} = 7.3 Mpc so that does not match with the theoretical value. However the gravitating mass calculated at 7.3 Mpc is only 17% bigger versus the data published at the same radius. Also there is a very good matching between calculus and data for mass calculated at 2·R_{VIR} = 3.4 Mpc being $M_G(< 2 \cdot R_{VIR}) = 1.335 \cdot M_{VIR}$

In addition, for the formula of total mass $M_{TOTAL}(< R_{ZG}) = U \cdot M_{VIR} \approx 2.7 M_{VIR}$ is postulated that the DMbG theory is able to solve the current discrepancy between the local density matter $\Omega_m = 0.08$ and the global density matter $\Omega_m = 0.24$

10. CONCLUDING REMARKS

In the framework of DMbG theory in [1] Abarca, M. 2023 the Direct mass formula for total mass associated to galaxies (baryonic plus DM) was extended up to clusters.

Thanks this formula and using the approximation of virial mass for M_{200} and the virial radius for R_{200}, it is possible to calculate, as universal formulas, the zero gravitating radius, R_{ZG}, being the ratio $R_{ZG}/R_{VIR} \approx 7.3$ and its total mass associated, being $M_{TOTAL}(<R_{ZG}) = 2.7 \cdot M_{VIR}$ Notice that by definition the gravitating mass at R_{ZG} is zero.

It is calculated the zero velocity radius, R_{ZV}, being the ratio $R_{ZV}/R_{ZG} \approx 0.8$, its gravitating mass $M_G(<R_{ZV}) \approx 1.0$ M_{VIR} The R_{ZV} is the real border of the cluster by reasons explained in its epigraph.

It is calculated the radius R_M, where the gravitating mass function reach a maximum, being the ratio $R_M/R_{ZG} \approx 0.49$ and its value at this radius results to be $M_G(<R_M) = 1.57 \cdot M_{VIR}$ Notice that the gravitating mass is decreasing at interval (R_M, R_{ZV}) and goes on decreasing up to R_{ZG} where $M_G = 0$

Although it has been stated that the R_{ZV} is the real border of the cluster, the gravitational field is not zero up to R_{ZG}, therefore it is possible to calculate the DM up to R_{ZG}, in fact one of the most important result of this paper is that $M_{TOTAL}(<R_{ZG}) = 2.7 \cdot M_{VIR}$,as universal law, and this result could be able to compensate the current low value for Local mass density $\Omega_m = 0.08$ up to quite close the global $\Omega_m = 0.24$ because $0.08 \cdot 2.7 = 0.216$ quite close to 0.24

As it is known the low density of matter at the Local Universe is a hard problem at the present cosmology. In the last chapter, the theory is checked in Virgo cluster with data from [13] O. Kashibadze.2020. The gravitating mass at $2 \cdot R_{VIR}$ $M_G(< 2 \cdot R_{VIR}) = 1.335 \cdot M_{VIR}$, this calculus match fully with data and the gravitating mass at 7.3 Mpc differs only a 17% bigger versus published data.

To conclude, I have selected three important formulas got in this work: The first one because allow increasing by the factor 2.7 the local matter density, so in my opinion may solve an important problem at the current cosmology.

The second one because is closely related to the distances between cluster neighbours. This result got by DMbG theory might be a test to check the rightness of the theory. Examining the distances between neighbour clusters is an affordable method to check this theory. Do not forget that the nature of DM is an open problem yet.

The third one is referred to gravitating mass i.e. the real mass that is possible to measure by dynamical measures. This formula shows that despite a half of R_{ZG} is 3.6 times the virial radius, the gravitating mass is only $0.5 \cdot M_{VIR}$ bigger versus the total mass enclosed by the virial radius. This theoretical calculus is compatible with results got by [12] Karachentsev I.D.,R. Brent Tully.2014.

$$M_{TOTAL}(<R_{ZG}) = \left(\frac{100}{\Omega_{DE}}\right)^{1/5} \cdot M_{VIR} \approx 2.7 \cdot M_{VIR}$$ The total mass associated to a cluster is 2.7·M_{VIR}, as a universal law.

The ratio $$\frac{R_{ZG}}{R_{VIR}} = \left(\frac{100}{\Omega_{DE}}\right)^{2/5} \approx 7.277$$ The ratio of zero gravity radius versus virial radius is 7.3 as universal law.

$$M_G^{MAXIMUM}(< 0.5 \cdot R_{ZG}) \approx 1.5 \cdot M_{VIR}$$ The maximum of mass gravitating is reached at a half of R_{ZG} and its value is only 1.5 M_{VIR}

11. BIBLIOGRAPHYC REFERENCES

[1] Abarca, M. 2023. viXra:2312.0002
 A dark matter theory by gravitation for galaxies and clusters-v2

[2] Abarca,M.2019. viXra:1909.0594
 A dark matter formula for Milky Way and M31 halo got by Buckingham theorem.V4

[3] Abarca,M.2019. viXra:1912.0233
Dark Gravitation Theory for M31 and Coma Cluster

[4] R.Ragusa et al.2022 arXiv:2212.06164v1
Does the virial mass drive the intra-cluster light? The relationship between the ICL and Mvir from VEGAS

[5] Sofue, Y.2015. arXiv:1504.05368v1   *Dark halos of M31 and the Milky Way.*

[6] Sofue,Y.2020.MDPI/Galaxies
Rotation curve of the Milky Way and the dark matter density.

[7] Seong –A Oh, Myung Gyoon Lee, In Sung Jang. ApJ,944:51.2023
Globular Clusters in NGC4839 Falling into Coma: Evidence for the Second Infall?

[8] Abarca,M.2016. viXra:1606.0279v1
Dark matter density on big galaxies depend on gravitational field as Universal law.

[9] Chernin,A.D. et all.2013. arXiv:1303.3800
Dark energy and the structure of the Coma cluster of galaxies

[10] Abarca,M.2016.vixra:1601.0242
A New DM Density Profile for M31 Galaxy to Demonstrate that DM is generated by the Gravitational Field.

[11] Biswajit Deb. 2022. arXiv:2207.10610v1
Constraining logarithmic f(R; T) model using Dark Energy density parameter and Hubble parameter H0

[12] Karachentsev I.D.,R. Brent Tully, et al. 2014. arXiv:1312.6769v2
Infall of nearby galaxies into the Virgo cluster as traced with HST

[13] Olga Kashibadze, Igor Karachentsev et al.2020. arXiv:2002.12820v1
On structure and kinematics of the Virgo cluster of

[14] Abarca,M.2023 viXra: 2304.0172
A Dark matter by gravitation theory got in M31 and Milky Way

[15] A. Simionescu, N. Werner, A. Mantz, S. W. Allen, O. Urban.2017. arXiv:1704.01236v1
Witnessing the Growth of the Nearest Galaxy Cluster: Thermodynamics of the Virgo Cluster Outskirts

[16] Abarca,M.2023 viXra: 2305.0170
A Dark Matter Theory by Gravitation Able to Calculate the Mass of Local Group of Galaxies-V2

[17] Jeff Shen , G. M. Eadie et al.2022.The ApJ, 925:1 (19pp). https://doi.org/10.3847/1538-4357/ac3a7a
The Mass of the Milky Way from the H3 Survey

[18] E.V. Karukes, M.Benito, et al.2020. arXiv:1912.04296v2
A robust estimate of the Milky Way mass from rotation curve data

[19] Xiangwei Zhang, Bingqiu Chen et al.2024.arXiv:2401.01517v1
The rotation curve and mass distribution of M31