

A Quantum Gravity Gedanken Experiment and Its Consequences

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Abstract

A simple gedanken experiment was proposed that imagines an observer shrinking down and entering the inner space of the atom. This led to five postulates which was justified mathematically using a quantum potential-modified spacetime structure within the atom. The new spacetime structure demonstrated that space was expanded within the atom, time was slowed down and inertial mass was increased. The paper also demonstrated that the uncertainty principle may have its origins in the modification of spacetime within the atom. Application of general relativity to the intraatomic space showed that charge emerges in a natural way from the changes in the spacetime structure. Within the atom, the term $8\pi G$ could be replaced with $\frac{2(\frac{e}{m})^2}{\epsilon_0}$ in the gravitational field equation. The absence of nuclear radiations could be attributed to the presence of a black-hole-like horizon around the nucleus, which also could explain the extraordinary stability of the electron within the atom that contains a positively charged nucleus. The spacetime transformation would appear to make the atomic world self-similar to, or symmetric with, the macroscopic world. The product of the space and time intervals in spacetime was invariant, which is in effect a law of conservation of spacetime. Thus, it appeared that spacetime may not be just a field of coordinate points, but a real entity that could be associated with both mass and energy. Inertial mass could be directly related to the proportion of space and time within spacetime.

Keywords: quantum gravity, spacetime curvature, quantum potential, general relativity, Schwarzschild metric, Einstein field equation, gauge symmetry

1. Introduction and the Gedanken experiment

Unlike within the atom, wherein forces are described by the standard model, which is a quantum field theory model, in the macroscopic world, gravitational forces are described by the effect of mass on spacetime curvature as in Einstein's theory of general relativity. These mathematical approaches, in the two domains of the very small and the very large, are so radically different that attempts to reconcile them have afflicted us for at least seven decades with still no clear end in sight. Quantum gravity refers to heroic, and no doubt brilliant, attempts to bridge the two domains [1]. In AIP Publishing alone there are over 20,000 papers in

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quantum gravity, and a full list may run well over 100,000 papers. Yet the consensus is that we are not even close to bringing together the quantum and gravitational domains. I feel it is time for a disruptive concept, a simple idea that has somehow been overlooked. In 1905, when Einstein proposed his Special Theory of Relativity (STR) which derived the Lorentz formulations from a gedanken (thought) experiment, the world could finally agree with the intuitive truth of that thought experiment, transform physics as we knew it then, and extricate us finally from Newton's constraints. The disruptive idea was not, in my opinion, the constancy of the speed of light per se, though of course that is what led to the rest, but the idea of observer dependence, that no two observers need to agree on measurements of mass, space and time. In 1996, Rovelli [2] first advocated observer dependence in quantum measurements spawning the field of relational quantum mechanics, but it was still not disruptive enough and still did not go into the heart of the conflict. I wish to propose a new disruptive gedanken experiment that has not so far been considered and which has potential to intuitively integrate both domains in a natural way. There are some profound consequences of this gedanken experiment, that can be mathematically established and is done so in this paper, and of considerable philosophical significance, that are likely as transformative as the mass-energy equivalence that arose from the STR.

The gedanken experiment is as follows. Imagine that a macroscopic observer shrinks down in size continuously. At first the room will appear very large and extremely massive. As this shrinking continues imagine that eventually the observer finds himself/herself inside an atom. It stands to reason that an atom will appear vaster and much more massive than it would to a normal observer at our scale. The important point is this. It is intuitively reasonable to think that the atom is now no longer a quantum object, but a normal macroscopic object behaving as normal objects do in the macrocosm. Simply by reducing the size of the observer we have potentially transformed the quantum world into a macroscopic world. I will later show that distances and masses are larger, and time slows down for this lilliputian when compared to a macroscopic normal human observer. Clearly there must be a spacetime transformation that transforms the quantum world into the macrocosm and vice versa. In fact, Rovelli [3] has said that there is no fundamental flaw in considering observers as anything that interacts with the environment. Fundamental particles are also observers, and there is nothing to stop us from attaching a reference system to any random observer of any size scale.

1.1 The Postulates

The following postulates are made based on the gedanken experiment, and this paper will develop the mathematical foundation for each of these postulates.

Postulate 1: A microscopic observer within the atom does not see the atom as quantum in nature, but rather macroscopic and classical in nature.

Postulate 2: There is a spacetime transformation that can map the microcosm into the macrocosm and hence transform electromagnetism into gravity; or vice versa.

Postulate 3: Mass, space and time are not absolute and is dependent on the size scale of the observer.

Postulate 4: The general relativistic field equation can be rederived to include charge so that electromagnetism becomes a form of gravity.

Postulate 5: Charge is a manifestation of gravity at quantum scales.

To address this mathematically what is required is a parameter that can influence spacetime structure, and one candidate is the potential energy of a particle that applies in the quantum regime whose negative gradient multiplied by its mass is equal to the force experienced by the particle. A proper choice is the quantum potential energy, derived in the 1950s by David Bohm [3].

This paper only considers the simplest case of a single electron in a hydrogen atom in the 1s state and demonstrates how the electromagnetic force is a manifestation of gravity at quantum scales.

2. Validating the postulates

2.1 Postulate 1

This is a restatement of the gedanken experiment as a postulate and hence does not require proof. One could say that validation of the rest of the postulates would provide a validation of the first postulate.

2.2 Postulate 2

There are two parts to validating this postulate. One is selecting a suitable spacetime metric and the second is obtaining a form for the quantum potential that enters into this metric. From this, one needs to demonstrate that the electromagnetic force can be obtained from general relativity considerations alone.

2.2.1 Quantum potential energy field and quantum forces

The quantum potential energy associated with a quantum state was shown by Bohm [3] to be of the form:

$$V_{\psi}(m_{\psi}) = -\frac{\hbar^2}{2m_{\psi}} \frac{\nabla^2 \psi}{\psi} \quad (1)$$

wherein ψ is the electron wave function and m_{ψ} is the quantum particle mass, which can be defined as the inertia associated with the quantum wavefunction for the particle. A detailed treatise about the quantum potential can be found in the book by Robert Carroll [4].

For the ground state of the electron in the hydrogen atom, ψ is only r -dependent and is given by [5]

$$\psi(r) = \frac{1}{\sqrt{\pi b^3}} e^{-\frac{r}{b}} \quad (2)$$

where b is a characteristic distance, namely the most probable distance of the electron from the nucleus in its ground state. Combining Eq. (1) and Eq. (2) it is straightforward to show that:

$$V_\psi(m_\psi, r) = \frac{\hbar^2}{m_\psi b r} - \frac{\hbar^2}{2m_\psi b^2} \quad (3)$$

Because the second term is a constant the potential energy change required to bring an electron from infinity to the point, r is then:

$$V(m_\psi, r) = \frac{\hbar^2}{m_\psi b r} \quad (4)$$

We next define the parameter $\phi_V(m_\psi, r)$ as the quantum confinement potential field, namely, the potential energy per unit mass $\left(= \frac{V(m_\psi, r)}{m_\psi} \right)$, so that:

$$\phi_V(m_\psi, r) = \frac{\hbar^2}{m_\psi^2 b r} \quad (5)$$

In Eq. (3) to Eq. (5), m_ψ refers to the electron rest mass. For the purely gravitational case:

$$m_i a = -m_g \nabla \phi_g \quad (6)$$

where, m_i is the inertial mass, m_g is the gravitational mass and ϕ_g is the gravitational potential (total gravitational potential energy per unit mass); then the statement of the equivalence principle is $m_i = m_g$ which has been verified by all experiments to date. Hence acceleration of the mass is the same as the gradient of the gravitational potential which allowed Einstein to develop a geometric law of gravity whereby it is spacetime that is curved, and bodies take the shortest, or straightest, path (geodesic paths) in this curved space which manifests as gravitational acceleration.

In analogy with Eq. (6) we can write for the quantum case:

$$m_i a = -m_\psi \nabla \phi_V(m, r) \quad (7)$$

Here m_i is the inertial mass. We apply a new equivalence principle in the quantum regime, like that used by Einstein, namely:

$$m_i = m_\psi = m \quad (8)$$

If we accept this as a hypothesis one gets:

$$a = -\nabla\phi_V(m, r) = \frac{\hbar^2}{m^2 b r^2} \quad (9)$$

The quantum confinement force is then:

$$F = ma = -m\nabla\phi_V(m_\psi, r) = \frac{\hbar^2}{m b r^2} \quad (10)$$

F is a Heisenberg force which follows from the uncertainty principle and associated with the repulsion experienced by a particle that is confined to a radius, r. Because of the following equality:

$$\frac{\hbar^2}{m b} = \frac{e^2}{4\pi\epsilon_0} \quad (11)$$

the quantum Heisenberg force is equal in magnitude to the electrostatic force, that is:

$$F = \frac{\hbar^2}{m b r^2} = \frac{e^2}{4\pi\epsilon_0 r^2} \quad (11a)$$

An observer inside the atom does not experience any quantum confinement as per the gedanken experiment, and hence there would not be a quantum forces present for this observer. The attractive force for such an observer would only be a gravitational attraction between two massive particles balanced by a centrifugal force. In contrast, the macroscopic observer interprets this as an electromagnetic attraction balanced by the quantum force of repulsion (Eq. (11a)). They are both equivalent descriptions of the same reality.

From Eq. (11), the energy levels of the hydrogen atom can be given by

$$E_n = -\frac{\left(\frac{\hbar^2}{2mb^2}\right)}{n^2} = -\frac{R}{n^2} \quad (12)$$

with R as the Rydberg Constant. Thus, one can write:

$$e = \sqrt{\frac{4\pi\epsilon_0\hbar^2}{m_\psi b}} \quad (13)$$

as a derived quantity from b . The Bohr radius, b , is a form of space quantization which Eq. (12) suggests could be more fundamental than charge. For the macroscopic observer, the perceived charge, e , is a derived quantity from b based on eq. (13).

Taking the negative potential energy to simulate attractive gravity, from Eq. (5) and Eq. (12), the kinetic energy term is:

$$E_{KE} = \left(\frac{1}{2}\right) \frac{\hbar^2}{mb^2} \quad (14)$$

Letting the kinetic energy be of the order $\left(\frac{1}{2}\right)mv^2$, Eq. (14) results in $pb = \hbar$ (p is the momentum) which is a statement of the uncertainty principle. The quantum potential is consistent with the uncertainty principle.

Based on Eq. (5), a mass- potential energy equation in the quantum regime can be obtained as follows:

$$m\phi_V(m, r) = V(m, r) \quad (15)$$

$$V(m, r) = V_0(m) \left(\frac{b}{r}\right) \quad (16)$$

$$V_0(m) = \frac{\hbar^2}{mb^2} = R \quad (17)$$

$$m\phi_V(m, b) = V_0(m) = R \quad (18)$$

We define V_0 here as a fundamental quantum energy associated with the quantum confined ground state, neglecting all kinetic energy effects. The inverse relationship of energy to mass, Eq. (17), has significance in relativistic models of the atom (see following section).

The total energy content of the mass (based on the equivalence principle, $m_i = m_\psi = m$) can be written for the ground state using Eq. (17) as:

$$E = \frac{\hbar^2}{mb^2} + mc^2 = R + mc^2 \quad (19)$$

The first term is the quantum contribution from the Heisenberg energy, Eq. (17), and the second term is the Einstein term. Clearly as mass goes to zero rest energy does not go to zero

but to a value equals the Rydberg constant, which is essentially a quantum ground state for the rest energy.

The fundamental equation of quantum particle dynamics recognized by Bohm [3] was:

$$ma = -\nabla[V_e + V] \quad (20)$$

wherein, for the electron, V_e is the classical electromagnetic potential (V is the quantum potential):

$$V_e(r) = -\frac{e^2}{4\pi\epsilon_0 r} \quad (21)$$

Differentiating the one-dimensional Schrödinger equation with respect to x on both sides and multiplying by ψ^* one gets

$$i\hbar\psi^* \frac{\partial}{\partial x} \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \psi^* \frac{\partial}{\partial x} \frac{\partial^2 \psi}{\partial x^2} + \psi^* \frac{\partial V_e}{\partial x} \psi. \quad (22)$$

Integrating both sides over all space with respect to x the result is

$$\left\langle \frac{\partial p}{\partial t} \right\rangle = -\nabla[\langle T \rangle + \langle V_e \rangle] \quad (23)$$

knowing that $\langle p \rangle = \int \psi^* \left(\frac{\hbar}{i} \frac{\partial}{\partial x} \right) \psi dx$ and $\langle T \rangle = -\frac{\hbar^2}{2m} \int \psi^* \frac{\partial^2 \psi}{\partial x^2} dx$, wherein $\langle \quad \rangle$ refer to the expectation value of the parameter between the brackets and T is the translational kinetic energy. Comparing with Eq. (20) and Eq. (23):

$$\frac{\hbar^2}{mbr} = \langle T \rangle = (1/2)m\langle v \rangle^2 \quad (24)$$

Thus, we find that the lower limiting value of r , or r_H , corresponding to a lower quantum horizon for r is, by the principle of special relativity, at $\langle v \rangle = c$:

$$r_H = \frac{2\hbar^2}{m^2bc^2} \quad (25)$$

The electron can never occupy positions $r < r_H$. At this point, though the electron rest mass is unaffected, its inertial mass should tend to infinity by the special theory of relativity. This limiting condition prevents atomic instability. The value of r_H is 5.64×10^{-15} m is only three orders of magnitude smaller than the Bohr radius and should be well outside the nucleus.

Based on special relativity, the inertial mass, m , compared to the rest mass, m_0 should be given by (using Eq. (24) and Eq. (25)):

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{\langle v \rangle}{c}\right)^2}} = \frac{m_0}{\sqrt{1 - \frac{2\hbar^2}{m_0^2 b r c^2}}} = \frac{m_0}{\sqrt{1 - \frac{r_H}{r}}} \quad (26)$$

The singularity in mass at $r = r_H$ is also predicted by the uncertainty relation when special relativity applies[‡].

The quantum potential energy value in Eq. (16) allows one to write the classical electromagnetic potential energy as:

$$V_e = -V_\psi \quad (27)$$

Since the potential is the same as the potential energy per unit mass one can write from Eq. (14):

$$\phi_e = -\phi_\psi = - \frac{\hbar^2}{m^2 b r} \quad (28)$$

and the acceleration as:

$$a = -\nabla\phi_e = +\nabla\phi_\psi \quad (29)$$

2.2.2 Deriving the spacetime field structure at quantum scales

A spacetime structure can be defined by a spacetime metric. It is well known that a potential of mass would create a curvature in this field. The quantum potential is indeed a potential of mass because the mass times the negative gradient of the potential is a force. The Schwarzschild metric is an ideal spacetime structure to start with as it is a black hole metric and

[‡] The usual momentum-position uncertainty relation is:

$$\Delta(mv)\Delta r \geq \frac{\hbar}{2}$$

yielding

$$(v\Delta m + m\Delta v)\Delta r \geq \frac{\hbar}{2}$$

As the electron approaches the horizon, $v \rightarrow c$ and hence by special relativity constraint it follows that $\Delta v \rightarrow 0$. This gives:

$$\Delta m \Delta r \geq \frac{\hbar}{2c}$$

Further, at the horizon, special relativity constrains Δr to be tending to zero also or else the velocity can exceed c . This means the inertial mass must tend to infinity though its rest mass is said to remain constant.

exhibits strong quantum effects. I start with this structure as illustrative of the intra-atomic situation. This metric defines a spacetime field, ϕ_s , represented as a spacetime interval, ds :

$$\phi_s^2 = ds^2 = -(c dt)^2 \left(1 + \frac{2\phi}{c^2}\right) + dr^2 \left(1 + \frac{2\phi}{c^2}\right)^{-1} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (30a)$$

If we replace the potential, ϕ , by the negative of the quantum potential one gets:

$$\phi_s^2 = ds^2 = -(c dt)^2 \left(1 - \frac{2\hbar^2}{m^2 b r c^2}\right) + dr^2 \left(1 - \frac{2\hbar^2}{m^2 b r c^2}\right)^{-1} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (30b)$$

This is justified as the quantum force is equal and opposite to the attractive force (Eq. (11a)). The real question is whether the attractive force can emerge as a gravitational force from the spacetime structure and be of equal magnitude to the electrostatic force of attraction using Eq. (30b). Appendix 1 shows that this is indeed the case based on the mathematical formalism of general relativity and predicts accelerations and forces from the spacetime geodesic that exactly match the electrostatic accelerations and forces of Eq. (9) and Eq. (10). This approach is different from the approach taken by Tavernelli [6,7] earlier who derived a geometric model for spacetime based on the Finsler geometry wherein the geometry is a function of both position and momentum.

The above analysis validates postulate 2.

2.3 Postulate 3

2.3.1 Relativity of space and, time

A fundamental consequence of proposing curved spacetime within the atom is the result that there would be a relativity of space, time and mass compared to the yardsticks in the macroscopic world we live in. The metric of Eq. (30b) clearly indicates that time moves slower and space is expanded from a perspective within the quantum spacetime field, compared to what would be measured by a remote macroscopic human observer outside this field. This is consistent with the finding that muons have longer decay lifetimes [8]. These equations are given below:

$$d\tau = dt \sqrt{\left(1 - \frac{2\hbar^2}{m^2 b r c^2}\right)} \quad (31)$$

$$dr_p = dr \left(1 - \frac{2\hbar^2}{m^2 b r c^2}\right)^{-1/2} \quad (32)$$

A larger value of $d\tau$ means that time is speeded up inside the atom. Eq. (31) shows the opposite, that is, time is slowed down inside the atom.

2.3.2 Relativity of mass

In special relativity, the concept of relativistic mass is accepted in contrast to the rest mass. If m_0 is the rest mass, then from special relativity, the total energy of mass is given by [9]:

$$E = p_t = m_0 c^2 \frac{dt}{d\tau} = \frac{m_0 c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad (33a)$$

wherein here the velocity, v , refers to the velocity of the mass relative to that of the observer. Hence the relativistic mass, m , is:

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad (33b)$$

The relativistic mass is the true inertia of the particle. To say that the inertial mass is different from the actual mass appears to be against the equivalence principle. Why have we insisted on this separation between inertial and rest mass? Primarily because we did not wish to abandon the basic notion of the conservation of mass. If relativistic inertial mass is the actual mass, then mass is not conserved. We show below that what is conserved is not actually mass but spacetime itself at a more fundamental level. Mass is observer dependent. If we use the same approach as in Eq. (33a), then for general relativity also:

$$E = p_t = m_0 c^2 \frac{dt}{d\tau} = \frac{m_0 c^2}{\sqrt{\left(1 - \frac{2\phi_g}{c^2}\right)}} \quad (33c)$$

wherein here ϕ_g is the gravitational potential. This equation has the same form as Eq. (25b) and one can say that this is a definition of relativistic mass in general relativity. However, in general relativity the concept of relativistic mass is not used and, instead, mass is considered absolute and treated only as the “rest” mass, which is basically the mass at zero gravitational potential. This is because the equations of general relativity only has zero-potential rest mass in it and the equations of motion of the mass in general relativity are determined by this “rest” mass. This is certainly correct, but the fact remains that there is still an “effective” inertial mass that is larger than the so-called zero-potential rest mass because of the gravitational field. I define mass as the inertial mass at a particular location and state of motion of the mass. In this

definition, mass is not conserved in general but is conserved only at constant velocity and potential. I will show that spacetime is conserved in general, and that mass is conserved only when the ratio of proper time to proper distance (space) is constant.

In the case of the quantum potential the relativistic mass is:

$$m = \frac{m_0}{\sqrt{1 - \frac{2\hbar^2}{m_0^2 b r c^2}}} \quad (34)$$

where m_0 refers to the electron mass technically at $r \rightarrow \infty$ where the quantum potential is zero. Eq. (34) is identical to Eq. (26) from a quantum potential analysis. Hence a curved spacetime within the atom appears to be a justifiable postulate.

The above analysis validates postulate 3.

2.4. Postulate 4

2.4.1 Gravity as electromagnetism: the new field equation

The Einstein field equation is the well-known:

$$\mathbf{G} = k\mathbf{T} \quad (35)$$

Where, \mathbf{G} is the Einstein tensor, \mathbf{T} is the stress-energy-momentum tensor and the constant, k , for the macroscopic gravitational case is given by

$$k = \left(\frac{8\pi G}{c^4} \right) \quad (36)$$

wherein G is the gravitational constant. This constant, k , was obtained by comparing the weak field case with the Newtonian gravitational limit, namely, the gravitational Poisson's equation. Clearly, this constant, k , would not apply to the quantum or electromagnetic case within the atom, hence the original Einstein's equation cannot be the correct relativistic field equation for the atomistic case. We need to establish the value of the constant k for the quantum case; in particular, we shall evaluate it for the case of the electron in the ground state of the hydrogen atom. Once again, we need to compare the quantum weak field equation with the electrostatic Poisson's equation and reevaluate the new constant k for the quantum case. The Newtonian electrostatic Poisson's equation is

$$\nabla^2 \varphi = \frac{\rho_c}{\epsilon_0} \quad (37)$$

where φ is the electrostatic potential of charge, ρ_c is the charge density and ϵ_0 is the vacuum permittivity.

Starting with Eq. (4) we can write the quantum potential as

$$\phi_\psi(m) = \frac{\hbar^2}{m^2 br} = \left(\frac{1}{m}\right) \frac{\hbar^2}{mbr} \quad (38)$$

Then using Eq. (9) one obtains

$$\phi_\psi(m) = \left(\frac{1}{m}\right) \frac{e^2}{4\pi\epsilon_0 r} = \left(\frac{e}{m}\right) \frac{e}{4\pi\epsilon_0 r} = \left(\frac{e}{m}\right) \varphi. \quad (39)$$

Now to evaluate the static weak field we use the static weak field geometry given by [11]

$$ds^2 = \left(1 - \frac{2\phi_\psi}{c^2}\right) (dx^2 + dy^2 + dz^2) - \left(1 + \frac{2\phi_\psi}{c^2}\right) (cdt)^2 \quad (40)$$

For this geometry, it is already established [8] that in an orthonormal basis, in the weak field limit to a linear order in ϕ_ψ the time component of the Einstein tensor is

$$G_{tt} = \left(\frac{2}{c^2}\right) \nabla^2 \phi_\psi \quad (41)$$

We know that the time component of the stress energy momentum tensor is

$$T_{tt} = \rho c^2 \quad (42)$$

This equation is still valid because the mass of the electron is still very much larger than m_{cr} (Eq. 18). We thus obtain from Eq. (35), Eq. (41) and Eq. (42):

$$\left(\frac{2}{c^2}\right) \nabla^2 \phi_\psi = k(\rho c^2) \quad (43)$$

Because $\rho_c = \rho \left(\frac{e}{m}\right)$, one obtains using Eq. (43)

$$\left(\frac{2}{c^2}\right) \left(\frac{e}{m}\right) \nabla^2 \varphi = k \frac{\rho_c}{\left(\frac{e}{m}\right)} c^2 \quad (44)$$

Inserting Eq. (35) into Eq. (44) we get the result that:

$$k = \frac{2 \left(\frac{e}{m}\right)^2}{\epsilon_0 c^4} \quad (45)$$

so that the final modified Einstein field equation for the quantum case of a single electron in the ground state of the hydrogen atom is

$$\mathbf{G} = \frac{2 \left(\frac{e}{m}\right)^2}{\epsilon_0 c^4} \mathbf{T} \quad (46)$$

Here e/m is the charge to mass ratio for the electron. Eq. (46) is the general relativistic field equation within the atom at least for the 1s orbital case of the hydrogen atom. It directly demonstrates that the electrostatic force is a gravitational force thus validating postulate 4. If we compare Eq. (45) with the corresponding gravitational case, Eq. (36), one can see that the value of $\frac{2 \left(\frac{e}{m}\right)^2}{\epsilon_0}$ is over 40 orders of magnitude larger than $8\pi G$ for the case of the electron. This explains why electromagnetic forces are so high in the atom; indeed, these forces are known to be exactly that much higher than conventional gravitational forces inside the atom.

The above validates postulate 4.

2.5. Postulate 5

The analysis given in 2.4 also validates postulate 5 as we have shown that $\frac{2 \left(\frac{e}{m}\right)^2}{\epsilon_0}$ can replace $8\pi G$ in the field equation.

3. Other consequences and inferences from the gedanken experiment

3.1 Conservation of spacetime and space-like and time-like universes

Eq. (31) and Eq. (32) suggest another important result:

$$d\tau dr_p = dt dr \quad (47)$$

Thus, the product of space and time intervals are an invariant, that is, spacetime is conserved. When space expands time intervals contract, or vice-versa. Space and time intervals convert into each other in spacetime but maintaining their product as a constant. In fact, this universal constancy of the product, reflecting conservation of spacetime, is not limited to this gedanken or to quantum gravity effects, but is also true for special relativity and for general relativity. When proper time intervals tend to zero, and space-intervals tend to infinity, we have a space-like universe, where there is no time (that is, time does not advance), only space exists. In a time-like universe the space-intervals tend to zero and time intervals expand towards infinity.

Only time moves at a single point in space and there are no other points in space. It is clear from the previous discussion that the atom is a more space-like universe compared to our human macroscopic world because time moves more slowly inside the atom. When spacetime is purely space-like, this represents quantum entanglement, or perfect non-locality, represented by the fact that a photon can be everywhere at the same time [12]. In fact, Bell's inequality [13] and its proven experimental validation [14,15], proves non-locality at the quantum level. Perhaps the spacetime fabric is itself the hidden variable referred to in the famous EPR paper [16] and also by Bohm [3].

It also follows, combining Eq. (31), Eq. (32) and Eq. (34) that:

$$d\tau m_p = dt m \quad (48)$$

And likewise:

$$\frac{m_p}{dr_p} = \frac{m}{dr} \quad (49)$$

Eq. (48) shows that it is time that converts into mass as we enter a more space-like universe ($d\tau$ decreases). Thus, conservation of mass applies only when the ratio of space interval to the time interval, $dr_p/d\tau$, is constant. In fact from Eq. (48) and Eq. (49) it follows that:

$$\frac{m_p}{m} = \sqrt{\frac{\left(\frac{dr_p}{d\tau}\right)}{\left(\frac{dr}{dt}\right)}} \quad (50)$$

If we consider units where $m = dr = dt = 1$ (the reference macroscopic world is taken as the unity reference) then

$$m_p = \sqrt{\frac{dr_p}{d\tau}} \quad (50a)$$

and hence the mass is conserved when the ratio of proper space interval to proper time interval is a constant.

3.2 Origin of uncertainty and the uncertainty principle

We discussed above that within the atom proper time moves slower relative that for the macroscopic observer. This means that proper time for the macroscopic observer is faster than the proper time for the atom. In the same manner proper distance is smaller for the

macroscopic observer corresponding to the same proper distance for the atomic observer. Space is contracted for the macroscopic observer. The macroscopic observer sees a much larger space contracted into the small space of the atom and time is also speeded up from his perspective. Thus, particles will appear to move considerably faster inside the atom thereby creating a larger uncertainty of position. As an analogy, if we fast forward a movie at high rates, the characters will become blurred or lose physical appearance and the positional location of the character become more uncertain. Vice versa, within the atom, wherein space is expanded, and time slows down, velocities will appear to be slower for the observer within the atom.

The uncertainty principle therefore appears to have its origin in the spacetime structure and the transformation of this structure between the atomic and macroscopic domains. From a perspective within the atom, on the other hand, uncertainty disappears because time slows down and masses are larger and thereby positional uncertainty is diminished. What is quantum for a macroscopic observer is not quantum in nature from a perspective within the atom.

2 Summary and Conclusions

All the five postulates of the gedanken experiment have been shown to have mathematical validation and the explanations draw on the understanding that the quantum potential can curve spacetime within the atom. This enhances forces and energies within the atom. From the perspective within the atom, the atomic universe is vast and considerably slowed down, and is in fact not a quantum world at all. Quantum behavior appears to arise directly from a Schwarzschild-like spacetime within the atom such that the nucleus is at the center of an incredibly miniature atomic-sized “black hole”. This is consistent with the fact that we are screened from powerful nuclear radiations.

Spacetime, it appears, is not a coordinate system, or just the “arena” in which masses and events interact, but itself is a real entity obeying conservation laws. The fundamental law of spacetime conservation applies to special relativity, general relativity and to quantum gravity, namely, that the product of the space interval and the time interval is a constant. Mass is conserved only when the relative proportion of space and time within spacetime is fixed, or a constant. Thus, mass conservation law is only a special case of the law of spacetime conservation for the particular case wherein the relative proportion of space and time in spacetime is a constant.

The replacement of the term $8\pi G$ with $\frac{2\left(\frac{e}{m}\right)^2}{\epsilon_0}$ in the gravitational field equation within the atom demonstrates that charge arises from the spacetime transformation between the macroscopic and atomic realms.

These findings will open enormous new experimental opportunities in quantum gravity. These may include minute changes in the wavelength of light emitted from the atom, or minute

changes in the electron mass because of the spacetime perturbations. Future work will require considering higher quantum states, multiple electrons and electromagnetic behavior to include consistency with all of Maxwell's equations.

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Appendix 1: Geodesic Accelerations and Forces Calculated from the Quantum Spacetime Metric

The geodesic acceleration from the new metric of Eq. (29) is obtained using

$$a_{rr} = -\Gamma_{tt}^r \left(\frac{\partial t}{\partial \tau} \right)^2 \quad (A1)$$

Here, $d\tau$ in Eq. (A1) is the proper time given by $\frac{ds^2}{c^2}$ so that $\left(\frac{\partial t}{\partial \tau}\right)^2$ is just $\left(1 - \frac{2\hbar^2}{m^2 b r c^2}\right)^{-1}$ from Eq. (30). Γ_{tt}^r is the Christoffel symbol defined in the general theory of relativity. Eq. (47) can be written as

$$a_{rr} = -\Gamma_{tt}^r \left(1 - \frac{2\hbar^2}{m^2 b r c^2}\right)^{-1} \quad (A2)$$

By deriving the Christoffel symbol from the metric (see below) one can show that

$$\Gamma_{tt}^r = -\frac{\hbar^2}{m b r^2} \left(1 - \frac{2\hbar^2}{m^2 b r c^2}\right) \quad (A3)$$

Thus, it follows that, combining Eq. (A2) and Eq. (A3),

$$a_{rr} = -\frac{\hbar^2}{m^2 b r^2} = -\frac{e^2}{4\pi\epsilon_0 m r^2} \quad (A4)$$

which is which the required electron acceleration, Eq. (9), (since we changed the sign of the potential). The force is then

$$F_{rr} = m a_{rr} = -\frac{\hbar^2}{m b r^2} = -\frac{e^2}{4\pi\epsilon_0 r^2} \quad (A5)$$

which is an attractive electromagnetic force. These equalities support the equivalence principle and the notion of curved spacetime inside the atom. It appears that accelerations can be erased locally and replaced by geometric considerations when the spacetime metric is given by Eq. (29). This consistency is a validation of the Schwarzschild spacetime metric in Eq. (29) for the spacetime within atoms.

Christoffel symbols

The metric, $g_{\mu\nu}$ based on Eq. (29) in the text is

$$g_{\mu\nu} = \begin{bmatrix} \left(1 - \frac{2\phi}{c^2}\right)^{-1} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2 \sin^2 \theta & 0 \\ 0 & 0 & 0 & -c^2 \left(1 - \frac{2\phi}{c^2}\right) \end{bmatrix} \quad (A6)$$

The Christoffel symbols, $\Gamma_{\beta\gamma}^\delta$, are obtained from the metric using

$$g_{\alpha\delta}\Gamma_{\beta\gamma}^{\delta} = \frac{1}{2} \left(\frac{\partial g_{\alpha\beta}}{\partial x^{\gamma}} + \frac{\partial g_{\alpha\gamma}}{\partial x^{\beta}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\alpha}} \right) \quad (A7)$$

Using the above equation, one readily obtains Eq. (A3).

The Christoffel symbols can also be readily obtained from the metric using a software such as Maple®.