## Einstein's Clock Observation While Riding a Train

Richard Kaufman (rdkaufman01 at gmail dot com)
In this note, we revisit Einstein's thought experiment for viewing Bern's clock tower with riding on a train. Einstein imagined observing Bern's clock tower while travelling on a train at the speed of light away from the clock. He concluded that he would not see the clock hands move. As the story goes, this inspired Einstein's work on Special Relativity, where he used an assumed constant speed of light, $c$, to show that time dilation and length contraction must occur for different observers moving at uniform velocity (greater than 0 ) relative to each other. This is a different interpretation of Lorentz's equation, where the formulas still hold.
Typically, time dilation is discussed in terms of considering the clocks for each of the two inertially moving observers. However, in the one-page paper that follows, we shall consider what a clock would show at different distances to the clock. Here, we will only consider the observations of the observer (us) stationary to the clock, and still show that the other observer would see a different time according to the light from the clock. We will observe that the moving observer will see the time on the clock slow down when moving away from the clock at a constant speed of $0.5 c$ (for example) and that this observer would see time on the clock speed up when returning to the clock at the same speed. Everything works out so that both observers see the same time on the clock at the beginning and end of the trip (at the clock's location).

Light travels to the right from a clock at the constant speed of light c. The times on the clock are shown next where equidistant locations are 1 light-hour apart. An observer at one of these locations would look back and see the time shown.


An hour later, the following times would be shown at each location:


If Einstein had been travelling on a train starting at position 12 p and moving to the right at the speed of light, then he would always see the clock showing 12 p.

Instead, suppose that Einstein was travelling at half the speed of light ( $0.5 c$ ) starting at 12 p (in the first figure) and moving to the right. Then after two hours on the clock, he would be at the position shown at 1p:


Although two hours would have passed on the clock, Einstein would see that the clock went from showing 12 p to 1 p. So, the clock would appear to be running slow (by 1 hour) according to what an outsider observer (us) observes that Einstein would see on the clock.

Now suppose that Einstein heads back to the clock at $0.5 c$. This time, the clock appears to move faster than it did before. After two hours on the clock, Einstein is at the location showing 4p.


Although two hours would have passed on the clock, Einstein would see that the clock went from showing 1 p to 4 p. So, the clock would appear to be running faster (by 1 hour) according to what an outside observer (us) observes that Einstein would see on the clock.

If Einstein had started at the location of the clock and travelled at $0.5 c$ to the right, we would observe that Einstein would see the clock appear to run slower. For his return to the clock at the same speed, we would observe that Einstein would see the clock appear to run faster. Everything would equal out so that Einstein sees the same time on the clock that it shows at the beginning and end of the trip (at the clock itself).

