$E = mc^2$ Used Indiscriminately Despite Its Derivations by Einstein Being Incorrect and Rudimentary

Karunesh M. Tripathi,

Retd. (vol.) Chief Engineer, Western Railway, Mumbai, India.

Author Email: karuneshtripathi@hotmail.com

M. Tech. Received June 1984, from I.I.T, Delhi, India.

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The relation $E = mc^2$ is considered as one of the triumphant colours of science and the general public treat it as an indelible signature of Einstein on the canvas of knowledge. The physicists have used the equivalence in almost all the domains of modern physics viz. atomic physics, nuclear physics, particle physics, quantum physics, astrophysics and cosmology etc.

A natural question arises whether the relation's derivation is so broad based as to cover the diverse types of particles/energies bound/released by diverse types of forces/energies under diverse conditions. To find the answer, there cannot be a better document than the derivation carried out by the father of the relation i.e. Einstein. He published two papers presenting the derivations – one in 1905 and the second in 1935 – with the titles as follows.

1. "Does the Inertia of a Body Depend upon its Energy Content?" (27-Sep-1905)
2. "Elementary Derivation of the Equivalence of Mass and Energy" 1935

Both are discussed briefly as follows.

1. The Paper of 1905:

The derivation is found to be too sketchy, apart from being questionable, to represent all kinds of mass-energy conversions. It proceeds by assuming that a body emits two light waves of equal energy in opposite directions which are oblique to x-axis. An observer moving with a uniform velocity along x-axis would find the energies of the two waves changed on account of Lorentz transformation (from stationary to moving frame). Since the two waves are of equal energy and emitted in opposite directions, the total transformed energy in the moving frame becomes independent of the angle of emissions due to cancelling out of terms dependent on the angle, and its value understandably turns out to be Lorentz Factor times that in the stationary frame. The difference in energy value due to change of frame is then equated with the supposed change in kinetic energy of the (light emitting) body, which in turn is assumed to be fully on account of diminution of its mass.

With such an assertion, where is the place for other kinds of energy which we keep exchanging with mass so extensively in nuclear and particle physics, and in General Relativity and cosmology? The question needs to be debated upon to correct our course in modern physics.

Further, such a scheme makes the derived relation vulnerable to modifications with slight change in the setup such as, if only one ray was emitted at the given angle, the relation became a function of the angle of emission. Further, if there was no moving observer, there would not be any change in energy of the emitted light waves on account of Lorentz transformation, rendering the diminution of mass of the emitting body to zero. This contradicted the very theory itself.

Even with such grave contradictions in the exercise, the author (Einstein) has dreamt at the end of the paper that "the theory may be successfully put to the test" for even the radioactive salts/elements. Carrying on the dream, we have extensively been using it in modern physics and also been making all out efforts to show conformity, rather than reviewing it in the contexts applied.

2. The Paper of 1935:

By this time, the concept of spacetime had significantly been popularized due to its cardinal role in the General Relativity.

For this reason probably, Einstein chose to present another derivation based on it. As will be shown ahead, he had taken the acceptance of the mass-energy equivalence for granted and simply pushed down his interpretation, though incorrectly, with the terms emanating from the Lorentz invariant of spacetime.

The salient steps are given below with my observations intermittently placed between two dotted lines.

The paper starts with the "*fundamental invariant of the Lorentz transformation*" as follows.

$$\boldsymbol{ds^2 = dt^2 - dx^2 - dy^2 - dz^2}$$

or

$$\boldsymbol{ds = dt(1-u^2)^{1/2}}$$

where

$$\boldsymbol{u^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

Further, It mentions - If one divides the components of the contravariant vector $(\boldsymbol{dt, dx, dy, dz})$ by $\boldsymbol{ds}$, one obtains the vector

$$\boldsymbol{\left(\frac{1}{(1-u^2)^{1/2}}, \frac{u_1}{(1-u^2)^{1/2}}, \frac{u_2}{(1-u^2)^{1/2}}, \frac{u_3}{(1-u^2)^{1/2}}\right)}$$

Observations 1:

It is pointed out here that the vector obtained just above has its elements only as numbers without any dimension/unit. This is because $\boldsymbol{dt}$ carries a scale of $\boldsymbol{1/c}$ which makes it a distance equal to $\boldsymbol{cdt}$, and the space elements already carry a scale of $\mathbf{1}$. Thus, $\boldsymbol{ds}$ also carries the same unit. Therefore, division by $\boldsymbol{ds}$ cancels out the units and makes the vector dimensionless. Similarly, the terms $\boldsymbol{u_1}$, $\boldsymbol{u_2}$, $\boldsymbol{u_3}$ and $\boldsymbol{u}$ are also dimensionless, as these are only fractions (with respect to $\boldsymbol{c}$).

It is clarified here that though some may name the above vector as a four-velocity vector in a different sense, there is no element of velocity (distance covered per unit time) in any of its elements, and these are mere dimensionless numbers.

Further, it has been argued that if the vector belonged to a material particle of mass $\boldsymbol{m}$, "*we obtain a vector connected with the motion of the particle by multiplying by* $\mathbf{m}$ *the four-vector of velocity that we have just written*...."

The first term of the vector, after multiplication by $\boldsymbol{m}$ becomes $\dfrac{\boldsymbol{m}}{(\mathbf{1}-\boldsymbol{u^2})^{1/2}}$, which on expansion and neglecting the third power of velocity becomes

$$\boldsymbol{m} + \frac{1}{2}\boldsymbol{mu^2}$$

It has further been argued that the second term is kinetic energy of the particle. Since the entire term was of energy, it is natural to "*ascribe to the mass-point in a state of rest the rest-energy* $\mathbf{m}$ *(with the usual time unit,* $\boldsymbol{mc^2}$*)*".

Observations 2:

As explained in my previous observations, the vector has no dimensions and calling it a four-velocity vector, in a different sense, does not make it a velocity for the purpose of calculating kinetic energy. Thus calling $\frac{1}{2}\boldsymbol{mu^2}$ as kinetic energy is a mistake. Further, the "*usual time unit*" $\boldsymbol{c^2}$ has incorrectly been assigned to first term $\mathbf{1}$, which is dimensionless, as the unit is already cancelled on division by $\boldsymbol{ds}$.

It remains a mystery as to why he did not extend the exercise to other three members of the vector. For when it is done, one gets incomprehensible terms for the three elements individually and for the entire vector as a whole.

The paper, however, does not stop here and maintains that a more complete proof can be presented by considering a system of particles and maintaining conservation of their energy and momentum in stationary as well as moving frames, and before-collision as well as after-inelastic-collision of two equal masses which changed after collision by equal amounts.

The total energy of the system is expressed as sum of the rest energy $\boldsymbol{E_0}$ and kinetic energy expressed in terms of the first element of the above vector (incorrectly as before). It is then shown that the change in mass due to the inelastic collision is equal to the change in the rest energy of the system. Therefore, the rest energy is the same as mass and vice versa.

The self-prophesy of equating the rest energy with the rest mass in this manner can hardly be accepted. The mistake of arbitrarily applying the unit of $\boldsymbol{c^2}$ to mass $\boldsymbol{m}$ has also been explained in Observations 2 above.

The Message:

Even if these mistakes are ignored for a moment, how can we apply a mass-kinetic-energy equivalence to all kinds of mass-energy equivalence, starting from nuclear to astronomical levels?

What about the models of Universe based on General Relativity which uses the current rudimentary and mistakes-ridden mass-energy equivalence in its metrics?

We need a derivation that caters to such diverse situations, or declare the relation as only an approximate starting point. This would keep the research and exploration on the right track. We may then possibly get rid of such frights as dark energy and dark matter.