cubic ellipsoid nucleus - part 1: the model and its mass formula

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Abstract

This study deals with the hypothesis that the nuclear structure determines the atomic shape and properties; this idea led to the construction of a tangible nuclear model, that provides a visual interpretation to nuclear and atomic physics and possibly enables drawing additional conclusions and new insights.

The model suggests that the nucleus has, in general, an ellipsoid shape with a cubic system of proton-neutron bonds and that the nuclear and atomic shells correlate with each other.
The cubic nuclear system perhaps provides clues that nucleons are composed of three sub-particles.

A simplified mass formula was developed to compare between the model and the experimental data; the formula is based on the model's theory and depends only on two parameters, unlike the common formula which is semi-empirical and contains five to six parameters, that are not an inherent part of the theory.
The two parameters, which are an integral part of the model are:

- d_0: the minimum distance between two neighboring nucleons.
- e_b: the binding energy between these neighboring nucleons.

It is assumed, for simplicity, that the values of these parameters are the same for all nuclei.

The calculation was implemented on 82 stable nuclei from Argon, Ar_{18}^{40} to Lead, Pb_{82}^{210} and provided relative errors smaller than 2% (average: 0.6%, standard deviation: 0.5%).

A byproduct of the process, that strengthened the assumption of the model, was the estimation of the combined radii of the proton and the neutron through $d_0 = (r_n + r_p)$, the distance between two neighboring nucleons, with a relative error smaller than 1.5%.

Following studies shall verify the results on more than 350 nuclei from Nitrogen, N_7^{14} to Plutonium, Pu_{94}^{244} and use the model to explain and calculate other nuclear topics as:

- the instability of heavy (beyond $Z \approx 82$) and superheavy nuclei (from $Z \approx 104$).
- the nuclear fission mechanism and the prediction of its most probable products.
- the larger nuclear stability of nuclei around Iron ($Z \approx 26$).
- the charge radius and the location of the excess neutrons in the nuclear envelope.
- the nuclear rotation and its conclusions regarding the angular velocity of pulsars (neutron stars) and their magnetic field inclination.

Further studies discuss also phenomena such as the nuclear magic number of protons, intermolecular bond angles, ferromagnetism, superfluidity and others in the light of the model.

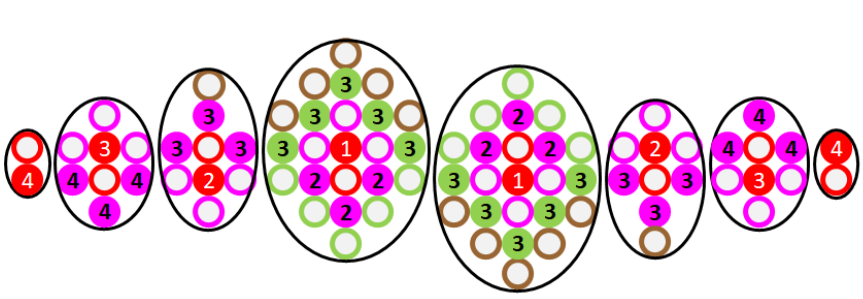
Content

The model at a glance

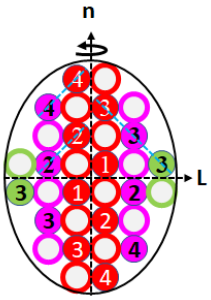
According to the model these are the shape and properties of the nucleus:

- the nucleus has an ellipsoid shape.
- the bonds between the nucleons form a cubic system.
- protons are connected only to neutrons (**p**-**n**).
- neutrons are connected mainly to protons.
- the protons are populated and organized in shells in the nucleus in analogy to those of the electrons in the atom.
- the energy layers of the protons (**n** - the principal quantum number) grow with their distance from the origin (the center of the nuclear ellipsoid).
- the perpendicular distance from the **z**-axis in the **x**-**y**-plane reflects the angular momentum (**L** - the sub-orbitals).
- we refer to each x-y layer as having either spin-up or spin-down. At this stage the spin is not crucial for the model development, we discuss it briefly in the appendix.
- the nucleus possibly rotates around its **z**-axis. Also this topic is not crucial at this point and will be discussed in subsequent studies.

As an example we observe the nucleus of Krypton Kr_{36}^{84}.
Following illustrations show the nuclear cross sections of its x-y planes along the z-axis and its cross section in the x-z plane.



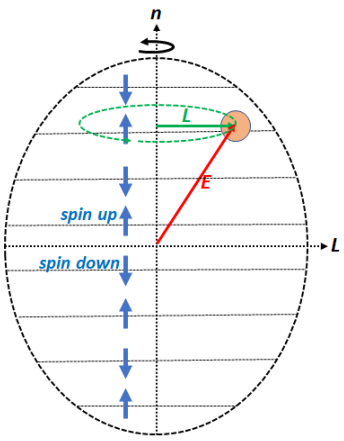
***x-y** planes along the **z** axis*



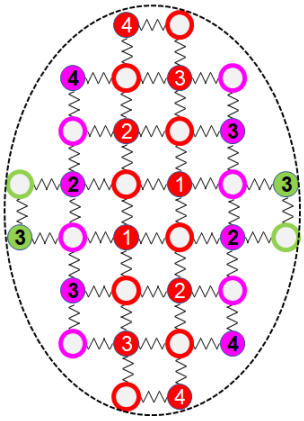
***x-z** plane*

***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**.*
***Numbers**: energy levels.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**, beyond the number equal to the protons (unpaired neutrons).*

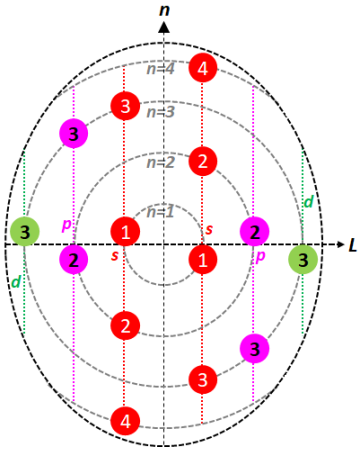
Following drawings describe the model via cross sections in the **x**-**z**-plane of the nucleus.



***1**: a nucleon in the nucleus*



***2**: the bonds between the nucleons*



***3**: the energy levels of the nucleus*

1. a nucleon (**circle**) is observed inside the ellipsoid (dashed line) that encloses the nucleons and schematically defines the nuclear surface:
   - the distance from the origin represents its energy **E**.
   - the distance from the **z**-axis depicts it angular momentum **L**.
   - **arrows**: spin of the nucleons in the x-y layer.

2. the bonds between the nucleons are shown for visibility as springs.
   - **protons**: full circles of the **s**, **p** and **d** sub-orbitals.
   - **numbers**: energy levels.
   - **neutrons**: hollow circles.

3. the circles of equal energy states **n** in the ellipsoid.
   - the lines mark the development of the **s**, **p** and **d** sub-orbitals along the **z**-axis.
   - the **s** line crosses all **n** circles from 1 to 4 (**s1** to **s4**).
   - the **p** line begins by **n**=2 and reaches till **n**=4 (**p2** to **p4**).
   - the **d** line begins by **n**=3 and reaches the ellipsoid border, before it reaches the **n**=4 circle, therefore there are no **d4** states at this stage (only **d3**).

Introduction

The nucleus and the atom are governed by different forces, have a size difference of about 5 orders of magnitude and according to current physics the nuclear shells have a different order than the atomic shells [10] in addition the nucleus is viewed as a point charge, so that its structure has no influence the shape and properties of the atom.
The hypothesis, that this research investigates, is that the nuclear structure determines the one of the atom; therefore an attempt was made to find a geometric model that could describe this and, at the same time, meet the requirements and constraints of the current theories of nuclear and atomic physics to justify this new perspective without contradictions.
The starting point was that the hypothesis holds and so, in the opposite direction, it is possible to learn and deduce from the atomic properties about the nuclear structure. Once such a model was obtained, a mass formula was created to test the feasibility of the model.

Before we continue with the research, we would like to provide a brief overview of the current state of physics in this field.

Nuclear models

There are several common nuclear models. Here we discuss them briefly and claim, that the suggested model doesn't contradict any of them [18].

The liquid drop model - semi classical fluid

As the name of this model hints, the nucleus is assumed to have a shape that makes sense from a physical point of view; a semi-empirical mass formula, with five (or six) parameters describes its nuclear energy [12, 13, 14]. In our model the physical shape is guaranteed via the ellipsoid; a mass formula is offered as well. The formula depends on only two parameters, which makes its implementation even more difficult. In contrast to the common semi-empirical approach, in our model the parameters are directly linked to the model.

The shell model

The nuclear shell model describes shells, that are different than those of atomic physics [10]. The nuclear shells are marked by the so-called magic numbers of proton and neutron, which are characterized by greater stability. In our research the nuclear shells have equal population and order as the atomic ones. This apparent contradiction is removed, if the predicted nuclear energy is provided correctly. The explanation, that will be discussed in following studies, is that the nuclear shape of the nuclei with magic numbers is more compact and possibly leads to alpha clusters, that increase nuclear stability.

The cluster model

The cluster model describes the nucleus as a molecule-like collection of proton-neutron groups [18]. This doesn't contradict our model, that sets proton-neutron pairs in its core and thus different clusters may be built inside it.

The nuclear shape

In general we assume in this research that the nuclear shape is of an ellipsoid, which is quite similar to the common assumption of a prolate spheroid (a rotational ellipsoid) [19].
The apparent difference is minor, especially if we keep in mind that the model is preliminary and our goal is to build a general picture that will lead to a new research direction.

Requirements

The nuclear properties

- The nuclear shape should make sense from a physical point of view.
- The system of bonds between the nucleons is assumed to be homogeneous and periodic; this means that the nuclear density (the distance between two neighboring nucleons) is assumed to be (at least nearly) constant.
- In a stable nucleus a proton is connected only to neutrons (**p-n** bond) because it is assumed that the **p-p** bond has a too strong electric repulsion; otherwise we could expect to observe a stable He_2^2 atom for instance (diproton) [16].
- In a stable nucleus a neutron is preferably connected with protons (**p-n** bond) because it is assumed that the proton stabilizes the neutron and that the **n-n** bond alone (with no protons involved) is unstable; otherwise we could expect to observe a stable **n-n** nucleus (neutronium) [17].
- The nucleus shall have the correct total nuclear spin.

A reflection of the atomic properties

If the research hypothesis holds, then the nuclear structure shall reflect the atomic properties:

- the atomic energy levels or shells.
- the orbitals and sub-orbitals and their population sequence.
- the same electronic properties for all isotopes of the same element.
- Pauli's exclusion principle.

A comparison with experimental data

A theoretical mass formula suitable for the model (unlike the common semi-empirical one) shall be constructed to test the matching between the theoretical and experimental data.

Results

The model

We get the following model, which is developed and explained in detail in the appendix below:

- The nuclear structure:
  - the shape of the nucleus is in general an ellipsoid.
  - it is composed of a cubic system of proton-neutron bonds.
  - the excess neutrons, beyond those that are paired with the protons, are located in the envelope of the ellipsoid.
- Properties:
  - the energy levels of the nucleons grow with their distance from the origin (the center of the ellipsoid).
  - the perpendicular distance of the nucleons from the **z**-axis (i.e. in the **x**-**y**-plane) depicts the angular momentum (and so the sub-orbitals).
  - each x-y layer has either spin-up or spin-down.
  - the nucleus possibly rotates around its main axis (the **z**-axis).
- The model attempts to assert the following:
  - an explanation of the structure of the periodic table: a justification of the atomic shells, the energy levels, the orbitals and sub-orbitals.
  - the correct nuclear population of protons and neutrons.
  - reasoning why different isotopes of the same element have equal electronic properties.
  - the correct nuclear spin.
  - it agrees with Pauli's exclusion principle.
- Examining the model:
  - the ellipsoid shape is reasonable from a physical point of view.
  - a theoretical mass formula that matches the model was created and delivered good results.

The Mass formula

The mass formula was built in accordance with the theory of the model (unlike the semi-empirical one):

$$m_{calc_x} = Z_x \cdot m_p + N_x \cdot m_n - \frac{(E_{b_x} - E_{c_x})}{c^2}$$

- A_x: atomic mass (number of nucleons) of the nucleus x.
- m_{calc_x}: the calculated mass of the nucleus x.
- Z_x: atomic number (number of protons) of the nucleus x.
- m_p: the mass of the proton.
- N_x: the number of neutrons ($N_x = A_x - Z_x$) in the nucleus x.
- m_n: the mass of the neutron.
- E_{b_x}: the total energy of the bonds between nucleons in the nucleus x.
- E_{c_x}: the total electric energy (between all protons) in the nucleus x.
- c: the speed of light.

$E_{b_x} = e_b \cdot n_{b_x}$:

- e_b: the energy of a single nucleon-nucleon bond in the nucleus (assuming they are equal for all bonds in all nuclei).
- n_{b_x}: the number of nucleon-nucleon bonds in the nucleus x.

$$E_{c_x} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{d_0}\left\{\frac{1}{2}\sum_i^{Z_x}\sum_{j\neq i}^{Z_x}\frac{1}{d_{i,j}}\right\} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{d_0}e_{c_x} \quad \text{where} \quad e_{c_x} \coloneqq \frac{1}{2}\sum_i^{Z_x}\sum_{j\neq i}^{Z_x}\frac{1}{d_{i,j}}$$

- d_0: the minimum distance between two neighboring nucleons (assuming all nuclei have the same cubic structure and distance between their nucleons).
- $d_{i,j}$: the unitless distance between the protons of the indices *i* and *j* measured in multiples of d_0: $d_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$
- e_{c_x}: the unitless total electric energy of the nucleus (sum of the reciprocal distances).

The absolute relative error of the calculation for the nucleus x is:

$$rel\_err_x = \left| \frac{m_{calc_x} - m_{meas_x}}{Z_x \cdot m_p + N_x \cdot m_n - m_{meas_x}} \right| = \left| \frac{m_{calc_x} - m_{meas_x}}{mass\_defect_x} \right|$$

- m_{meas_x}: the measured mass of the nucleus x.
- $mass\_defect_x$: $Z_x \cdot m_p + N_x \cdot m_n - m_{meas_x}$ is the mass defect of the nucleus x.
  Remark: rel_err_x is represented here in percentage.

The mass formula depends thus only on the two variables:

- e_b: the energy of a single nucleon-nucleon bond.
- d_0: the minimum distance between two neighboring nucleons.

The implementation requires two preliminary calculation steps for all nuclei:

- Drawing the nucleus and counting the number of nucleon-nucleon bonds n_{b_x}.
- Calculating the relative total electric energy of the nucleus e_{c_x}.
  Remark: these two steps are described in the appendix.

The mass formula calculation and results

The mass formula calculation was performed on isotopes of elements with larger abundant from Lithium, Li_3^7 to Plutonium, Pu_{94}^{244} (several elements have more than one isotope). Experimental data were taken from [1].
Nuclei till approximately Argon, Ar_{18}^{40} show larger relative errors than those of heavier nuclei, so we discuss in this work nuclei from Ar_{18}^{40}, as common while using a mass formula. In following studies we expand this to include the lighter elements and basically most nuclei. We also limit here the results for stable nuclei, meaning up to Lead although this is not required. The results of the relative errors of the mass formula calculation for 82 stable nuclei from Argon, Ar_{18}^{40} to Lead, Pb_{82}^{210} are:

max.	mean	st. dev.	rel. err. ≤ 2%	≤ 1%	≤ 0.5%
1.9%	0.6%	0.5%	100%	81%	58%

the relative errors of the mass formula calculation for nuclei from Ar_{18}^{40} to Pb_{82}^{210}

These results are within reasonable range [12], [15].
The calculation parameters were found as:

- $d_0 = 1.62 \pm 0.03\ fm$
- $e_b = 5.72 \pm 0.03\ MeV$

this seems to be within range as well [5].

We get through $d_0 \approx (r_n + r_p)$ a rough estimation for the radii of the proton and neutron:

- $r_n \approx 0.80\ fm$ [3] *(Neutron radius)*, $r_p \approx 0.84\ fm$ [4] *(Proton radius)*
- $r_n + r_p \approx 1.64\ fm$
- relative deviation for d_0: $\left|\frac{d_0 - (r_n + r_p)}{(r_n + r_p)}\right| = \left|\frac{1.62 - 1.64}{1.64}\right| = \left|\frac{0.02}{1.64}\right| \approx 1.3\%$

This strengthens the hypothesis of the model.

Discussion of the results and conclusion

The cubic ellipsoid nuclear model offers a perspective on nuclear and atomic physics, that is different than the common one, and justifies this idea via calculations. It does not appear to contradict these theories, but rather to expand their understanding and open new research directions of nuclear and atomic physics and possibly other fields.

The model aims to obtain the following:

- a tangible link between the nuclear structure and the atomic properties.
- a reflection of the structure of the periodic table in terms of the shells, the number of protons and neutrons for each isotope, and qualitatively for the energy levels and orbitals; it was built that way from the beginning, but here it was shown to be possible.
- the chemical properties of an element are independent of its isotopes; we therefore assume that its protons have the same spatial structure for all of its isotopes; this justifies the model assumption, that the excess neutrons are located in the nuclear envelope.
- the model is constructed also to delivers the correct total nuclear spin.
- the model complies with Pauli's exclusion principle.

The mass formula:

- it relates directly to the theory of the model (as opposed to the common semi-empirical mass formula) and therefore offers advantages from a physics perspective.
- it has only two parameters (as opposed to the common one with five to six parameters) and therefore delivers success even under more difficult conditions.
- the mass formula calculations are in good agreement with the experimental data. The results are in a range that is not far from the results of the common mass formula [12], [15], yet again, here the results are directly linked to the model.
- the distance d_0 between two neighboring nucleons agrees with the sum of the neutron and proton radii; this strengthens the model assumption and the concept of this mass formula.

The structure of the nucleons - a suspicion as a conclusion of the model:

- the nuclear cubic system may be explained due to a symmetry that the nucleus strives to achieve, but perhaps also due to three perpendicular waves or sub-particles, that make up the nucleon itself; this may be a hint that the nucleon is made up of three quarks.
- this idea is qualitatively described in the appendix of this article.

Issues:

The results of the model include issues that are not fully reasoned, described or solved in this article. We raise them here for a brief discussion.

- The nuclear core (the inner part with the proton-neutron pairs): at this stage its "exact" organization is offered only for nuclei with complete sub-orbital and more precisely only for the noble gases, but it seems to provide a good estimation also for the other nuclei.
- The excess neutrons (that are located, according to the model, in the nuclear envelope): their exact location determines their number of bonds and so the results of the mass formula. This issue shall be better discussed in the future. Currently the results seem to be good enough to believe that the model makes sense.
- In future studies (e.g. of the nuclear larger stability in the vicinity of $Z \approx 26$ (Iron), the instability of heavy nuclei and the nuclear fission) the nuclear core plays a major role and the position of the excess neutrons is less crucial.
- Light nuclei (smaller than Argon and especially smaller than Nitrogen): have a larger deviation from the mass formula calculation; the reason is possibly their structure that is not perfectly cubic or their density, meaning the distance between neighboring nucleons, that is slightly larger than it is by "well ordered" nuclei. Further research shall try to deal with this.
- The deployment of the spin of the nucleons is only briefly discussed in the appendix. This subject shall be analyzed in future research.
- The nuclear rotation is not discussed at this stage. It could lead to different distances between the nucleons along the z-axis and in the x-y plane in dependency of the sub-orbital. Also this is a subject of future studies.
- In connection with the rotation, the angular momentum is also not exact, since the sub-orbitals are not precisely radial. This topic will also be discussed in future studies.
- Additional assumptions that limit the model:
    - only proton-neutron bonds: this can be justified, if we assume that during nuclear growth, each new proton binds only with a neutron and vice versa.
    - The following points are not necessarily accurate, but place additional limitations on the implementation of the model and thus strengthen its statements and results.
        - a constant distance between neighboring nucleons for all nuclei.
        - the same nuclear cubic shape for all nuclei.
        - a constant binding energy between two neighboring nucleons for all nuclei.
        - the same nuclear core for all isotope of the same element: this is a result of the last paragraphs.

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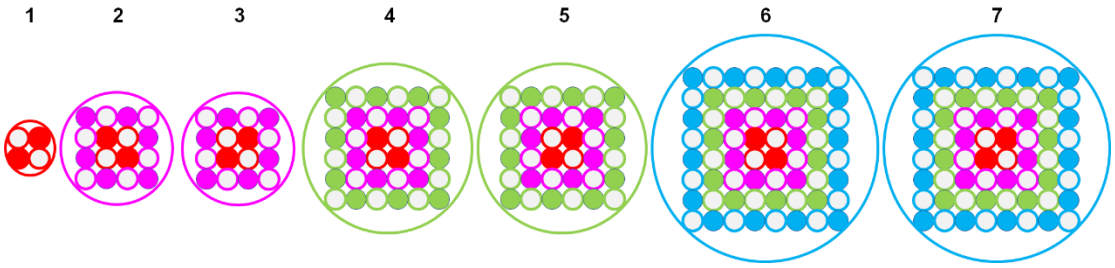
16. *Helium-2 (diproton) - Wikipedia*
17. *Neutronium - Wikipedia*
18. *Nuclear structure - Wikipedia*
19. The shape of the atomic nucleus - Wikipedia

Appendix

The development of the model

We describe in the following the development of the model in a visual way in order to make the idea more tangible.
The number of protons for closed layers of the periodic table is: **2**, **8**, **8**, **18**, **18**, **32**, **32**.
We assume that the protons and neutrons are ordered in pairs, so the number of nucleons is: **4**, **16**, **16**, **36**, **36**, **64**, **64**, or as square powers: $\mathbf{2^2}$, $\mathbf{4^2}$, $\mathbf{4^2}$, $\mathbf{6^2}$, $\mathbf{6^2}$, $\mathbf{8^2}$, $\mathbf{8^2}$.
We get square layers in a cubic structure placed on top of each other to form a square pyramid shape as shown in the following figure:



***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**.*
***neutrons**: hollow circles with colors according to their orbital.*

or as a table (**p**-protons, **n**-neurons, **A**=**p**+**n**: nucleons):

layer	p	n	A=p+n	p total	A total	S	P	D	F
1	2	2	4	2	4	2			
2	8	8	16	2+8=10	16+4=20	2	6		
3	8	8	16	10+8=18	20+16=36	2	6		
4	18	18	36	18+18=36	36+35=72	2	6	10	
5	18	18	36	36+18=54	72+36=108	2	6	10	
6	32	32	64	54+32=86	108+64=172	2	6	10	14
7	32	32	64	86+32=118	172+64=236	2	6	10	14

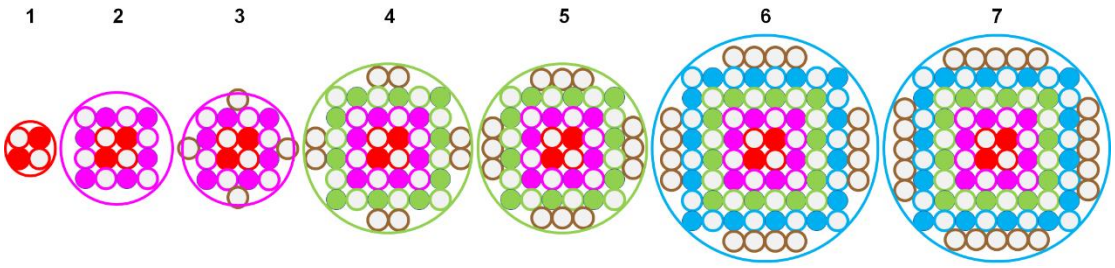
The interim result received for the noble gases is:
He_2^4, Ne_{10}^{20}, Ar_{18}^{36}, Kr_{36}^{72}, Xe_{54}^{108}, Rn_{86}^{172}, Og_{118}^{236}.

- The number of nucleons **A** (atomic mass) is correct for Helium and Neon.
- Argon has a stable isotope with 36 nucleons, but its most common isotope consists of 40 nucleons. [1]
- For nuclei larger than Argon the number of nucleons is larger than what we found.

Next assumption is that as the nucleus grows, excess neutrons are required to stabilize it. The model shall thus be expanded to explain the number of excess neutrons in each layer. This begins after Neon, i.e. from the third layer of the periodic table.

Completing the nuclear layers

In order to stabilize the nucleus, excess neutrons are added from the third layer.
We demand that each layer has more neutron than its predecessor under the assumption that this increases the nuclear stability. In each side there is one extra neutron with respect to the predecessor layer.
We receive the following pyramid shape:



***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**: beyond the number of the protons.*

The number of excess neutrons for each layer is **0**, **0**, **4**, **8**, **12**, **16**, **20** respectively, meaning a total of **4**, **16**, **20**, **44**, **48**, **80**, **84** nucleons at each layer.

layer	p	n=p+excess	A=p+n	p tot.	A total	S	P	D	F
1	2	2	2+2=4	2	4	2			
2	8	8+**0**=8	8+8=16	10	4+16=20	2	6		
3	8	8+**4**=12	8+12=20	18	20+20=40	2	6		
4	18	18+**8**=26	18+26=44	36	40+44=84	2	6	10	
5	18	18+**12**=30	18+30=48	54	84+48=132	2	6	10	
6	32	32+**16**=48	32+48=80	86	132+80=212	2	6	10	14
7	32	32+**20**=52	32+52=84	118	212+84=296	2	6	10	14

The results received for the noble gases are:

He_2^4, Ne_{10}^{20}, Ar_{18}^{40}, Kr_{36}^{84}, Xe_{54}^{132}, Rn_{86}^{212}, Og_{118}^{296}.

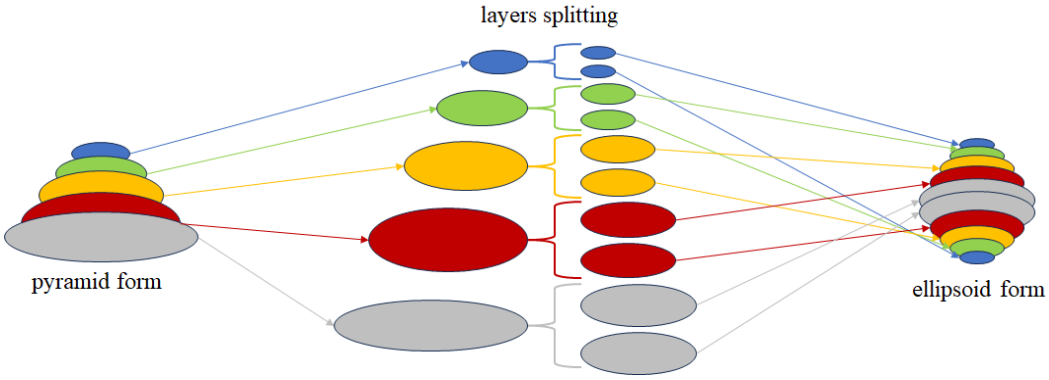
- for the stable nuclei (up to Xenon) these results are correct.
- Radon and Oganesson are radioactive and are large.
- For Radon Rn_{86}^{212} is an isotope with a half-life in the range of minutes [1].
- About Oganesson there is not much data, but Og_{118}^{296} is expected to be one of its isotopes [6] [7] [8].

Splitting the layers to form an ellipsoid

In the last section we reached a nuclear model with a pyramid form, that provides the correct number of protons and neutrons for each layer.
The pyramid shape has several disadvantages, such as:

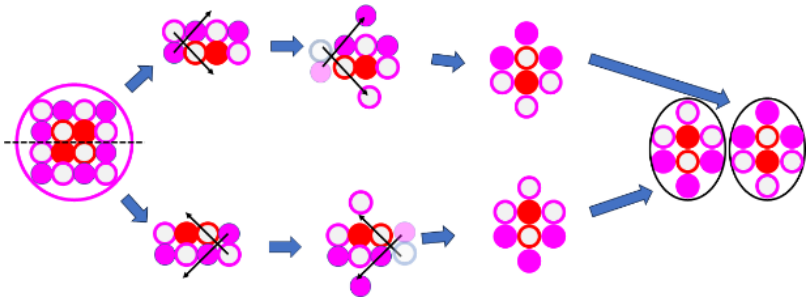
- it is not symmetrical with respect to the nuclear center.
- it does not fit the idea of the energy levels.
- its shape is not physical, unlike, for example, the liquid drop model.

To solve these issues, we split each layer of the pyramid in two parts, one at the positive side of the **z**-axis and the other at its negative side as shown is the next illustration:



splitting the pyramid and constructing an ellipsoid

the process of splitting a specific layer is shown in the next illustration. The layer is split in the middle; each part is reorganized to get an oval form; each layer has either spin-up or spin-down.



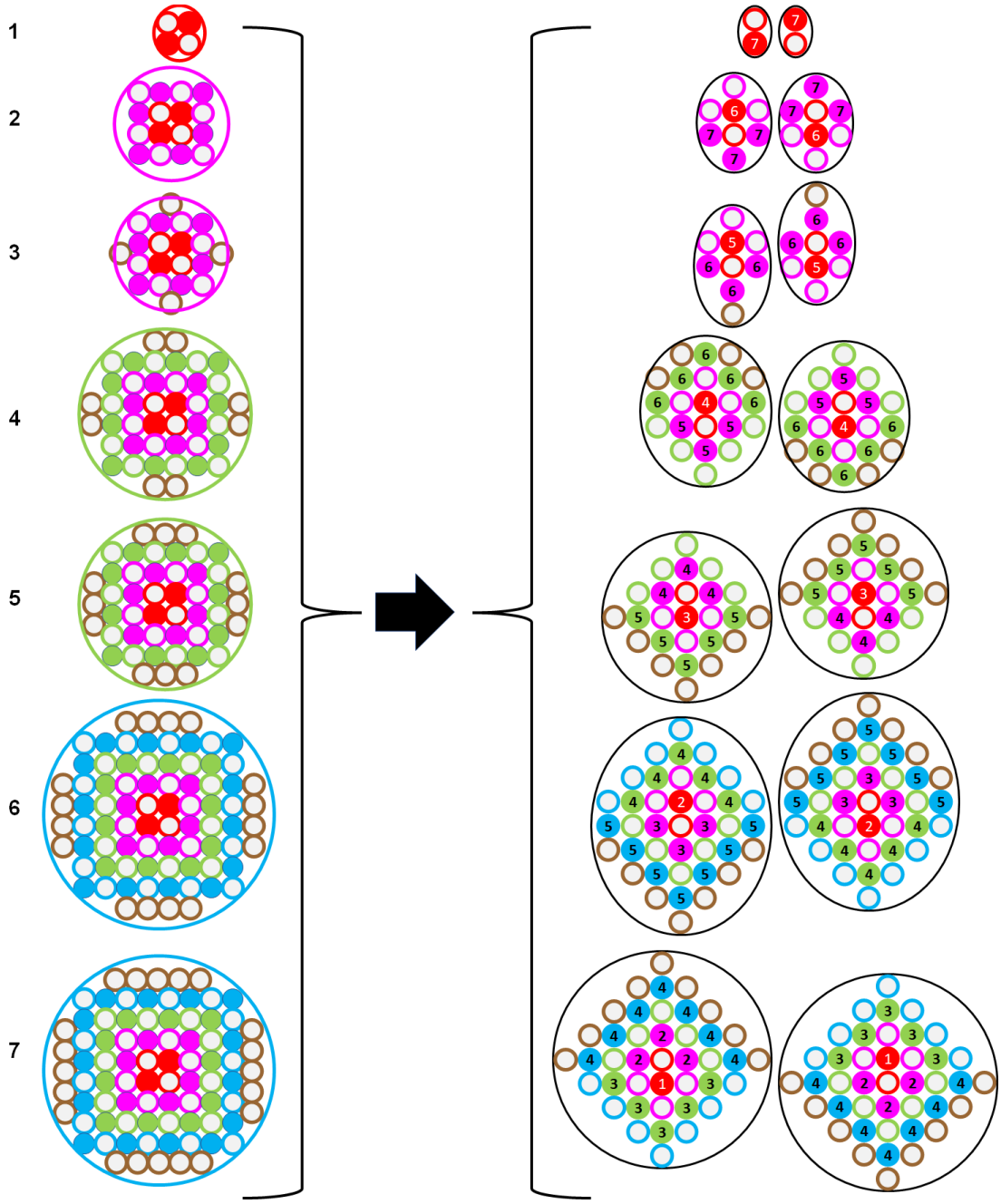
splitting a specific nuclear layer

This is the shape of the nucleus, that the model suggests.
We've reached:

- a shape that makes sense from a physical point of view.
- a model that delivers the correct number of protons and neutrons for each layer.
- an interpretation of the energy levels and the sub-orbitals.

The next illustration shows this in detail.

Splitting the pyramid leads to the following nuclear structure:



***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**.*

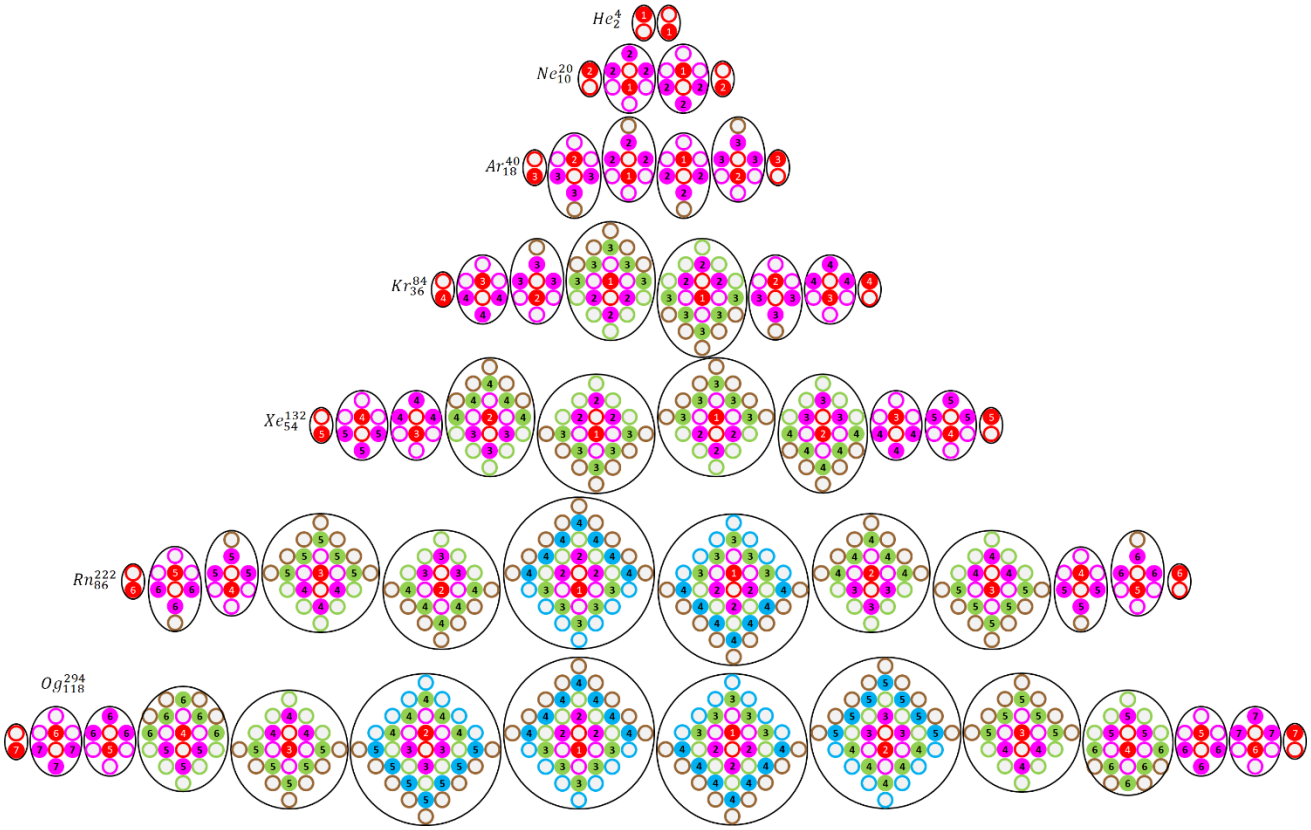
***Numbers**: energy levels.*

***neutrons**: hollow circles with colors according to their orbital.*

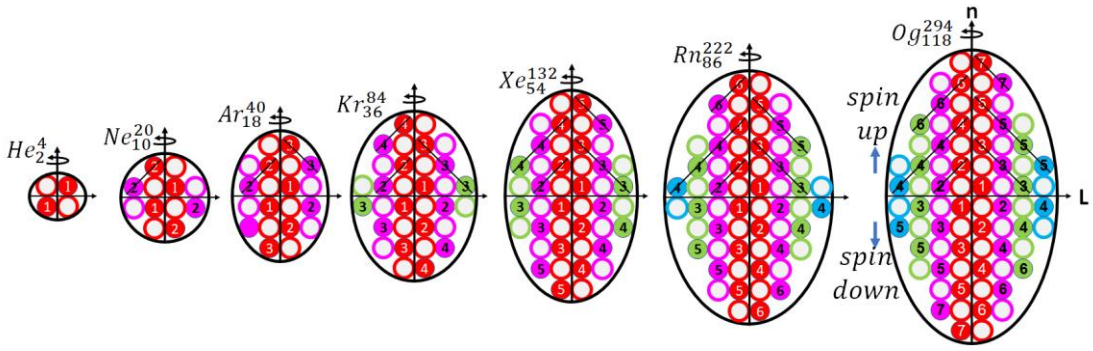
***excess neutrons**, beyond the number equal to the protons (unpaired neutrons).*

The nuclei of the noble gases

As a result of the model structure we've reached in the last section, we can now draw the nuclei of the noble gases:



***Nuclear layers**: cross sections in the **x-y** planes along the **z** axis.*
***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**. **Numbers**: energy levels.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**: beyond a numb equal to the protons.*

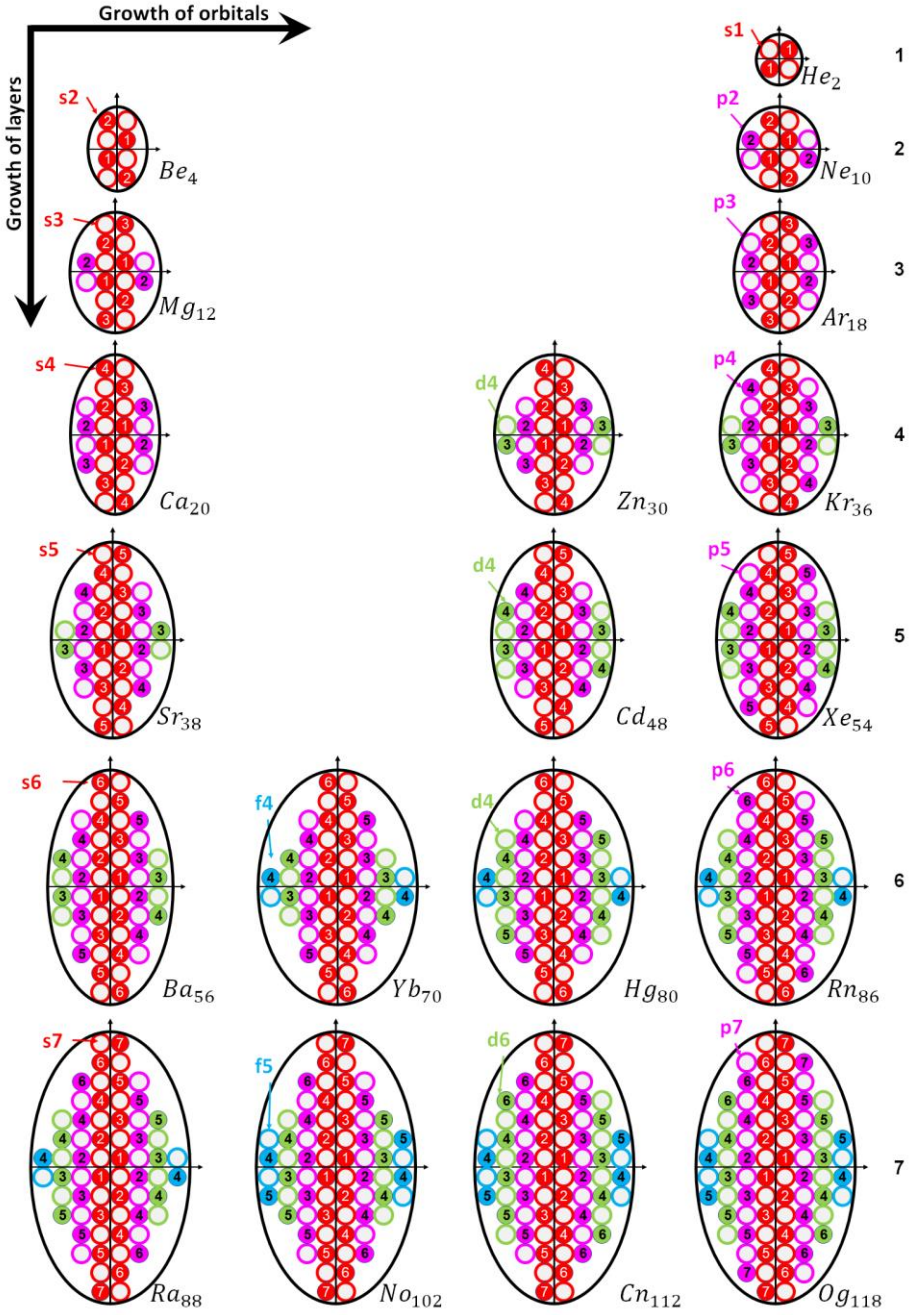


***The layers and energy levels of the nuclei:** a cross section in the **x-z** plane.*
***excess neutrons**, beyond a number equal to the protons, are not shown.*

The ellipsoids of the full sub-orbitals of the periodic table

In order to better understand the model, the ellipsoids of the full sub-orbitals are shown and ordered as they appear in the periodic table. The orbitals grow from left to right and the layers grow from top to bottom; the colored arrows refer to the last filled orbital.

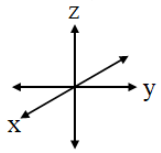
Cross sections in the x-z plane of the ellipsoids of the full sub-orbitals



***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**. **Numbers**: energy levels.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**, beyond a number equal to the protons, are not shown.*

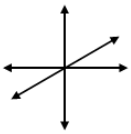
An assumption about the structure of nucleons as a conclusion of the model

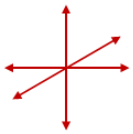
We want to analyze the structure of the nucleons, the protons and neutrons, as a consequence of the nuclear model. According to the model the nuclear structure is cubic, so our first assumption is that the nucleon consists of three mutually perpendicular waves or sub-particles, as shown in the next illustration.



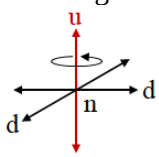
Nucleon

There are two types of nucleons, the proton and the neutron, and, according to the model, only p-n bonds are expected (and no p-p or n-n) so we suspect they are made up of different sub-particles, as demonstrated by the next illustration, where the different colors refer to the sub-particles:

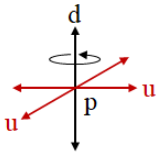

Neutron


Proton

This shape supports both requirements of a cubic system and of two different types of nucleons; we try to meet the requirements with a minimum number of sub-particles, i.e. only two, marked in black and red.
Restricting the model to p-n bonds means that only black-red bonds are allowed.
It is known that both nucleon types are very similar, and a neutron can be transformed into a proton, we therefore ask how can the two sub-particles be mixed between the nucleons, so that a single change of the neutron will transform it into a proton. The result is shown next:



Neutron



Proton

The result:

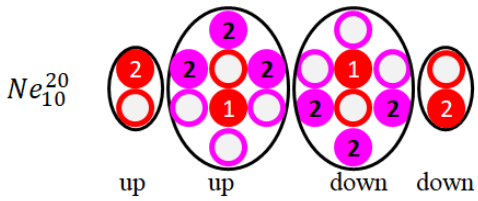
- the nucleon consists of at least two sub-particles, marked with black and red.
- the spin is marked as a rotation around its z-axis.
- the neutron is made up of two black and one red sub-particles.
- the proton has one black and two red sub-particles.

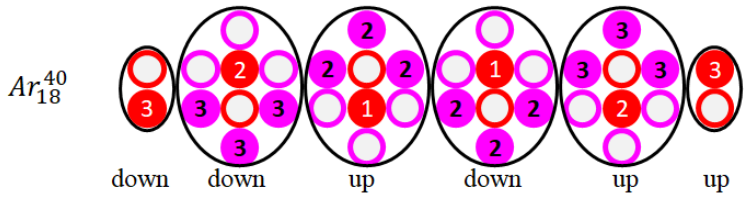
This agrees qualitatively with the standard model, if we choose black as down quark (***d***) and red as up quark (***u***).
This is not proof, just a first guess that needs to be developed further.

The spin of the nucleons

We want to analyze the spin deployment of the nucleons, although the conclusions don't make any difference at this stage of the study.
We begin with the Helium He_2^4 in the following illustration of two proton-neutron pairs:

We define the spins of the left pair, where the proton appears above the neutron, as spin-up, and the one on the right as spin-down.
Now we expand this idea to Neon Ne_{10}^{20} and Argon Ar_{18}^{36}:





In general, we see that each layer of the ellipsoid consists either of spin-up or spin-down nucleons.
Neighboring layers of similar outermost sub-orbitals have opposite spins; e.g. for Argon the layers from the second till the fifth have an outermost sub-orbital P and so their spins change d-u-d-u; this doesn't necessarily holds for the first and second layers (or the fifth and sixth), where the outermost sub-orbitals are S and P respectively.
This arrangement possibly enables the order of proton-neutron pairs in clusters of alpha particles and may also play a role in the determination the so-called nuclear magic numbers. This idea shall still be processed in following studies and possibly also be analyzed if it has to do with the Hund rules and the population sequence of atomic states, but at this point we don't further discuss it.

The implementation of the mass formula calculation

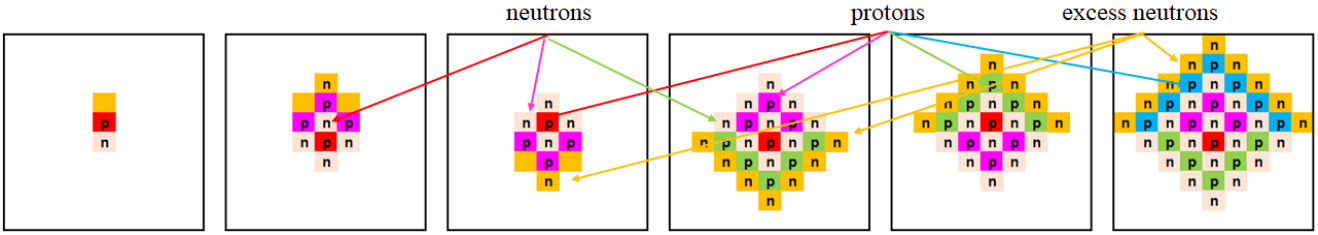
This section explains how the number of nucleon-nucleon bonds n_{b_x} and the relative total energy of the nucleus e_{c_x} are calculated.

Drawing the nucleus and counting the number of nucleon bonds n_{b_x}

Counting the number of nucleon-nucleon bonds n_{b_x} of each nucleus is implemented by drawing all the nuclei in Excel sheets that automatically run this.
The sequence of the process is the following:

- First the Oganesson nucleus Og_{118}^{294} is built, because it is the largest noble gas, meaning it has a closed orbital and as such is more likely to be drawn correctly according to the model.
- Then the nuclei with closed sub orbitals are derived from Oganesson:
  - Og_{118}^{294} is copied and the number of protons and neutrons is adjusted to create the next full sub-orbital nucleus below it, which is Copernicium Cn_{112}^{282}.
  - In a similar manner Cn_{112}^{282} is copied and Nobelium No_{102}^{256} is drawn.
  - This process continues to form all the nuclei with closed sub orbitals (S, P, D, F).
- Beginning with every nucleus that closes a sub-orbital, the nuclei below it are built in a similar process to the above till the nucleus above the next closed sub-orbital is reached:
  - Oganesson is copied to the next below it, which is Tennessine.
  - Then Oganesson is copied to build Livermorium and so on till Nihonium is reached.
  - Then a similar process is done by using the Copernicium Cn_{112}^{282} nucleus for all nuclei below it and above Nobelium No_{102}^{256}.
  - The process continues till every nucleus with a closed sub-orbital creates all the nuclei of its sub-orbital.

During this process the structure of the nuclei was analyzed and studied and ideas were developed on how to assess what configuration is more probable for the nucleons.
As an example next drawing shows the Excel sheet of Radon Ra_{86}^{222} (only the left half, spin-up, is shown due to its large size).



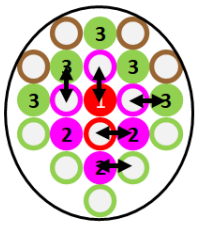
Radon Ra_{86}^{222}: only the left half, spin-up, is shown due to its large size.
***Legend:** protons (**p**) of the orbitals **S**, **P**, **D**, **F**. neutrons (**n**).*

Calculating the relative total energy of the nucleus e_{c_x}

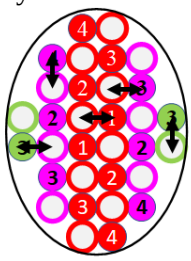
We first build an Excel sheet that calculates the sum of all proton-proton relative energies for Oganesson (the reciprocal distances):

$$e_{c_x} = \frac{1}{2}\sum_i^{Z_x} \sum_{j\neq i}^{Z_x} \frac{1}{d_{i,j}} \quad \text{with} \quad \frac{1}{d_{i,j}} = \frac{1}{\sqrt{(x_j-x_i)^2+(y_j-y_i)^2+(z_j-z_i)^2}}$$

then in a somewhat similar manner to the process of creating the Excel sheets that count n_{b_x}, as described in the section above, the closed sub-orbitals are created and each of them creates the nuclei in its sub-orbital below it.
Following drawings show the minimum distance between two neighboring nucleons d_0 through cross sections of the nucleus (marked by arrows in several positions):

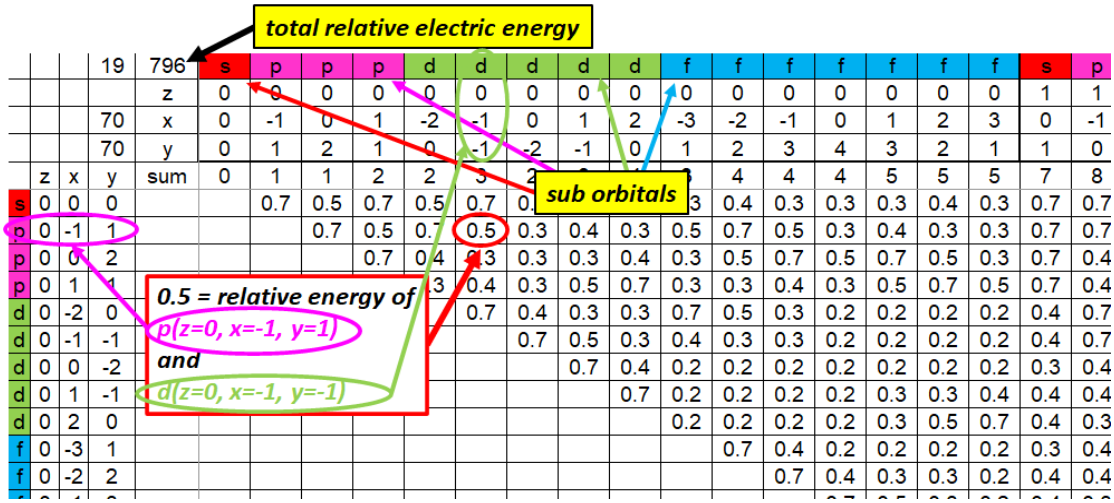


$\boldsymbol{d_0}$ *in the x-y plane*



$\boldsymbol{d_0}$ *in the x-z plane*

As an example the Excel sheet of Ytterbium Yb_{70} explains the calculation of the relative electric energy e_{c_x} (the reciprocal distances):



			19	796	s	p	p	p	d	d	d	d	d	f	f	f	f	f	f	f	s	p
				z	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
			70	x	0	-1	0	1	-2	-1	0	1	2	-3	-2	-1	0	1	2	3	0	-1
			70	y	0	1	2	1	0	-1	-2	-1	0	1	2	3	4	3	2	1	1	0
	z	x	y	sum	0	1	1	2	2	3	2				4	4	4	5	5	5	7	8
s	0	0	0			0.7	0.5	0.7	0.5	0.7	0.5			0.3	0.4	0.3	0.3	0.3	0.4	0.3	0.7	0.7
p	0	-1	1				0.7	0.5	0.7	0.5	0.3	0.4	0.3	0.5	0.7	0.5	0.3	0.4	0.3	0.3	0.7	0.7
p	0	0	2					0.7	0.4	0.3	0.3	0.3	0.4	0.3	0.5	0.7	0.5	0.7	0.5	0.3	0.7	0.4
p	0	1	1						0.3	0.4	0.3	0.5	0.7	0.3	0.3	0.4	0.3	0.5	0.7	0.5	0.7	0.4
d	0	-2	0							0.7	0.4	0.3	0.3	0.7	0.5	0.3	0.2	0.2	0.2	0.2	0.4	0.7
d	0	-1	-1								0.7	0.5	0.3	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.4	0.7
d	0	0	-2									0.7	0.4	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.4
d	0	1	-1										0.7	0.2	0.2	0.2	0.2	0.3	0.3	0.4	0.4	0.4
d	0	2	0											0.2	0.2	0.2	0.2	0.3	0.5	0.7	0.4	0.3
f	0	-3	1												0.7	0.4	0.2	0.2	0.2	0.2	0.3	0.4
f	0	-2	2													0.7	0.4	0.3	0.3	0.2	0.4	0.4

Ytterbium Yb_{70}: the coordinates of each proton appear once above and once from left; for each pair of (different) protons the relative electric energy (their reciprocal distance) is calculated at the point of their intersection.

Executing the mass formula calculation; analyzing and learning the model

The exact structure of the nucleus is crucial for the study of the model and for the correct execution of the mass formula. This structure determines:

- n_{b_x}: the number of bonds in the nucleus.
- e_x: the total relative electric energy of the nucleus (more precisely: the unitless sum of the reciprocal distances, where the distances are the unitless multiples of d_0).

The electric energy, e_x, is less sensible towards small changes of the nuclear structure, but for n_{b_x} small variations have larger impact (of one percent or more) on the relative error of the mass formula calculation, so while running the tests the "correct" value of n_{b_x} for each nucleus was sought. "Correct" means logical in the physical sense and compared with other nuclei, so this isn't something that can be proved yet by this research, but we improve the tests and results of the process in following research, where we test a larger number of nuclei.
The process of filling up the nucleus with nucleon runs as follows:

- The following nuclei are selected to begin with:
    - nuclei of filled-up sub-orbitals (S, P, D, F) because their real shape is assumed to be more probable to be created correctly:
      Be_4, Ne_{10}, Mg_{12}, Ar_{18}, Ca_{20}, Zn_{30}, Kr_{36}, Sr_{38}, Cd_{48}, Xe_{54}, Ba_{56}, Yb_{70}, Hg_{80}, Rn_{86}, Ra_{88} (Hydrogen and Helium are not discussed in this research due to their small number of bonds, that seem to deviate from the cubic structure; a separate future research shall deal with them).
- The nuclei are filled up with:
    - the protons: there is only one possible configuration.
    - the paired neutrons: also here there is only one possible configuration.
    - the excess neutrons are set in a way that reaches maximum symmetry and compared between various nuclei to assess the more probable configuration.
- The mass formula is calculated and the best parameters are found.
- The nuclei that show larger deviation of their relative errors are checked again and their structure is being studied and possibly changed via comparison with other nuclei.
- At a second step all other nuclei are being created and the process repeats itself.

After creating all nuclei, trials are run in iterations with the construction of the nuclei.
This process is delicate, because we shall avoid the tendency to adapt the structure so that better results are achieved.
The results are not necessarily accurate as we cannot be sure of the correct structure of the nuclei, but a rough estimation seems to be possible.

Mass formula calculation: data

This section shows the data according to which the mass of the nuclei was calculated via the mass formula.
The parameters with which the mass formula are shown here are those of the best results achieved while running the calculation:

- $d_0 = 1.62fm$
- $e_b = 5.72MeV$

The legend of the table below is given in the order of the columns from left to right:

- *nuc.*: the nucleus (name)
- Z_x: atomic number of the nucleus of the element x; the number of protons.
- A_x: mass number of the nucleus of the element x; the number of nucleons.
- $N_x := A_x - Z_x$: the number of neutrons of the nucleus of the element x.
- n_k: the number of nucleon-nucleon bonds in the nucleus as it was calculated by the relevant Excel file. *
- e_c: total relative electric energy of the nucleus as it was calculated by the relevant Excel file. *
- *measured [amu]*: measured mass of the nucleus in [amu].
- *base mass:* $Z_x \cdot m_p + N_x \cdot m_n$ *[amu]*: base mass in [amu]:

  number of protons · proton mass + number of neutrons · neutron mass

- *calculated mass [amu]*: calculated mass according to the mass formula in [amu]
  $m_{calc_x} = Z_x \cdot m_p + N_x \cdot m_n - \frac{(E_{b_x} - E_{c_x})}{c^2}$.
- Δ_1*: [amu] calculated - measured*: calculated mass *minus* measured mass in [amu]
- Δ_2*: [amu] base - measured*: base mass *minus* measured mass in [amu]
- *relative error* Δ_1*:* Δ_2: the relative error in percent:
  $relative\ error = \left| \frac{calculated\ mass - measured\ mass}{base\ mass - measured\ mass} \right|$

* see section: The mass formula

nuc.	Z_x	A_x	n_k	e_c	measured mass [amu]	calculated mass [amu]	relative error $\Delta_1 : \Delta_2$	base mass $Z_x \cdot m_p + N_x \cdot m_n$ [amu]	Δ_1: calculated minus measured	Δ_2: base minus measured
H	1	2	1	0.0	2.014	2.010	233.7%	2.016	-0.004	0.002
H	1	3	2	0.0	3.016	3.012	43.5%	3.025	-0.004	0.009
He	2	3	2	0.7	3.016	3.012	61.4%	3.023	-0.004	0.007
He	2	4	4	0.7	4.003	4.008	18.4%	4.032	0.005	0.029
Li	3	6	6	1.9	6.015	6.013	7.1%	6.048	-0.002	0.033
Li	3	7	7	1.9	7.016	7.015	1.6%	7.056	-0.001	0.040
Be	4	9	11	3.4	9.012	9.008	6.7%	9.072	-0.004	0.060
B	5	10	12	5.7	10.013	10.011	2.2%	10.080	-0.001	0.067
B	5	11	14	5.7	11.009	11.008	1.8%	11.088	-0.001	0.079
C	6	12	16	9.0	12.000	12.006	6.3%	12.096	0.006	0.096
C	6	13	18	9.0	13.003	13.002	1.0%	13.104	-0.001	0.101
N	7	14	20	12.7	14.003	14.001	2.0%	14.112	-0.002	0.109
N	7	15	21	12.7	15.000	15.003	2.8%	15.120	0.003	0.120
O	8	16	24	16.7	15.995	15.996	0.9%	16.128	0.001	0.133
O	8	17	25	16.7	16.999	16.999	0.3%	17.136	0.000	0.137
O	8	18	26	16.7	17.999	18.001	1.4%	18.145	0.002	0.146
F	9	19	29	20.8	18.998	18.994	2.9%	19.152	-0.004	0.154
Ne	10	20	32	26.1	19.992	19.988	2.7%	20.159	-0.005	0.167
Ne	10	21	33	26.1	20.994	20.990	2.0%	21.168	-0.003	0.174
Ne	10	22	34	26.1	21.991	21.993	0.8%	22.177	0.002	0.185
Na	11	23	36	30.0	22.990	22.992	0.9%	23.184	0.002	0.194
Mg	12	24	38	34.1	23.985	23.991	2.7%	24.191	0.005	0.206
Mg	12	25	40	34.1	24.986	24.987	0.5%	25.200	0.001	0.214
Mg	12	26	42	34.1	25.983	25.983	0.3%	26.209	0.001	0.226
Al	13	27	45	39.3	26.982	26.977	1.9%	27.216	-0.004	0.234
Si	14	28	46	44.8	27.977	27.984	2.7%	28.223	0.007	0.246
Si	14	29	48	44.8	28.976	28.980	1.3%	29.232	0.003	0.255
Si	14	30	50	44.8	29.974	29.976	1.0%	30.241	0.003	0.267
P	15	31	52	53.0	30.974	30.979	1.9%	31.248	0.005	0.274
S	16	32	56	59.7	31.972	31.968	1.4%	32.255	-0.004	0.283
Cl	17	35	63	66.4	34.969	34.956	4.0%	35.280	-0.013	0.311
Ar	18	36	66	74.4	35.968	35.953	4.7%	36.287	-0.015	0.319
Ar	18	40	70	74.4	39.962	39.963	0.1%	40.322	0.000	0.359
K	19	39	70	79.9	38.964	38.958	1.6%	39.312	-0.006	0.348
Ca	20	40	72	85.6	39.963	39.958	1.2%	40.319	-0.004	0.356
Ca	20	42	74	85.6	41.959	41.963	1.3%	42.336	0.005	0.378
Ca	20	44	78	85.6	43.955	43.956	0.2%	44.353	0.001	0.398
Sc	21	45	81	92.8	44.956	44.952	1.0%	45.361	-0.004	0.405
Ti	22	46	84	103.7	45.953	45.951	0.3%	46.368	-0.001	0.415

nuc.	Z_x	A_x	n_k	e_c	measured mass [amu]	calculated mass [amu]	relative error $\Delta_1 : \Delta_2$	base mass $Z_x \cdot m_p + N_x \cdot m_n$ [amu]	Δ_1: calculated minus measured	Δ_2: base minus measured
Ti	22	48	88	103.7	47.948	47.944	0.9%	48.385	-0.004	0.437
Ti	22	50	90	103.7	49.945	49.949	1.0%	50.403	0.004	0.458
V	23	51	93	112.4	50.944	50.946	0.5%	51.410	0.002	0.466
Cr	24	52	96	123.1	51.941	51.945	1.0%	52.417	0.005	0.477
Cr	24	54	100	123.1	53.939	53.938	0.2%	54.435	-0.001	0.496
Mn	25	55	103	132.2	54.938	54.936	0.5%	55.442	-0.002	0.504
Fe	26	54	102	141.5	53.940	53.941	0.2%	54.432	0.001	0.492
Fe	26	56	106	141.5	55.935	55.933	0.3%	56.449	-0.002	0.514
Co	27	59	112	153.3	58.933	58.932	0.1%	59.474	-0.001	0.541
Ni	28	58	112	164.5	57.935	57.933	0.4%	58.464	-0.002	0.528
Ni	28	60	114	164.5	59.931	59.938	1.3%	60.481	0.007	0.550
Ni	28	62	118	164.5	61.928	61.931	0.4%	62.498	0.002	0.570
Cu	29	63	120	174.6	62.930	62.935	1.0%	63.506	0.006	0.576
Zn	30	64	122	185.1	63.929	63.941	1.9%	64.513	0.011	0.584
Zn	30	66	126	185.1	65.926	65.933	1.2%	66.530	0.007	0.604
Zn	30	68	130	185.1	67.925	67.926	0.2%	68.548	0.001	0.623
Ga	31	69	132	195.0	68.926	68.931	0.8%	69.555	0.005	0.629
Ge	32	74	142	205.1	73.921	73.921	0.1%	74.597	0.000	0.676
As	33	75	144	218.5	74.922	74.928	1.0%	75.604	0.007	0.682
Se	34	80	154	230.0	79.917	79.920	0.5%	80.646	0.003	0.729
Br	35	79	154	241.7	78.918	78.921	0.4%	79.636	0.003	0.718
Br	35	81	158	241.7	80.916	80.914	0.3%	81.653	-0.002	0.737
Kr	36	82	160	253.5	81.913	81.920	0.9%	82.661	0.007	0.747
Kr	36	84	164	253.5	83.911	83.913	0.2%	84.678	0.001	0.766
Kr	36	86	166	253.5	85.911	85.918	0.9%	86.695	0.007	0.785
Rb	37	85	167	264.1	84.912	84.912	0.0%	85.685	0.000	0.773
Sr	38	84	164	271.1	83.913	83.927	1.8%	84.675	0.014	0.762
Sr	38	86	168	271.1	85.909	85.920	1.3%	86.692	0.010	0.783
Sr	38	88	172	271.1	87.906	87.912	0.9%	88.710	0.007	0.804
Y	39	89	176	283.2	88.906	88.907	0.1%	89.717	0.001	0.811
Zr	40	90	180	295.5	89.905	89.901	0.4%	90.724	-0.004	0.820
Nb	41	93	186	313.4	92.906	92.906	0.0%	93.749	0.000	0.843
Mo	42	98	196	329.6	97.905	97.902	0.4%	98.791	-0.003	0.885
Tc	43	98	197	343.5	97.907	97.908	0.1%	98.789	0.000	0.882
Ru	44	102	204	357.5	101.904	101.911	0.8%	102.823	0.007	0.918
Rh	45	103	208	373.9	102.906	102.910	0.5%	103.830	0.004	0.925
Pd	46	106	214	384.8	105.903	105.908	0.5%	106.855	0.004	0.951
Ag	47	107	218	405.6	106.905	106.910	0.6%	107.862	0.005	0.957
Cd	48	112	228	421.0	111.903	111.906	0.3%	112.904	0.003	1.001

nuc.	Z_x	A_x	n_k	e_c	measured mass [amu]	calculated mass [amu]	relative error $\Delta_1 : \Delta_2$	base mass $Z_x \cdot m_p + N_x \cdot m_n$ [amu]	Δ_1: calculated minus measured	Δ_2: base minus measured
In	49	115	234	433.9	114.904	114.906	0.2%	115.928	0.002	1.025
Sn	50	120	244	447.1	119.902	119.899	0.3%	120.970	-0.003	1.068
Sb	51	121	247	463.9	120.904	120.904	0.0%	121.978	0.000	1.074
Te	52	126	256	478.4	125.903	125.904	0.1%	127.020	0.001	1.116
I	53	127	260	493.4	126.904	126.901	0.3%	128.027	-0.003	1.122
Xe	54	132	268	508.0	131.904	131.908	0.3%	133.069	0.004	1.165
Cs	55	133	270	519.5	132.905	132.914	0.7%	134.076	0.009	1.171
Ba	56	138	278	531.0	137.905	137.918	1.0%	139.118	0.013	1.213
La	57	139	280	545.4	138.906	138.927	1.7%	140.125	0.020	1.219
Ce	58	140	284	566.6	139.905	139.930	2.0%	141.133	0.024	1.227
Pr	59	141	288	582.7	140.908	140.928	1.6%	142.140	0.020	1.232
Nd	60	144	298	603.2	143.910	143.910	0.0%	145.164	0.000	1.254
Pm	61	145	298	621.6	144.913	144.935	1.8%	146.172	0.023	1.259
Sm	62	152	310	636.1	151.920	151.935	1.1%	153.231	0.015	1.311
Eu	63	153	316	659.5	152.921	152.927	0.5%	154.238	0.006	1.317
Gd	64	158	328	683.3	157.924	157.918	0.4%	159.280	-0.006	1.356
Tb	65	159	330	700.8	158.925	158.930	0.4%	160.287	0.005	1.362
Dy	66	164	340	718.4	163.929	163.928	0.1%	165.329	-0.002	1.400
Ho	67	163	340	735.8	162.929	162.934	0.4%	164.319	0.005	1.391
Er	68	166	348	756.6	165.930	165.929	0.1%	167.344	-0.001	1.414
Tm	69	169	354	777.9	168.934	168.937	0.2%	170.369	0.003	1.434
Yb	70	172	362	796.0	171.936	171.930	0.4%	173.393	-0.006	1.457
Lu	71	175	368	815.1	174.941	174.936	0.3%	176.418	-0.005	1.477
Hf	72	178	374	834.0	177.944	177.942	0.1%	179.442	-0.002	1.499
Ta	73	181	383	859.3	180.948	180.936	0.8%	182.467	-0.012	1.519
W	74	184	390	882.6	183.951	183.939	0.7%	185.492	-0.012	1.541
Re	75	185	393	903.1	184.953	184.948	0.3%	186.499	-0.005	1.546
Os	76	192	404	923.6	191.961	191.959	0.1%	193.558	-0.002	1.597
Ir	77	193	410	947.2	192.963	192.952	0.7%	194.565	-0.011	1.602
Pt	78	194	412	971.0	193.963	193.970	0.4%	195.573	0.007	1.610
Au	79	197	420	992.4	196.967	196.966	0.1%	198.597	-0.001	1.631
Hg	80	198	426	1,014.0	197.967	197.957	0.6%	199.605	-0.010	1.638
Hg	80	200	430	1,014.0	199.968	199.949	1.1%	201.622	-0.019	1.654
Hg	80	202	432	1,014.0	201.971	201.955	1.0%	203.639	-0.016	1.669
Tl	81	205	436	1,032.2	204.974	204.972	0.1%	206.664	-0.002	1.689
Pb	82	208	442	1,050.5	207.977	207.977	0.0%	209.688	0.001	1.712
Bi	83	208	444	1,072.4	207.980	207.984	0.3%	209.687	0.005	1.707

nuc.	Z_x	A_x	n_k	e_c	measured mass [amu]	calculated mass [amu]	relative error $\Delta_1 : \Delta_2$	base mass $Z_x \cdot m_p + N_x \cdot m_n$ [amu]	Δ_1: calculated minus measured	Δ_2: base minus measured
Po	84	208	448	1,092.0	207.981	207.977	0.2%	209.686	-0.004	1.704
At	85	209	451	1,111.1	208.986	208.984	0.1%	210.693	-0.002	1.707
Rn	86	222	470	1,132.0	222.018	221.999	1.1%	223.804	-0.019	1.787
Fr	87	223	471	1,146.8	223.020	223.014	0.3%	224.811	-0.006	1.792
Ra	88	226	476	1,162.0	226.025	226.022	0.2%	227.836	-0.003	1.811
Ac	89	227	481	1,183.3	227.028	227.019	0.5%	228.843	-0.008	1.816
Th	90	232	488	1,204.8	232.038	232.039	0.0%	233.885	0.001	1.847
Pa	91	231	493	1,235.2	231.036	231.027	0.5%	232.875	-0.009	1.839
U	92	238	502	1,263.5	238.051	238.058	0.4%	239.935	0.007	1.884
Np	93	237	503	1,289.6	237.048	237.067	1.0%	238.924	0.019	1.876
Pu	94	244	520	1,309.9	244.064	244.041	1.2%	245.984	-0.023	1.920