# QED analysis of the two photons -- into -- a paraphoton inelastic scattering 

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#### Abstract

Paraphoton is the axion-like chameleon (no fixed rest-mass) particle introduced to the quantum chromodynamics to solve the strong CP-problem. Nowadays, it is the best candidate to be the quantum of the dark matter. In the paper, we present the quantum analysis of the two-photons process resulting to the paraphoton formation.


## 1. Introduction

The recent experiments on the Large Hadron Collider [1] discover the elastic photon-photon scattering $\gamma+\gamma \rightarrow \gamma+\gamma$. This way, there is also possible the inelastic process (fig.1) combines two photons to the paraphoton [2] by the channel $\gamma+\gamma \rightarrow a$. The outcome $a$ is a massive boson with spin $s_{a}=2$ and "chameleon" mass $m_{a}=2 h v_{\gamma} / c^{2}$, which interacts neither the "strong" nor a "weak" way but potentially reveals itself in the neutrino experiments [3]. Because of its symmetry paraphoton is also the best candidate to be the gravitational gauge boson [4] which is probably form the dark matter. These all make the process $\gamma+\gamma \rightarrow a$ very important, and the paper performs its quantum analysis.


Figure 1. Feynman diagram for the process of inelastic interaction of two photons $\gamma+\gamma \rightarrow a$

## 2. Analysis

The analysis can be performed by the standard routine, described in [5]. Hereinafter the paper uses the nuclear system of units (Dirac constant is 1 , speed of light is 1 ) and 4 -vectors.

Let $k_{1}$ and $k_{2}$ be photon momenta (see fig.1), $\lambda_{1}$ and $\lambda_{2}$ its polarizations and $q$ the momentum of paraphoton.

The initial state of the system (two photons) is

$$
\begin{equation*}
\left|k_{1} \lambda_{1}, k_{2} \lambda_{2}\right\rangle=a_{\lambda_{1}}^{+}\left(k_{1}\right) a_{\lambda_{2}}^{+}\left(k_{2}\right)|0\rangle \tag{1}
\end{equation*}
$$

where $|0\rangle$ is the ground state of a vacuum, $a^{+}$is the photon "birth" operator.
The final state of the system (paraphoton) is $|q\rangle$.
This way, the scattering matrix is following:

$$
\begin{gather*}
\langle q| S\left|k_{1} \lambda_{1}, k_{2} \lambda_{2}\right\rangle=e^{2} \frac{1}{(2 \pi)^{9 / 2}} \frac{1}{\sqrt{8 k_{10} k_{20} q}} \times  \tag{2}\\
\times e_{\mu}^{\lambda_{1}}\left(k_{1}\right) e_{\mu}^{\lambda_{2}}\left(k_{2}\right) M_{\mu \nu}\left(k_{1}, k_{2} ; q\right)(2 \pi)^{4} \delta\left(k_{1}+k_{2}-q\right)
\end{gather*}
$$

Here $e^{\lambda 1}\left(k_{1}\right)$ and $e^{\lambda 2}\left(k_{2}\right)$ are 4-polarizations of photons, $e$ is the elementary charge, $\delta$ is the Dirac $\delta$ function and $\mathrm{M}_{\mu \mathrm{v}}\left(k_{1}, k_{2} ; q\right)$ is unknown because of no good theory nowadays. Meanwhile, the analysis of the symmetry can get it with the single unknown constant.

To obtain $\mathrm{M}_{\mu v}$, lets start from the evident commutativity

$$
\begin{equation*}
\left[a_{\lambda_{1}}^{+}\left(k_{1}\right), a_{\lambda_{2}}^{+}\left(k_{2}\right)\right]=0 \tag{3}
\end{equation*}
$$

which gets

$$
\begin{equation*}
\langle q| S\left|k_{1} \lambda_{1}, k_{2} \lambda_{2}\right\rangle=\langle q| S\left|k_{2} \lambda_{2}, k_{1} \lambda_{1}\right\rangle \tag{4}
\end{equation*}
$$

means that photons must be identical.
This way,

$$
\begin{equation*}
M_{\mu \nu}\left(k_{1}, k_{2} ; q\right)=M_{\nu \mu}\left(k_{2}, k_{1} ; q\right) \tag{5}
\end{equation*}
$$

Next, Lorentz- and inversion-invariances makes $\mathbf{M}_{\mu v}\left(k_{1}, k_{2} ; q\right)$ be a pseudotensor of rank 2. At fig. 1 there are three 4 -vectors $\left(k_{1}, k_{2}, q\right)$ bound by the energy and the momentum conservation

$$
\begin{equation*}
k_{1}+k_{2}=q \tag{6}
\end{equation*}
$$

which makes only two $\left(k_{1}, k_{2}\right)$ of them independent ones. This two vectors can combine a rank 2 pseudotensor by the following way only:

$$
\begin{equation*}
M_{\mu \nu}\left(k_{1}, k_{2} ; q\right)=A \varepsilon_{\mu v \rho \sigma} k_{1 \rho} k_{2 \sigma} \tag{7}
\end{equation*}
$$

where $\varepsilon$ is the Levi-Civita epsilon and $A$ is a constant.
To construct scalar $A$ from 4 -vectors $k_{1}$ and $k_{2}$ there are two ways only:

1) $k_{1}^{2}=k_{2}^{2}=0$ which is trivial, and
2) $k_{1} k_{2}=1 / 2 q^{2}=-1 / 2 m_{a}^{2}$ where the paraphoton mass $m_{a}$ is figured.

Therefore, $A$ is determined by the rest-mass $m_{a}$ of the paraphoton. In the local inertial system of paraphoton's mass center the summation of (2) over $\lambda_{1}$ and $\lambda_{2}$ gives

$$
\begin{gather*}
d \Gamma=\frac{1}{(2 \pi)^{2}} \frac{1}{8 m_{a}} e^{4} \sum_{\lambda_{1}, \lambda_{2}=1,2}\left(e_{\mu}^{\lambda_{1}}\left(k_{1}\right) e_{\nu}^{\lambda_{2}}\left(k_{2}\right) M_{\mu \nu}\right) \times \\
\times\left(e_{\rho}^{\lambda_{1}}\left(k_{1}\right) e_{\sigma}^{\lambda_{2}}\left(k_{2}\right) M_{\rho \sigma}\right)^{*} \delta\left(k_{1}+k_{2}-q\right) \frac{d k_{1}}{d k_{10}} \frac{d k_{2}}{d k_{20}}=  \tag{8}\\
=\frac{1}{(2 \pi)^{2}} \frac{1}{8 m_{a}} e^{4} \sum_{\mu, \nu} M_{\mu \nu} M_{\mu \nu}^{*} \eta_{\mu} \eta_{\nu} \delta\left(k_{1}+k_{2}-q\right) \frac{d k_{1}}{d k_{10}} \frac{d k_{2}}{d k_{20}}
\end{gather*}
$$

with $\eta_{\lambda}=(+1)$ for $\lambda=1,2,3$, and ( -1 ) for $\lambda=4$. This is the differential cross-section of $\gamma+\gamma \rightarrow a$. To simplify (8), one can uses following almost-evidences:

$$
\begin{equation*}
\varepsilon_{\mu \nu \rho \sigma} k_{1 \rho}^{*} k_{2 \sigma}^{*} \eta_{\mu} \eta_{v}=-\varepsilon_{\mu \nu \rho \sigma} k_{1 \rho} k_{2 \sigma} \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
\varepsilon_{\mu \nu \rho \sigma} \varepsilon_{\mu \nu \rho \prime \sigma^{\prime}}=2\left(\delta_{\rho \rho^{\prime}} \delta_{\sigma \sigma^{\prime}}-\delta_{\rho \sigma^{\prime}} \delta_{\sigma \rho^{\prime}}\right) \tag{10}
\end{equation*}
$$

which give

$$
\begin{equation*}
\sum_{\mu, \nu} M_{\mu \nu} M_{\mu \nu}^{*} \eta_{\mu} \eta_{\nu}=\frac{1}{2} m_{a}^{4}|A|^{2} \tag{11}
\end{equation*}
$$

This way, we also can obtain the integral cross-section $\Gamma$. To do it, note that $d \Gamma\left(k_{1}, k_{2}\right)=d \Gamma\left(k_{1}, k_{2}\right)$. Therefore, to calculate $\Gamma$, one should use the following trick: to integrate $d \Gamma$ by all possible $k_{1}$ and $k_{2}$, and get a half of the result. During the integration one should also use the feature of the Dirac $\delta$ :

$$
\begin{equation*}
\int \delta\left(k_{1}+k_{2}-q\right) \frac{d k_{1}}{k_{10}} \frac{d k_{2}}{k_{20}}=2 \pi \tag{12}
\end{equation*}
$$

which after all gives the final result:

$$
\begin{equation*}
\Gamma=\frac{1}{4} \pi \alpha^{2} m_{a}^{3}|A|^{2} \tag{13}
\end{equation*}
$$

where $\alpha=e^{2} / 4 \pi=1 / 137$ is the fine structure constant, $m_{a}$ is the (chameleonic) rest-mass of the paraphoton, $A$ is a constant to be measured during the experiment.

## 3. Conclusion

The performed QED analysis gives the differential (8) and the integral (13) cross-sections of the photon-photon process resulting to the birth of the paraphoton. Because of T-symmetry, these also are the cross-sections of the invert process of paraphoton decay onto two photons. Therefore, we predict the two photons -- into --the paraphoton oscillations, which can be detected by the known distribution of the dark matter in the Universe [6,7]. These oscillations makes the Universe be transparent to observe ultra-deep field at the Hubble Space Telescope (HST anomaly, [8]). The $\gamma+\gamma \rightarrow a$ channel also influence to the registered star energy radiation resulting to the anomalous flux of the neutrino (solar neutrino anomaly [9]) and unexpectedly high positron ratio in the cosmic rays (PAMELA anomaly [10]). At last, one should notice the most recent experiments on the electromagnetically induced over-transparency of the crystalline ruby [11] and the Primakoff ("Light shining through a wall") effect in the photonic crystals [12].

## References

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