

An Expression of A in Power Law Equation for Resistivity and Temperature, Resistance or Color and Pressure

Yongrong Bao*

100 Renmin South Road, Luoding 527200, Guangdong, China

Abstract

We study the power law equation governing the relationship between resistivity and temperature, as well as the pressure-induced resistance and color change of a superconductor, through the generalized relational expression. By assuming a relationship between resistivity, effective mass and temperature, we find a generalized formula without gravitational effect and a generalized expression for the power law coefficient A . Through careful selection of the exponent N of the speed of light in vacuum, we obtain specific power law equations and a particular expression for A . In regards to the relationship between temperature and pressure, it has that the resistance varies inversely with pressure to the power of a third, sixth or half within a specific range. Similarly, in relation to color and pressure, the frequency varies directly with pressure to the power of a quarter, half, linearity, negative half, or inverse proportion. This indicates that the color shifts from blue to purple to blush as the pressure rises within a certain range. It shows that the expansion of the application of generalized relational expression can be achieved by manipulating the exponent of the constants.

1. Introduction

The power law equation [1] describes the relationship between the resistivity of a superconducting material and its temperature. There have been numerous studies on this topic, both theoretical and experimental. One of the most famous models is the Bloch-Grüneisen law [2]; another is Callendar-Van Dusen equation. Holographic method or gauge/gravity duality has made much progress [3] for the linear temperature and resistivity recent years. But they didn't give the specific expression for the power law coefficient A in most cases.

Room temperature superconductivity is a fascinating area of research because the large application prospect. Some materials become superconducting at higher temperatures under high pressure. For instance, C-H-S system becomes superconductor at 287K and 267 Gpa [4], LaH₁₀ [5] and YH₉ [6] become superconducting at around 250k under 200 Gpa in recent years. This is a milestone; simultaneously excessive pressures greatly limit their application. Recently room temperature superconductivity at near barometric pressure (about 1 Gpa) was claimed in the nitrogen-doped lutetium hydrides [7]. But several groups couldn't reproduce except the color changed by the pressure [8, 9, 10, 11]. Also no direct equation described color changing [12].

The paper is organized as follows. In Sec. 2, we find a generalized formula without gravitational effect and a generalized expression for the coefficient A . In Sec. 3, we obtain the resistance varying inversely with pressure to the power of a third, sixth or half within a specific range. In Sec. 4, we get that the color varies directly with pressure to the power of a quarter, half, linearity, negative half, or inverse proportion within a certain range. We conclude in Sec. 5.

2. Expression of A

The basic relationship [13] is

$$A \sim A_p = [\hbar^{(\delta+\varepsilon+\zeta+\eta)} G^{(\delta-\varepsilon+\zeta-\eta)} c^{-(3\delta-\varepsilon+5\zeta-5\eta)} \kappa^{-2\eta} e^{2\lambda}]^{1/2} \quad (1)$$

* Corresponding author: baoyong9803@163.com. Because there are many duplicate names called Yong Bao (包勇), the pen name is Yongrong Bao (包勇戎) to show the difference.

Where A is any physical quantity, $[A] = [L]^\delta [M]^\varepsilon [T]^\zeta [\Theta]^\eta [Q]^\lambda$ its dimensions, L, M, T, Θ and Q are the dimensions of length, mass, time, temperature and electric charge separately (here we use the LMTΘQ units [14]), A_P the corresponding Planck scale of A , $\delta, \varepsilon, \zeta, \eta$ and λ the real number, \hbar, G, c, κ and e the reduced Planck constant, gravitational constant, speed of light in vacuum, Boltzmann constant and elementary charge separately.

The generalized relational expression [13] is

$$\prod_{i=1}^n A_i^{\alpha_i} \sim \prod_{i=1}^n A_{iP}^{\alpha_i}, \quad i = 1, 2, 3 \dots n \quad (2)$$

where A_i is the physical quantity, α_i the real number, and A_{iP} the corresponding Planck scale.

The power law equation for resistivity and temperature [1] is

$$\rho = \rho_0 + AT^n \quad (3)$$

where ρ is the resistivity, ρ_0 the residual resistivity, A the coefficient, n the exponent, and T the temperature.

In here we define

$$\rho_n = A_n T^n \quad (4)$$

Consulting the Uemura's law [15], we assume that the resistivity ρ_n has relationship between effective mass m and temperature T , and obtain

$$\rho_n m^{\alpha} T^{\beta} \sim \left(\frac{\hbar^3 G}{c^3 e^4}\right)^{1/2} \left(\frac{\hbar c}{G}\right)^{\alpha/2} \left(\frac{\hbar c^5}{k^2 G}\right)^{\beta/2} = \hbar^{(3+\alpha+\beta)/2} G^{(1-\alpha-\beta)/2} c^{-(3-\alpha-5\beta)/2} k^{-\beta} e^{-2} \quad (5)$$

where $\rho_P = \left(\frac{\hbar^3 G}{c^3 e^4}\right)^{1/2}$ is the Planck resistivity (from (1)), $m_P = \left(\frac{\hbar c}{G}\right)^{1/2}$ the Planck mass, and $T_P = \left(\frac{\hbar c^5}{k^2 G}\right)^{1/2}$ the Planck temperature. Neglecting the gravitational effect, and ordering $1 - \alpha - \beta = 0$ and $3 - \alpha - 5\beta = N \rightarrow \alpha = (2+N)/4$ and $\beta = (2-N)/4$, where N is a number, we find

$$\rho_n \sim \frac{\hbar^2 k^{(N-2)/4} T^{(N-2)/4}}{e^2 c^{N/2} m^{(N+2)/4}} \sim A_n T^n, \quad N \neq 2 \quad (6)$$

It is the generalized formula for the resistivity, effective mass and temperature without gravitational effect. Therefore

$$A_n \sim \frac{\hbar^2 k^{(N-2)/4}}{e^2 c^{N/2} m^{(N+2)/4}} \quad (7)$$

This is the generalized expression of A .

(1) Ordering $N = -2$, we obtain

$$\rho_{-1} \sim \frac{\hbar^2 c}{e^2 k T} \sim A_{-1} T^{-1}$$

The resistivity exhibits inversely-proportional temperature dependence [16], but independence the effective mass.

(2) Making $N = 0$, that is ignoring the relativistic effect, get

$$\rho_{-1/2} \sim \frac{\hbar^2}{e^2 k^{1/2} m^{1/2} T^{1/2}} \sim A_{-1/2} T^{-1/2}$$

Resistivity exhibits negative subduplicate temperature dependence.

(3) Ordering $N = 4$, get

$$\rho_{1/2} \sim \frac{\hbar^2 k^{1/2} T^{1/2}}{e^2 c^2 m^{3/2}} \sim A_{1/2} T^{1/2}$$

Resistivity exhibits subduplicate temperature dependence [17].

(4) Making $N = 6$, obtain

$$\rho_1 \sim \frac{\hbar^2 k T}{e^2 c^3 m^2} \sim A_1 T$$

Resistivity of metal exhibits linear temperature dependence [4, 18, 19]. Taking it into the formula $\rho = \hbar k T m / n_n e^2$ [19], we obtain $m \sim \hbar^3 \sqrt{n_n} / c$, where n_n is the carrier density of quasi-particles.

(5) Ordering $N = 8$, get

$$\rho_{3/2} \sim \frac{\hbar^2 k^{3/2} T^{3/2}}{e^2 c^4 m^{5/2}} \sim A_{3/2} T^{3/2}$$

Resistivity exhibits temperature dependent on the power of three-half [18, 20].

(6) Making $N = 10$, obtain

$$\rho_2 \sim \frac{\hbar^2 k^2 T^2}{e^2 c^5 m^3} \sim A_2 T^2$$

Resistivity exhibits quadratic temperature dependence in the low temperature limit for the Fermi liquid like [21].

(7) Ordering $N = 12$, get

$$\rho_{5/2} \sim \frac{\hbar^2 k^{5/2} T^{5/2}}{e^2 c^6 m^{7/2}} \sim A_{5/2} T^{5/2}$$

Resistivity exhibits temperature dependent on the power of five-half [20].

(8) Making $N = 14$, obtain

$$\rho_3 \sim \frac{\hbar^2 k^3 T^3}{e^2 c^7 m^4} \sim A_3 T^3$$

Resistivity exhibits cubic temperature dependence [22].

(9) Ordering $N = 16$, get

$$\rho_{7/2} \sim \frac{\hbar^2 k^{7/2} T^{7/2}}{e^2 c^8 m^{9/2}} \sim A_{7/2} T^{7/2}$$

Resistivity exhibits temperature dependent on the power of seven-half [23].

(10) Making $N = 18$, obtain

$$\rho_4 \sim \frac{\hbar^2 k^4 T^4}{e^2 c^9 m^5} \sim A_4 T^4$$

Resistivity exhibits biquadratic temperature dependence [24].

(11) Ordering $N = 20$, get

$$\rho_{9/2} \sim \frac{\hbar^2 k^{9/2} T^{9/2}}{e^2 c^{10} m^{11/2}} \sim A_{9/2} T^{9/2}$$

Resistivity exhibits temperature dependent on the power of nine-half [22, 25].

(12) Making $N = 22$, obtain

$$\rho_5 \sim \frac{\hbar^2 k^5 T^5}{e^2 c^{11} m^6} \sim A_5 T^5$$

Resistivity exhibits quintic temperature dependence, which is the Bloch-Grüneisen law [2].

And so on.

3. Pressure caused resistance change

Similarly the power law equation for resistance and temperature [11] is

$$R = R_0 + AT^a \quad (8)$$

where R is the resistance, R_0 the residual resistance, and a the exponent.

We define

$$R_a = A_a T^a \quad (9)$$

The relationship between resistance R , temperature T and pressure p is

$$RT^\alpha p^\beta \sim \left(\frac{\hbar}{e^2}\right) \left(\frac{\hbar c^5}{k^2 G}\right)^{\alpha/2} \left(\frac{c^7}{\hbar G^2}\right)^\beta = \hbar^{(2+\alpha-2\beta)/2} G^{-(\alpha+4\beta)/2} c^{(5\alpha+14\beta)/2} k^{-\alpha} e^{-2} \quad (10)$$

where $R_p = \frac{\hbar}{e^2}$ is the Planck resistance, and $p_p = \frac{c^7}{\hbar G^2}$ the Planck pressure. Neglecting the gravitational effect, we make $2 + \alpha - 2\beta = N$ and $\alpha + 14\beta = 0 \rightarrow \alpha = 2(N-2)/3$ and $\beta = (2-N)/6$, and find

$$R_a \sim \frac{\hbar^{N/2} k^{2(2-N)/3} T^{2(2-N)/3}}{e^2 c^{(2-N)/2} p^{(2-N)/6}} \sim A_a T^a, \quad N \neq 2 \quad (11)$$

This is the generalized formula for the resistance, pressure and temperature without gravitational effect also.

(1) Ordering $N = 0$, that is ignoring the quantum effect, we obtain

$$R_{4/3} \sim \frac{kT}{ce^2} \sqrt[3]{\frac{kT}{p}}$$

So the resistance varies inversely with pressure to the power of a third [26].

(2) Making $N = 1$, get

$$R_{2/3} \sim \sqrt[6]{\frac{\hbar^3 k^4 T^4}{c^3 p}} / e^2$$

Resistance varies inversely with pressure to the power of a sixth [26].

(3) Ordering $N = -1$, obtain

$$R_2 \sim \frac{k^2 T^2}{ce^2 \sqrt{cp}}$$

Resistance varies inversely with pressure to the power of a half within a certain range.

4. Pressure induced color change

Referring to the Birch-Murnaghan equation [27], we consider the frequency ω has relationship between pressure p and volume V , and obtain

$$\omega p^\alpha V^\beta \sim \left(\frac{c^5}{\hbar G}\right)^{1/2} \left(\frac{c^7}{\hbar G^2}\right)^\alpha \left(\frac{\hbar G}{c^3}\right)^{3\beta/2} = \hbar^{-(1+2\alpha-3\beta)/2} G^{-(1+4\alpha-3\beta)/2} c^{(5+14\alpha-9\beta)/2} \quad (12)$$

where $\omega_P = \left(\frac{c^5}{\hbar G}\right)^{1/2}$ is the Planck frequency, $V_P = \left(\frac{\hbar G}{c^3}\right)^{3/2}$ the Planck volume. Neglecting the gravitational effect, we order $1 + 4\alpha - 3\beta = 0$ and $5 + 14\alpha - 9\beta = N \rightarrow \alpha = (N-2)/2$, $\beta = (2N-3)/3$, and find

$$\omega \sim \hbar^{(2-N)/2} c^{N/2} p^{(2-N)/2} V^{(3-2N)/3}, \quad N \neq 2 \quad (13)$$

This is the generalized equation for the frequency, pressure and volume without gravitational effect too.

(1) Ordering $3 - 2N = 0$, $N = 3/2$, we obtain

$$\omega_{1/4} \sim \sqrt[4]{\frac{c^3 p}{\hbar}}$$

(2) Making $N = 0$, that is ignoring the relativistic effect, get

$$\omega_1 \sim \frac{pV}{\hbar}$$

(3) Ordering $N = 1$, obtain

$$\omega_{1/2} \sim \sqrt[3]{V} \sqrt{\frac{cp}{\hbar}}$$

(4) Making $N = 3$, get

$$\omega_{-1/2} \sim \frac{c}{V} \sqrt{\frac{\hbar c}{p}}$$

(5) Ordering $N = 4$, obtain

$$\omega_{-1} \sim \frac{\hbar c^2}{pV^{5/3}}$$

Therefore, the frequency varies directly with pressure to the power of a quarter, half, linearity, negative half, or inverse proportion. This indicates that the color shifts from blue to purple to blush as the pressure rises within a specific range [11].

5. Discussion

In this paper, we study the power law equation governing the relationship between resistivity and temperature, as well as the pressure-induced resistance and color change of a superconductor, by the generalized relational expression. We find the following results.

- (1) The resistivity was assumed to have relationship between effective mass and temperature, and was found a generalized formula without gravitational effect and a generalized expression for the power law coefficient A.
- (2) Through careful selection of the exponent N of the speed of light in vacuum, was obtained specific power law equations such as from $A_{-1}T^{-1}$ to A_5T^5 , and a particular expression for A.
- (3) It was discovered a generalized equation neglecting gravitational effect for the resistance, temperature and pressure through selecting exponent N of reduced Planck constant correctly. Resistance varied inversely with pressure to the power of a third, sixth or half within a certain range.
- (4) Also a generalized equation without gravitational effect for the frequency, pressure and volume was found. Frequency varied directly with pressure to the power of a quarter, half, linearity, negative half, or inverse proportion. It indicates that the color shifts from blue to purple to blush as the pressure rises within a specific range.
- (5) The value of exponent N isn't arbitrary and requires being determined in conjunction with experiments.
- (6) It shows that the expansion of the application of generalized relational expression can be achieved by manipulating the exponent of the constants.

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