Introduction to the Theory of Time Frames 1) Time Flow

Branimir Špigel (Independent Researcher)

Sveti Ilija, Croatia

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Email: spiegel.lab1@gmail.com

Abstract

This paper marks the beginning of a series of papers dedicated to establishing the theoretical foundations of a novel theory of relativity called the Theory of Time Frames. Due to the extensive nature of the topic, the theoretical basis of time frames will be divided into multiple papers, each addressing a specific aspect. This initial work focuses on developing a deeper understanding of the flow of time, which serves as a fundamental concept for the theory of time frames. Examining time dilation through the prism of the flow of time revealed a unique opportunity to introduce novel concepts: the background time flow, the gravitational time flow, and the kinetic time flow. Moreover, this exploration allowed for the establishment of the unit of time flow. These pivotal investigations laid a robust groundwork for delving deeper into the impact of the flow of time on various physical phenomena, offering a promising direction for further research in this domain.

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1. Introduction

Einstein published the special theory of relativity in 1905 [1] and the general theory of relativity in 1915 [2]. Since their publication, these theories have maintained a prominent place in physics. Despite this, these theories have faced numerous criticisms, especially regarding their logical formulations and conceptual underpinnings [3].

Both special relativity and general relativity introduce the concept of spacetime, combining the three dimensions of space and time into a four-dimensional continuum.

According to Einstein's general theory of relativity, gravity is not a force that acts at a distance as described by Newtonian physics but instead arises from the curvature of spacetime caused by the presence of mass and energy. This knowledge should have provided us with insight into the nature of the gravitational field, enabling us, one day, to perform artificial manipulation of the gravitational field or to neutralize the influence of the gravitational field on mass in any way.

However, even though more than a century has passed since the publication of the general theory of relativity, we have not come even close to understanding how to achieve this goal. That's why, despite the acceptance of Einstein's theory of relativity, there remains the question of whether the spacetime model truly corresponds to the mechanisms of nature. In other words, while the spacetime model simulates nature relatively well, it is possible that it falls short of describing the actual processes by which nature operates. In that case, spacetime would be nothing more than a fictitious model, useless for a deeper understanding of the mechanisms of nature.

If nature does not create gravity by bending spacetime, then gravity becomes an unknown process to us, and it will remain a mystery as long as we rely on the concept of spacetime.

This raises the question of whether spacetime is leading us to a dead end. Is this the reason we are still dealing with the primitive principles of rocket propulsion (albeit not in the sense of technology) instead of developing spacecraft propulsion systems based on our deeper knowledge of the mechanisms of gravity?

Do space and time bind us to the Earth and prevent us from using the inexhaustible resources of the universe?

These questions justify the exploration of alternative models regarding space and time. That could ultimately lead to a better understanding of the gravitational field and the achievement of many goals necessary for the expansion of human civilization in space.

Therefore, our focus is not to improve upon Einstein's theory of relativity but to establish the fundamental principles of a new theory of relativity called the "Theory

of Time Frames." This paper marks the first step in a series of works aimed at achieving this goal.

The theory of time frames adopts a three-dimensional space and one-dimensional time framework, treating space and time as independent entities. Space is considered unchanged, exhibiting homogeneity and isotropy, while time is viewed as a variable.

The concept of variable time suggests that time can flow at different rates in various regions of the universe, leading to localized temporal variations. This phenomenon is known as time dilation. The experimental evidence supporting time dilation justifies introducing the concept of a variable flow of time.

The primary goal of this paper is to offer a profound explanation of time dilation through the prism of the flow of time, considering the flow of time as a real, measurable physical quantity. Additionally, we introduce the unit of the flow of time and the concept of the maximum possible flow of time, termed the "background time flow."

This initial paper serves as an introductory foundation; subsequent works will delve deeper into specific aspects, providing a coherent and logical progression of the new theory of relativity and gravity.

2. Time dilation, and the flow of time (q_t)

Einstein's theory of relativity revolutionized our understanding of time by revealing its relativity and dynamic nature. Time is not an absolute, fixed entity, but a relative concept influenced by factors such as relative motion and gravitational fields. This is evident in the phenomena of kinetic and gravitational time dilation:

- Kinetic Time Dilation [4]: Special relativity explains time dilation as a result of relative motion between observers. This means that observers moving relative to each other will measure time as running at different rates.
- Gravitational Time Dilation [5]: General relativity accounts for time dilation caused by variations in gravitational fields. Clocks in stronger gravitational fields run slower compared to clocks in weaker gravitational fields.

Numerous experiments have provided strong empirical evidence for both kinetic and gravitational time dilation, such as the muon experiment, the Pound-Rebka Experiment (1959) [6], and the Häfele-Keating Experiment (1971) [7]. In the meantime, highly accurate atomic clocks have been developed, allowing the gravity dilation of time to be verified at a height difference of less than one meter.

In Einstein's theory of relativity, the mechanism behind time dilation is understood within the framework of four-dimensional spacetime and the curvature of spacetime.

However, the theory of time frames departs from this four-dimensional spacetime concept, adopting a three-dimensional space and one-dimensional variable time framework. To explain variable time, the concept of a variable flow of time will be used.

The flow of time is a relative quantity that can be compared with another flow of time. We cannot determine the absolute rate at which time flows, but we can compare the measurements of time between different frames of reference or clocks.

It should be emphasized that the exact underlying mechanisms and the fundamental nature of time itself are still open questions in physics.

These reasons explain why the concept of time flow has not been assigned a specific symbol or notation, nor a dedicated dimension (unit). Nevertheless, for the purposes of hypothesis and discussion, let us assume that the flow of time is a real and changing physical quantity denoted by q_t .

3. "Stretchable" unit of time

The definition of one second, according to the International System of Units (SI), is as follows:

"The second is defined as being equal to the time duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the fundamental unperturbed ground state of the cesium-133 atom."

This definition implies that the duration of one second is considered constant and not subject to change. However, it is essential to note that this unit of time, 1 s, is defined within the gravitational field on the surface of the Earth. Physical phenomena, such as gravitational time dilation and the relativistic effects of high-speed motion, can influence the measurement of time.

Clocks in stronger gravitational fields run more slowly than clocks in weaker gravitational fields, meaning that their time units differ. Consequently, the unit of time can be regarded as a variable ("stretchable") quantity, contingent on the local flow of time in which measurements are taken. In this context, the unit of time emerges as a variable entity.

For instance, due to gravitational time dilation, one second on Earth will be significantly shorter than one second near the horizon of a black hole. To perform such measurements, the start and end of clock measurements must be synchronized. Even though they began and concluded measurements simultaneously, clocks located in areas with different gravitational field strengths will display different elapsed times due to time dilation.

3.1. Example: Different duration of time units

Consider the scenario shown in Figure 1, where two bounded regions of space, denoted as S_1 and S_2 , are present. Inside space S_1 , the time flow q_{t1} is active, while within space S_2 , another time flow qt2 is active. The curve C illustrates that q_{t1} is slower than q_{t2} . In spaces S_1 and S_2 , clocks are positioned to measure elapsed time. Clock A is located in space S_1 , and clock B is in space S_2 .

Therefore, the passage of time q_{t1} influences the measurement of the elapsed time of clock A, whereas the passage of time q_{t2} influences the measurement of the elapsed time of clock B.

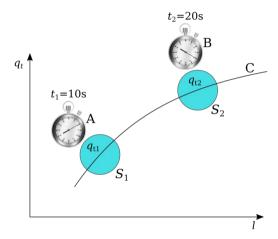


Figure 1 Time dilation measurement

A time diagram illustrating this example of time measurement is shown in Figure 2. Time measurements on both clocks start (t') and end (t'') simultaneously.

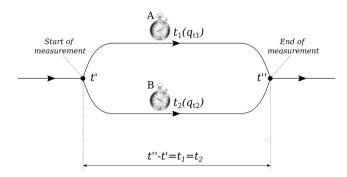


Figure 2: Measurements of time intervals t_g and t_0 start at time t' and end at time t''.

After the measurement, the clocks displayed the following elapsed times:

$$t_1 = 10 \text{ s} \text{ (clock A)},$$

 $t_2 = 20 \text{ s} \text{ (clock B)}.$

A discrepancy in measuring elapsed times arose, indicating time dilation. Clock A operated more slowly due to the slower time flow q_{t1} , whereas clock B ran more quickly due to the faster time flow q_{t2} .

Since the measurements started and ended simultaneously, the durations of t_1 and t_2 must be considered equal, hence:

$$t_1 = t_2 \tag{1}$$

It follows from the measurement values of the clocks that:

$$10 s = 20 s$$
.

This relationship is mathematically incorrect. However, it is important to note that clock A and clock B measured the elapsed time in different time flows, affecting the duration of their measurement units. That is why the elapsed time t_1 is expressed in seconds of a different duration than the seconds of the elapsed time t_2 . To distinguish these time units, they are marked with an index:

$$t_1 = 10 \text{ s}_1,$$

 $t_2 = 20 \text{ s}_2.$

1 s₁ is the time unit of clock A, affected by the flow of time, q_{t1} .

1 s₂ is the time unit of clock B, affected by the flow of time, q_{t2} .

The correct equality is now expressed as:

$$t_1[s_1] = t_2[s_2]$$
 (2)

 $10 \, s_1 = 20 \, s_2$

Consequently, it follows that:

$$1 s_1 = 2 s_2$$

The variability of time units is not a common practice in physics. However, in the context of time flow variability, it becomes essential.

Thus, units of time will be treated as variables, permitting mathematical operations to be performed with them. By substituting $1 s_1 = 2 s_2$ into the expression

 $t_1 = 10 \text{ s}_1$, we have:

$$t_1 = 10.2 \text{ s}_2$$

Thus, the time period t_1 , expressed in units of time s_2 , is:"

$$t_1 = 20 \text{ s}_2$$

Comparing $t_1 = 20$ s₂ with $t_2 = 20$ s₂, it is evident that both time periods are identical, confirming relation (1).

4. Background time interval (t_0) and gravitational time interval (t_g)

Gravitational time dilation occurs in regions with varying gravitational fields, resulting in different rates of time passage. A stronger gravitational field slows down time, while a weaker field allows time to flow faster.

The Karl Schwarzschild solution (3) of gravitational time dilation for a non-rotating spherical mass is a result of Einstein's general theory of relativity. It calculates the discrepancy in measured time intervals between two observers in regions with different gravitational fields:

$$t_g = t_0 \sqrt{1 - \frac{2GM}{rc^2}} \tag{3}$$

Where:

- t_g is the **gravitational time interval** measured by an observer (clock) in a strong gravitational field (near a massive object). The gravitational field slows down the flow of time. This affects the duration of the time unit of the clock that measures the elapsed time within that gravitational field.
- t_0 is the **background time interval** measured by an observer (a clock) who is far away from any massive object (in a weak gravitational field). The weak gravitational field can be ignored and considered to have no effect on the background time interval measurement. The background time interval is the reference time interval used for comparison with the time interval t_g measured in a stronger gravitational field.
- M stands for the mass of the object causing gravitational dilation.
- r is the distance from the center of the object causing gravitational dilation.
- G is the gravitational constant ($G = 6.6743 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$).
- c is the speed of light in a vacuum (c = 299,792,458m/s).

Measurements of the time intervals t_g and t_0 are shown in the time diagram in Figure 3. In order for both time intervals to be compared, it is necessary that the beginning of both measurements (t'), as well as the end of both measurements (t'), be simultaneous.

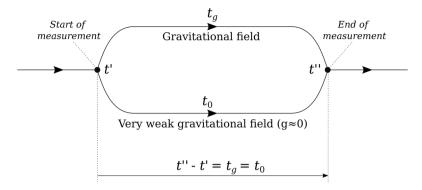


Figure 3 Measurement of t_g and t_0

Remark: The equation for gravitational time dilation (3) is formulated in accordance with the principles of the general theory of relativity. Since the general theory of relativity doesn't incorporate variations in time units for t_g and t_0 , it's necessary to use the same time unit (1 s) for both.

Let's consider, for example, that $t_0 = 5$ s₀. Instead of using 5 s₀ in equation (3), it's necessary to substitute it with 5 s. After the calculation, the time interval t_g will be obtained in the same time unit (1 s).

However, in further calculations that adhere to the principles of the theory of time frames, different units of time will be applied to t_g and t_0 , specifically 1 s and 1 s₀. This concept is elaborated upon in greater detail in the subsequent sections of this paper.

5. Background time flow (q_{t0}) and gravitational time flow (q_{tg})

As explained in the previous chapter, measurements of background time interval (t_0) and gravitational time interval (t_g) are performed in different strengths of the gravitational field, which also means in different time flows. Therefore, it is about two different flows of time:

• Background time flow (q_{t0})

The background time (t_0) , is particularly interesting as it can be regarded as a reference frame situated at an infinite distance from any mass. Consequently, the influence of gravity in this reference frame is effectively zero, leading to the absence of any slowing of the flow of time. This non-slowing flow of time will be called "background time flow," and denoted by q_{t0} . It is the fundamental flow of time that acts in the background of all space in the universe.

The background time flow serves as a reference for comparing it with other time flows that are influenced by gravitational fields or kinetic time dilation. The background time flow is the fastest possible rate of time flow in the universe.

• Gravitational time flow (q_{tg})

Gravitational time flow (q_{tg}) is the flow of time in a region of space under the influence of a gravitational field. The gravitational field acts to slow the flow of time.

The fastest gravitational time flow occurs when the gravitational field is approximately equal to zero. In this case, the gravitational time flow is equivalent to the background time flow ($q_{tg} = q_{t0}$). As the gravitational field increases, the difference between the background time flow and the gravitational time flow also increases.

The time flow diagram

The time flow diagram (Figure 4) illustrates the time flows q_{tg} and q_{t0} , along with their correlation to the strength of the gravitational field.

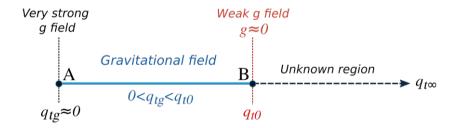


Figure 4 The time flow diagram

The area of action of gravity, and thus the gravitational slowing down of the flow of time (q_{tg}) , is between points A and B. At point A, where the gravitational field is exceptionally strong, time almost comes to a standstill, resulting in the gravitational time flow $q_{tg} \approx 0$. As we progress from point A to point B, the gravitational field gradually diminishes, leading to a corresponding increase in the time flow, q_{tg} . At point B, the gravitational field is approximately equal to zero, allowing q_{tg} to reach its maximum. The flow of time at that point corresponds to the background flow of time (q_{t0}) .

A hypothetical question arises regarding whether the time flow continues to increase beyond point B, eventually reaching an infinitely large time flow, $q_{t\infty}$.

6. Background second (1 s_0) and gravitational second (1 s)

The time intervals t_0 and t_g have different time units since they are measured in different time flows (q_{t0} and q_{tg}). The following time units will be used for this purpose:

Background unit of time, or background second (1 s₀)

The background time flow q_{t0} determines the duration of the time unit by which the background time interval t_0 is measured.

The time unit of the background time interval t_{θ} will be denoted as 1 s₀ and referred to as the "background second" or the "background unit of time." It serves as a measure for comparison with all other units of time.

• Gravitational unit of time, or gravitational second (1 s)

Gravitational time flow q_{tg} determines the duration of the time unit by which the gravitational time interval t_g is measured.

The time unit of the gravitational time interval t_g will be denoted as 1 s and referred to as the "gravitational second" or the "gravitational unit of time."

If several measurements are performed in different gravitational field strengths, the time units must be labeled with indexes to distinguish them (for example, $1 s_1$, $1 s_2$, $1 s_3$, etc.).

A time diagram with time flows, time intervals, and time interval units is shown in Figure 5.

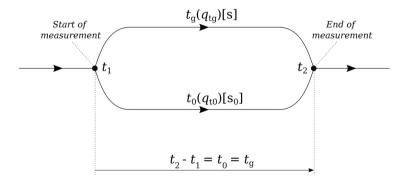


Figure 5

The background time flow (q_{t0}) is the fastest time flow. Therefore, gravitational time flow (q_{tg}) is always slower than background time flow. It follows that the duration of the background unit of time $(1 s_0)$ is always shorter in relation to the gravitational

unit of time (1 s).

7. Time flow unit (1 s/s_0)

In the realm of physics, a specific unit for the passage of time has not been formally defined. However, with the introduction of the background flow of time, q_{t0} , and its corresponding background time unit, 1 s₀, such a concept becomes attainable.

To address this, we can use the background unit of time, $1 s_0$, as the reference unit for all other time units.

Consequently, the unit for the time flow (q_t) is:

$$1\frac{s}{s_0}$$
, respectively $q_t[\frac{s}{s_0}]$

The notation used is as follows:

- 1s denotes a variable unit of time that changes under the influence of gravitational dilation of time (gravitational second) or due to kinetic dilation of time (kinetic second).
- 1 s₀ denotes the background unit of time (the background second). The background second is an immutable unit of time.

The flow of time is a relationship that describes how much time has passed in a given area of space in proportion to one background second $(1 s_0)$.

The following relation will be used to describe the flow of time in the gravitational field:

$$q_{tg} = \frac{t_g}{1 s_0} \left[\frac{s}{s_0} \right] \tag{4}$$

 t_g represents the elapsed time measured by a clock within a gravitationally time flow, measured in local time units, 1s.

At the same time, $t_g[s] = 1 s_0$, which means that the measurements of both time intervals started and ended simultaneously and lasted one background second (1 s₀). This is visible in the following time diagram in Figure 6.

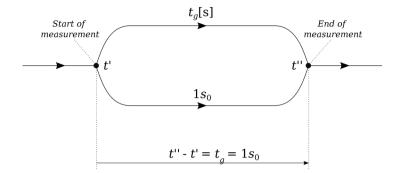


Figure 6

The flow of time in the gravitational field (q_{tg}) can also be expressed by the ratio of time intervals t_g and t_0 (see Figure 5). And in this case, measurements at both time intervals must start and end simultaneously, so that:

$$t_g[\mathbf{s}] = t_\theta[\mathbf{s}_0] \tag{5}$$

The flow of time in the gravitational field is:

$$q_{tg} = \frac{t_g}{t_0} \left[\frac{s}{s_0} \right] \tag{6}$$

The relationship 1 s = 1 s₀ is valid in the background time flow (q_{t0}) . Therefore, the background time flow amounts to:

$$q_{t0} = \frac{1 \, s}{1 \, s_0} = 1 \frac{s}{s_0} \tag{7}$$

The gravitational or kinetic flow of time is always less than the background time flow. That's why 1 s is always less than 1 s_0 .

Remark: It is evident from the above explanation that the numerical value of the flow of time defines the relationship between time units. Let's say, for example, that the flow of time in the gravitational field amounts to:

$$q_{tg} = 0.8 \frac{s}{s_0}$$

This means that during one background second $(1 s_0)$ in the area of the observed gravitational field, 0.8 s will pass. That is why the relationship between time units is valid:

$$1s_0 = 0.8 s$$

8. Time Dilation Example

Let's take the following example, shown in Figure 7.

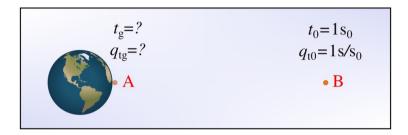


Figure 7 At point A, the Earth's gravity slows down the background time flow to the value q_{tg} . At distant point B, the background flow of time ($q_{t0} = 1 \text{ s/s_0}$) is active.

Point A is located on the surface of the Earth, where the Earth's gravity acts to slow down the flow of time. This gravitationally slowed flow of time is denoted by q_{tg} .

Point B is located far away from all gravity sources. There is a weak (negligible) gravity field at that point, but it has no effect on the flow of time. Therefore, the background flow of time, $q_{t0} = 1 \text{ s/s_0}$, acts at point B.

Let's assume that the clock at point B shows one background second ($t_0 = 1 s_0$). The time diagram in Figure 5 illustrates this situation. Our objective is to determine:

- 1. How much time has passed ($t_g = ?$) simultaneously at point A on Earth during one background second ($t_0 = 1 \text{ s}_0$) at point B?
- 2. The dilation of time ($\Delta t = ?$) between the elapsed time at point A compared to the elapsed time at point B.
- 3. The gravitational flow of time ($q_{tg} = ?$) on the earth's surface (at point A).
- 4. What is the difference between the flow of time ($\Delta q_t = ?$) at point A in relation to the flow of time at point B?

The calculation was performed according to the points mentioned above:

1. Gravitational time interval at point A (t_g =?) for the elapsed time $t_0 = 1 \text{ s}_0$ at point B

Using the time dilation formula (3), we can determine the elapsed time (t_g) , taking into account the following values:

- Mass of the Earth $M = 5.972 \times 10^{24} \text{ kg}$
- Radius of the Earth $r = 6371 \text{ km} = 6.371 \times 10^6 \text{ m}$

The length of the background time interval $t_0 = 1$ s₀ is given. However, in equation (3), instead of 1 s₀, the value 1 s must be included, as discussed in chapter 4 (remark).

$$t_g = (1s)\sqrt{1 - \frac{2G(5.972 \cdot 10^{24}kg)}{(6.371 \cdot 10^6 m)c^2}}$$

 $t_g = 0.9999999993 \text{ s}$

The following relationship corresponds to this:

$$t_0 = 1 \text{ s}_0 = 1 \text{ s}.$$

2. The time dilation ($\Delta t = ?$) between points A and B

It is questionable what the time difference (time dilation Δt) between t_g and t_0 is in order to achieve the relationship 1 s = 1 s₀.

In this case, the relationship applies:

$$t_{e} + \Delta t = 1 \text{ s} = 1 \text{ s}_{0}.$$

The time dilation Δt between the two points (A and B) can be calculated as follows:

$$\Delta t = 1 \text{ s}$$
 - $t_{g} = 1 \text{ s}$ - 0.9999999993 s = 7 x 10⁻¹⁰ s.

 $\Delta t = 0.7$ nanoseconds.

The clock at point A will run 0.7 nanoseconds slower for every background second on the clock at point B due to gravitational time dilation.

3. The time flow at point A $(q_{tg}=?)$

The gravitational flow of time (q_{tg}) due to the gravitational field on the Earth's surface (point A) can be calculated using relation (6).

$$q_{tg} = \frac{t_g}{t_0} = \frac{0.9999999993 \, s}{1 \, s_0} = 0.999999993 \, \frac{s}{s_0}$$

4. The difference between time flows at points A and B ($\Delta q_t = ?$)

We calculated the gravitational time flow $q_{tg} = 0.9999999993$ s/s₀ at point A. The background time flow $q_{t0} = 1$ s/s₀ operates at point B.

The difference between these two time flows is:

$$\Delta q_t = q_{t0} - q_{tg} = 1 \frac{s}{s_0} - 0.999999993 \frac{s}{s_0} = 7 \cdot 10^{-10} \frac{s}{s_0}$$

$$\Delta q_t = 0.7 \frac{ns}{s_0}$$

The flow of time on the Earth's surface is 0.7 ns/s_0 slower than the background flow of time.

9. Total time flow

Background time flow is the fastest time flow in our universe. As a result of the action of the gravitational field or the relative motion of an object, there is a decrease in the total flow of time (q_t) in the observed space.

Note: Every point on an object, including its atoms and particles, is in contact with space. The object does not displace space; rather, it exists within space, with all its components permeated by it. This is why it can be considered that the flow of time experienced by an object corresponds to the flow of time within the space it occupies.

The following relationship applies to the total flow of time:

$$q_t = q_{t0} - q_{tg} - q_{tk} (9)$$

where:

- q_t represents the total time flow experienced by an object,
- q_{t0} background time flow
- q_{tg} gravitational time flow
- q_{tk} kinetic time flow

Even when an object is not influenced by the gravitational or kinetic aspects of time's flow, the background time flow continues to exert an influence on it.

The background time flow constitutes a consistent element within the universal time flow. The origin of this background time flow is a subject of inquiry. Given its widespread presence throughout the universe, it is conceivable that it originated under the influence of the matter of the entire universe.

10. Summary and conclusion

In this paper, we laid the foundations of the theory of time frames. We rejected the concept of four-dimensional space-time in favor of a simpler approach based on three-dimensional space and one-dimensional variable time.

The central and pivotal concept in this theory is the flow of time, considered a genuine physical quantity that operates within space. To further clarify this concept and the relationships between different time flows, we introduced new physical notions such as the background time flow, gravitational, kinetic, and total time flows. We also defined the unit of time flow.

The background flow of time is understood as the fastest time flow in the universe, which can be slowed down due to the action of gravitational fields or the relative motion of objects.

Time units are now perceived as variable quantities, dependent on the local flow of time in which measurements are conducted.

This paper serves as an initial exploration of the theory of time frames, revealing numerous unanswered questions that require further investigation. In particular, a significant challenge lies in explaining the effect of the flow of time on light and moving objects within the theory of time frames. This is a task to be addressed in subsequent papers.

References

- [1] Einstein, A. (February 2004). <u>Relativity: the Special and General Theory by Albert Einstein</u>. Project Gutenberg.
- [2] Einstein, Albert, 1916. The Foundation of the General Theory of Relativity (Die Grundlage der allgemeinen Relativitätstheorie). Annalen der Physik 49 (7): 769–822.
- [3] Criticism of the theory of relativity. (2023, August 26). In Wikipedia. https://en.wikipedia.org/wiki/Criticism of the theory of relativity
- [4] Time dilation. In Wikipedia. https://en.wikipedia.org/wiki/Time dilation
- [5] Gravitational time dilation. In Wikipedia. https://en.wikipedia.org/wiki/Gravitational time dilation
- [6] Pound, R. V.; Rebka Jr. G. A. (November 1, 1959)."Gravitational Red-Shift in Nuclear Resonance". Physical Review Letters. 3 (9): 439-441.
- [7] Hafele, J. C.; Keating, R. E. (July 14, 1972). "Around-the-World Atomic Clocks: Predicted Relativistic Time Gains". Science. 177 (4044): 166–168.