

How Quantum is Quantum Gravity?

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Abstract

Nature does not compartmentalize its happenings into different theories or disciplines. The theories are put forth by us to approximately understand the finer workings of nature. All theories are mere mathematical models built to understand nature. Any or all of them can be superseded by a better model or combination of models. The current paper analyses the formulation of Planck scale quantities, the workings of the temporal gauge, the concept of time other related fundamental issues. In particular, the paper points out that the force at the Planck scale is non-quantum.

Keywords: Planck scale, vorticity, circulation, canonical, loop quantum gravity, string theory.

The Planckian domain of physics known as the realm of quantum gravity is the most vexing puzzles of this century. What happens in that domain is a formidable challenge to figure out for any theoretical physicist and especially more stronger a challenge for the experimenter since the scale is so small ($10^{-33}cm$), and the corresponding energy scale is so staggeringly large ($10^{19}GeV$), that it will require an atom smasher bigger than the Milky Way galaxy. There are many tentative theories at hand making incredible predictions, such as string theory, loop quantum gravity, the double copy theory with its color-kinematics duality, the twistor theory, the theory of causal dynamical triangulations, and many more. String theory predicts the existence of 6 extra dimensions. Loop quantum gravity works in three dimensions and shows that the excitations of quantum space are 1-dimensional. Causal dynamical triangulations posits that spacetime is woven out of 1-dimensional threads of quantum geometry. Unlike the dissimilarities, there are also similarities. For example, in string theory, when one approaches the Planck scale of length, one passes right through it and ends up in a cosmic landscape into a strange domain of the multiverse. Similarly, loop quantum cosmology, an offspring of loop quantum gravity, has shown that when one pushes towards spacetime singularity, one undergoes a “bounce” and squeezes right through and enters into a new universe.

However, all these predictions are bizarre. One gas not taken into account as to how quantum is quantum gravity. There is an anomalous quantity – in fact, two anomalous quantities that rear up their ugly heads rendering quantum gravity non-quantum.

We address this problem and its various ramifications in this paper, as well as other considerations.

The quantum theory is settled on a firm philosophical and physical foundations by the Copenhagen Interpretation (CI) enunciated by Niels Bohr almost a 100 years ago. The theory underpinning the CI entails bizarre propositions such as properties as,

1. **Nonlocality:** which means that different parts of a quantum system appear to influence each other even when they are a long way apart and even although there is no known interaction between them.
2. **The Measurement Problem:** which arises from the idea that quantum systems possess properties only when these measured, although, there is apparently nothing outside quantum physics to make the measurement.

Now we can pose this question to the Big Bang Singularity. When one makes an observation at the past of the universe, one gets hints of a hot big bang universe. What happens when no one looks at the past of the universe? What happens when no one peers into the deep Hubble Field? After all, the universe can be described by a universal wave function ψ , that satisfies the zero-energy Schrödinger equation a.k.a. the Wheeler-DeWitt equation;

$$\hat{\mathcal{H}}(q)\psi(q) = 0 \quad (1)$$

Which actually is

$$\left(\frac{G_{abcd}}{(16\pi G)^2} \frac{\delta}{\delta q_{ab}} \frac{\delta}{\delta q_{cd}} + \frac{\sqrt{q}}{16\pi G} {}^3\mathcal{R} \right) \psi(q) = 0 \quad (2)$$

The theory behind the Wheeler-DeWitt equation is one of temporal gauge, viz.,

$$t = 0 . \quad (3)$$

Consider now a ket $|\psi\rangle$ and a bra $\langle q|$, so that the inner product reads

$$\langle q|\psi\rangle = e^{ipq} \quad (4)$$

Similarly, we have other eigen functions such as e^{-iEt} , $e^{-im\Gamma}$, etc.,

Now, analytical continuity between these complex eigen functions leads one to

$$Et = m\Gamma \quad (5)$$

Or

$$Et = \frac{E}{c^2} \Gamma$$

Or, therefore

$$t = \frac{\Gamma}{c^2} \quad (6)$$

Since, in the temporal gauge, $t = 0$, we have $\Gamma = 0$.

Here, Γ is the strength of the vorticity also called the circulation. So in the Wheeler-DeWitt case, or rather the Arnowitt-Deser-Misner (**ADM**) case, there is no vorticity, since there is no time. Now, according to the Landau-Raychudhuri equation, there is a vorticity term in the equation, which indicates expansion. If there is no vorticity, there is no expansion. How can the universe be born and undergo inflation if there is no expansion to start with. One could hope for some quantum gravitational fluctuation to cause the Bang. But here comes another major difficulty: know this, that the Planck scale is characterised by relativistic quantum gravity and hence is composed of the fundamental constants, c , \hbar and G .

So the Planck length is

$$\lambda_{Pl} = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-33} cm \quad (7)$$

The Planck energy by

$$E_{Pl} = \sqrt{\frac{c^5\hbar}{G}} \approx 10^{19} GeV \quad (8)$$

And the Planck time by

$$\tau_{Pl} = \sqrt{\frac{G\hbar}{c^5}} \approx 10^{-43} sec \quad (9)$$

Let us now construct a force out of the fundamental constants so as to correspond with the Planck scale, as after all, all interactions happen via a force. So we know that work done or equivalently, the energy spent is $E = F \times l$, where l is the displacement. So, $F = E/l$, which the Planck domain translates from eqs. (7) and (8), as

$$F_{Pl} = \frac{E_{Pl}}{\lambda_{Pl}} = \frac{c^4}{G} \quad (10)$$

Now when is a term quantum mechanical? When there is \hbar entering the term, right? As we can see above, there is no \hbar , i.e., the Planck's constant in the Planck scale expression for force is missing.

So, Planck's force classical. It is relativistic but still classical. Thus there is a scale for quantum gravity but the force involved in the Big Bang is purely classical relativistic.

This is the first anomaly in the theoretical physics underpinning quantum gravity. So now, going back to the quantum gravitational fluctuation that could have caused the Big Bang, there is no quantum in quantum gravity as the force given by eq. (10) is purely classical. What can cause such a huge force $\approx 10^{54} GeV/m$. Is this force enough to bring into effect, the universe that we see around us? Suppose now that the wave function of the universe admits local circulating structures, viz., for energy E and circulation Γ , we have a Fourier transform decomposition, as

$$\psi(\Gamma) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \psi(E) \exp(iE\Gamma/c^2) dE \quad (11)$$

so that the energy content Fourier decomposes into locally circulating structures. How can the quantal gravity which is non-quantum, be without time, i.e., obey the temporal gauge. It does so simply by allowing local vortices to exist inside $\psi(E) \rightarrow \psi(\Gamma)$.

Now, if the nett vorticity is zero, i.e., the vortices cancel each other out, then the temporal gauge will still hold. Let us look more closely at the Planck energy, E_{Pl} . It translates in the face of relativistic explorations to a Planck mass $m_{Pl} \approx 10^{-6} gm$. So,

$$\psi(\Gamma) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \psi(m) \exp(im\Gamma/c^2) dm \quad (12)$$

what is this fundamental mass? Why is it so large compared to the other elementary particle masses? This is the second anomaly. If vorticity and spin are related then the above Fourier

decomposition should translate to wave functions corresponding to different fundamental particles with $m \leftrightarrow E$ and $S \leftrightarrow \Gamma$.

The String Theory Scenario

In string theory, this corresponds to the Regge trajectories of $S \rightarrow m^2$. And by our Fourier transform technique, this should correspond to an isomorphism with $\Gamma \rightarrow E^2$. Thus, the vorticity should allow for a string with tension T , which we know is related to the Regge slope α' by $T = 1/2\pi\alpha'$, and with different modes of vibration to which may correspond our aforementioned Fourier analysis. Thus all the fundamental particles are the modes of a sting with vorticity Γ . Because all fundamental particles are produce by the different modes of vibration of an open ended string, whereas, gravity is the only one produced by a closed loop string. Now, what about the vorticity of a closed loop string? Since gravity is conservative, the closed loop string should have a zero vorticity, i.e., the circulation around the closed loop gives zero:

$$\Gamma = \oint \vec{v} \cdot d\vec{l} = 0 \quad (13)$$

Now, do not forget that our quantal gravity is in reality, classical. So, the graviton corresponding to the closed loop string is not a quantum of the gravitational field in the quantum field theoretical sense. So, the spacetime is something very special. You see, the Wheeler-DeWitt equation also known as the Hamiltonian constraint causes actual Lorentz rotations i.e., physical rotations in spacetime, whereas the constraint corresponding to the other three forces, viz., the electromagnetic, strong and the weak force, called the Gauss constraint causes rotations corresponding to gauge transformations which have nothing to with actual dynamics in spacetime where th Wheeler-DeWitt equation figures. So, the spacetime is not like the other gauge felds. It is “real”. It actually does warp but only when something really heavy or massive tuns up to warp it. This makes gravity different from the other forces.

At the beginning, we spoke of nonlocality. This entails entanglement. If the wave function of the universe $\psi(E)$ Fourier decomposes $\psi(\Gamma)$, yielding fundamental particle wave functions, then, relative to some superior observer, all the particle states and the state of the universe are entangled in some way or another. This entanglement could be translated as gravity. You see, the force of gravity, though conceived in the Planckian scale, is still classical as we have see in our above analysis. So it is not a fundamental force but a vestige of something more fundamental.

Analysis of the Gravitational constant G and further ramifications

What about the constant of gravitation. Firstly, the Planck force screens the Einstein curvature G_{ab} versus the energy-momentum tensor T_{ab} as follows: from eq (10), and the Einstein Field Equations of general relativity, viz.,

$$G_{ab} = \frac{8\pi G}{c^4} T_{ab} \quad (14)$$

$$G_{ab} = \frac{8\pi}{F_{Pl}} T_{ab} \quad (15)$$

Or $10^{54}G_{ab} = 8\pi T_{ab}$. Thus whatever T_{ab} be, it entails an energy force $10^{54}GeV/m$ times G_{ab} . Let us call $10^{54}G_{ab} = K_{ab}$, so that $K_{ab} = T_{ab}$.

What happens to K_{ab} when $G \rightarrow \infty$? We ask this question because of the following reason: near the Big Bang Singularity, the energy-momentum simply cannot go to infinity. There is no physical sense in saying such things. The universe is finite and the energy content in it has to be finite, otherwise there would have been stars and galaxies everywhere in the sky filling up the entire dome and there would be brightness all the day and the night. This is the Olber's paradox. So coming back to our argument, when $G \rightarrow \infty$, c stays constant, so

$$K_{ab} = \frac{c^4}{G} G_{ab} \rightarrow 0$$

As long as c stays the same or does not scale. Thus, the screened curvature $K_{ab} \rightarrow 0$. What does this speak for the interior of a blackhole, precisely for a singularity? Does the screened curvature entail some sort of non-singular event? Could be....

Let's explore the consequences for a singularity involving the entirety of the spacetime viz., the Big Bang Singularity. Notice the Wheeler-DeWitt equation Eq. (2). When one approaches the Big Bang Singularity, if G starts to scale upwards, at precisely $t = 0$, there will be no gravity. That's the strangeness in the proportion. Thus, at the Big Bang, the Wheeler-DeWitt equation annihilates the gravitational degrees of freedom. This is the scaling law interpretation of the Wheeler-DeWitt equation. What happens when $G \rightarrow 0$? K_{ab} apparently tends to a mathematical singularity. So the energy-momentum becomes indeterminate. The physics falls apart. There is thus a skin depth in time for $t = 0, G \rightarrow \infty$ and $K_{ab} = 0$, so gravity momentarily drops to zero and then picks up from there to go to the other end. Something like a bouncing universe or a cosmic landscape could be easily envisaged there to occur in the respective scenarios (loop, strings, etc.) so, there is a bubble centered at $t = 0, \lambda = 0$ and radius $\lambda = \lambda_{pl}, \tau = \tau_{pl}$, a $d = 3 + 1$ bubble, around which is the probable Planck force operable to yield an expanding universe in both the directions, or as string theorists may have their way – a cosmic multiverse landscape or a bouncing universe in loop quantum cosmology format!

Conclusions

The various theories of physics describing the so called holy grail – the quantum theory of gravity and the concomitant unified field theory are mere mathematical models just as the Newtonian and Einstein theories of relativity, the theory of quantum mechanics and the quantum field theory. By no means are these to be understood as the nature itself. Nature goes on working on its own oblivious of the approximate descriptions of it. Nature was quantum before quantum theory was discovered. Spacetime curved before general theory of relativity was conceived. The Wheeler-DeWitt equation, one of the most beautiful equations in theoretical physics doesn't have a proper interpretation. Same goes for the candidate wave functionals satisfying the Wheeler-DeWitt equation. Under such circumstances, the arguments in this paper should be taken seriously. Planck's force is not a quantum force. This justifies the high magnitude impulse required for the creation of the universe. A lot more thought has to be put in by the physics community into these issues before making claims about the existence of a quantum theory of gravity or a possibility of constructing one.

