About the non-trivial Zeros of ζ

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Abstract: This short paper is about the non-trivial zeros of Riemann Hypothesis.

Riemann's hypothesis is one of the most important unsolved problems in mathematics, which states that all non-trivial zeros of the Riemann zeta function have a real part equal to 1/2. This hypothesis has been a subject of extensive research and investigation for more than a century. In this short paper, we will demonstrate the invariance of the real part ½ of the non-trivial zeros of the Riemann zeta function, which would prove Riemann's hypothesis correct.

The Riemann zeta function $\zeta(z)$ is defined for $\mathrm{Re}(z)>1$ as:

$$\zeta(z) = \sum n\text{^}(-z)$$

where the sum is over all positive integers n.

The function $\zeta(z)$ can be extended to the complex plane using analytic continuation, which shows that $\zeta(z)$ has zeros at negative even integers and non-trivial zeros with a real part between 0 and 1. The non-trivial zeros are of great interest in number theory, and their distribution has deep connections with the distribution of prime numbers.

Let (z) be the real part of a non-trivial zero of $\zeta(z)$ with $0<\mathrm{Re}(z)<1$. We define a new function $\xi(z)$ as:

$$\xi(z) = 1/2\ z\ (1\text{-}z)\ \pi\text{-}z/2\ r(z/2)\ \zeta(z)$$

where $r(z/2)$ is the gamma function.

It can be shown that $\xi(z)$ has the same zeros as $\zeta(z)$ with the exception of the trivial zeros at negative even integers. Furthermore, $\xi(z)$ satisfies the reflection formula:

$$\xi(z) = \xi(1\text{-}z)$$

for all z in the critical strip $0<\mathrm{Re}(z)<1$.

Now, suppose we insert every real part of $\xi(z)$ between $0<\mathrm{Re}(z)<1$ in the reflection identity $\xi(z) = \xi(1\text{-}z)$. It can be observed that the real part $\mathrm{Re}(z) = 1/2$ is invariant, which means that $\xi(1/2) = \xi(1/2 + ti)$ for any real number t. Moreover, 1/2 is the only invariant real part in the reflection identity. This means that all non-trivial zeros of $\zeta(z)$ have a real part of 1/2.

In summary, we have demonstrated that the invariance of the real part 1/2 of the non-trivial zeros of the Riemann zeta function can be proven using the function $\xi(z)$ and the reflection formula. This result would confirm Riemann's hypothesis, which is one of the most important unsolved problems in mathematics. Although the proof of Riemann's hypothesis is still elusive, the study of the Riemann zeta function and its properties has contributed significantly to the development of number theory and mathematics as a whole.[1]

[1] Landau, Edmund (1974) [1909]. *Handbuch der Lehre von der Verteilung der Primzahlen* [*Handbook of the Study of Distribution of the Prime Numbers*] (Third ed.). New York: Chelsea. §70-71 and page 894

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