Einstein’s 1905 Paper rides on Incorrect Angular Frequency to Forecast Redshift

Karunesh M. Tripathi,

Retd. (vol.) Chief Engineer, Western Railway, Mumbai, India.

Author Email: karuneshtripathi@hotmail.com

M. Tech. Received June 1984, from I.I.T, Delhi, India.

Abstract:

The relativistic Doppler Effect is explained from the theory of Relativity by applying an incorrect factor (γ) for the time dilation to the time period of the electromagnetic waves arriving the Earth from celestial sources. This leads to a forecast of redshift. However, if the correct ratio of time transformation i.e. $\sqrt{\frac{1-v/c}{1+v/c}}$ is applied, one gets a blueshift because the frequency, being inverse of the time period, changes in the ratio of $\sqrt{\frac{1+v/c}{1-v/c}}$. Thus Relativity, when applied correctly, leads to results just reverse of what is professed today.

A question may, however, arise as to how to reconcile the above statement with the redshift worked out by Einstein in his 1905 Paper, Section 7 titled “Theory of Doppler’s Principle and of Aberration”. The answer is he committed a mistake in working out the angular frequency of light in the moving (observer’s) frame.

Key Words: Special Relativity, Lorentz Transformation, Astrophysics, Cosmology

PACS Nos, 03.30.+p, 11.30.Cp, 42.25.Fx, 42.25.Bs, 98.62.Py, 95.30.Ky, 95.30.Sf, 98.80.Jk, 95.30.Ky

Introduction:

The Lorentz transformation, for the case of the primed frame moving with a uniform velocity v in (+)ve x-direction with respect to the non-primed frame (stationary), is as follows [1][2].

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$

In case of electromagnetic waves, since $x = ct$, the above relations get reduced to the following.

$$x' = \sqrt{\frac{1 - v/c}{1 + v/c}}\, x$$

$$t' = \sqrt{\frac{1 - v/c}{1 + v/c}}\, t$$

The relations show that the transformed distance as well as time traversed by the EM wave, in the frame co-directionally moving (observer) with the wave, get lesser than those existing in the stationary frame (source).

Applying the same result, the time period of the EM wave too would get reduced in the moving frame. This leads to conclusion that the frequency, being inverse of the time period, would increase for the moving frame i.e. observer, meaning a blueshift.

The same results are obtained by considering the source as the moving frame and the observer/receiver as the stationary frame.

The current practice, however, disregards the above facts and instead chooses to incorrectly apply a time dilation by a factor γ to the time period of the wave emitted by source (considered stationary) to work out its value in the moving (observer's) frame. This leads to redshift, which is taken as an evidence of correctness of the method. However, the fact remains that if the source was considered moving instead of the observer (in line with Relativity), the results change enormously, leading to a paradox, and this questions the validity of the existing practice.

Now, a question arises as to how Einstein managed to get a redshift on application of Relativity in his 1905 Paper, Section 7 titled "Theory of Doppler's Principle and of Aberration" [1]. The answer is he committed a mistake in working out the angular frequency of light in the moving (observer's) frame.

The same is explained below.

Discussion:

First, let us recall the parameters used in the paper.

The (t, x, y, z) are the parameters of the light wave in the stationary frame (source), and the corresponding parameters in the moving (observer's) frame are $(\tau, \xi, \eta, \varsigma)$.

The angular frequency parameters ω and ω' are similarly for the stationary and the moving frames respectively.

Similarly, (l, m, n) are the direction cosines of the wave normal (direction of ray) along the x, y and z axes respectively, and their corresponding values in the moving frame are (l', m', n').

Now, coming directly to the spot of the mistake, the following two relations (in bold) have been stated, which are correct **except the relation between $\boldsymbol{\omega'}$ and $\boldsymbol{\omega}$**.

[**Note:** Please note that β has been used here for the Lorentz Factor, in place of the current notation of γ]

$$\boldsymbol{\Phi = \omega\left\{t - \frac{1}{c}(lx + my + nz)\right\}}$$

$$\boldsymbol{\Phi' = \omega'\left\{\tau - \frac{1}{c}(l'\xi + m'\eta + n'\varsigma)\right\}}$$

where

$$\boldsymbol{\omega' = \omega\beta(1 - lv/c)}$$

$$\boldsymbol{l' = \frac{l - v/c}{1 - lv/c}}$$

$$\boldsymbol{m' = \frac{m}{\beta(1 - lv/c)}}$$

$$\boldsymbol{n' = \frac{n}{\beta(1 - lv/c)}}$$

$$\textbf{where } \boldsymbol{\beta = 1/\sqrt{1 - v^2/c^2}}$$

Observations:

The given expression of ω' is incorrect, as brought out below.

The times t and τ, in the stationary frame and the moving frame respectively, are related by

$$\tau = \beta(1 - lv/c)t$$

The same relation has also been used in the expressions for l', m' and n' whose denominators are $c\tau$, or $\beta(1 - lv/c)ct$.

Now, let the time period of the light wave in the stationary frame K and the moving frame k be T and T' respectively. Therefore, $\omega = 2\pi/T$ and $\omega' = 2\pi/T'$.

The transformation of time would apply to all timespans, including the time period T. Therefore, similar to the transformation $\tau = \beta(1 - lv/c)t$, the following relation is also true.

$$T' = \beta(1 - lv/c)T$$

Therefore,

$$\omega' = \frac{2\pi}{T'} = \frac{2\pi}{\beta(1 - lv/c)T} = \frac{\omega}{\beta(1 - lv/c)}$$

Thus the correct ratio of the angular frequencies in the two frames i.e. ω'/ω is just the inverse of what is worked out by Einstein.

Implications:

As a result, when the corrections are applied to the frequency, the forecast of a redshift in the light arriving from receding stars and galaxies, as experienced by the observers on the Earth, turn into a blueshift. The same is further corroborated below.

Einstein in his paper has gone ahead to reduce the general result to a case where the observer was moving in the direction of light received (from a receding luminous body) i.e. for $l = 1$.

Since he took $\omega'/\omega = \beta(1 - lv/c)$, it reduced to the following relation.

$$\nu' = \nu\sqrt{\frac{1 - v/c}{1 + v/c}}$$

Where ν' and ν are the frequencies in the observer's (moving) frame and the light-emitting-body's frame (stationary) respectively. That meant a redshift for observers on the Earth.

However, since the correct ratio of ω'/ω is just the inverse of what has incorrectly been taken, the above relation turns into

$$\nu' = \nu\sqrt{\frac{1 + v/c}{1 - v/c}}$$

which means a blueshift.

Conclusion:

The above exercise establishes beyond doubt that if the Special Relativity was correctly applied to the relative motion between observers on the Earth and the light emitting celestial bodies, the results were always blueshift. However, by incorrect application of a time dilation factor of γ, a redshift is shown to occur. While maintaining the theory of Relativity, if the source was considered moving and one applied the factor γ to its time, the results are entirely different. This leads to a paradox, declaring that the application of the theory is faulty.

On the other hand, if Relativity was correctly applied to either of the two frames, the results were the same.

It may be recalled that Einstein did not apply only the Lorentz Factor to the time of the stationary frame, to get the time of the moving frame. Instead, he has used the correct relation for time transformation, fully in accordance with the theory and its product, the Lorentz transformation. However, he failed to take note of the inverse proportionality between the angular frequency and the time period of waves.

The above discussions brings two options to the fore i.e. either the theory of Relativity is incorrect, or the notion of the redshift being on account of the theory is incorrect. The chances of the former are negligible, in view of the established constancy of light speed in uniformly moving frames. Therefore, we have to look for factors other than those of Relativity to find the answer for the redshift, which is currently thought of being on account of the so called relativistic Doppler Effect.

References:

1. A. Einstein, "On the Electrodynamics of Moving Bodies", June 30, 1905, distributed by http://www.fourmilab.ch/
2. A. Einstein's book "Relativity: The Special and The General Theory" 1916