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“Algebraic proof of Goldbach’s Conjecture”

Contents:

I-Introduction:

This is a work I did within the framework of the famous Goldbach conjecture proof, which states that every even natural integer greater than 3 can be written in the form of the sum of two prime numbers. I hope this proof is sufficient and helpful for all concerned. If you have any comments or notes regarding the work, you can contact me via my e-mail: muhammad2357ameen@gmail.com

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II-Abstract:

This is an algebra-based proof of Goldbach's conjecture.First, we define the formula for prime numbers greater than 3, then we prove that every even number greater than 8 can be written in the form of the sum of two primes, and finally, we deal with the rest of even numbers less than 10 , which are 8, 6 and 4.

III- Formula for a prime number greater than 3:

●We know that every prime number greater than 2 is an odd number, that 2 and 3 are consecutive and primary, 3+2×b odd (sum of odd and even).

● I'm going to suggest the sequence So where every number in parentheses is prime, this sequence starts from the first prime number greater than (2), which is of course (3), the increment factor of this sequence in each part is 2, I determined it due to the logical relationship that links it to the number 3 (prime numbers cannot appear confined between two elements of the series because they will be in the form: 3 + 2 x b-1 = 2 + 2 x b

So = (3), (5), (7), 9, (11), (13),15, (17), (19), 21...

●You can notice here that starting from the number 3, when every addition to the number 2 is arranged in a multiple of 3 (In the form: 3q×2), the number is not prime (if it's greater than 3), because it's multiple of 3. EX: 9= 3 + 2 × 3.

● So, any prime number can be written in the form: 3 + 2q where q is natural integer not a multiple of 3 (q must be natural integer, because otherwise, it be in the form: $\frac{n}{2}$, with n an odd number, so in this case the formula will be: $3 + 2 \times \frac{n}{2} = 3 + n$ and become an even number which can't be a prime number.)

●Returning to our sequence, we can notice the emergence of non-prime odd numbers that respond to the previously suggested formula:

So = (3), (5), (7), 9, (11), (13),15, (17), (19), 21...25,27, (29) ...

EX: 25 = 3 + 2 × 11

●That because they are multiples of prime numbers, where: If b is odd; b >1 and not a multiple of 3, then:

$b \times (3 + 2k) = 3b + 2kb = 3 + 3b - 3 + 2kb = 3 + 3(b-1) + 2kb$

And we have b odd, so, 3 × (b-1) is even, $3 \times \frac{b-1}{2}$ is natural, and the equation is completed through: $3 + 2(3 \times \frac{b-1}{2} + kb) = 3 + 2(3 \times \frac{b-1}{2} + kb)$

And the number is not a multiple of 3 because bk is not a multiple of 3 (as we said previously).

● Therefore, we need to change the formula of the prime number that we are working on, and we note in this context:

$3 + 2q$ in this case can be decomposed into a combination of factors, where: $3 + 2q = n - 2x + 2y = n + 2(y-x)$ with y, n, and x are natural integers and n and $2(y-x)$ have a common factor in some cases, meaning that n and $(y-x)$ have a common factor in some cases, because n is equal to $3 + 2y$, and therefore it's odd and has no common factor with 2.

● So according to the above, any prime number greater than 3 can be written in the form: $3+2x$ if x is a natural number that does not belong to the set of multiples of 3 and $\gcd(p,2(x-z)) = 1$ whatever p, x and y are if $3 = p - 2z$. (If This Always True)

● So, any prime number greater than 3 is written in this form:

$3 + 2z + 2 \times (q \times (3 + 2z) + r)$ with z, q and r natural numbers, one of them at least greater than 0**.

IV-Even numbers greater than 8 and prime numbers greater than 3:

● According to the second part, the sum of two prime numbers greater than 3 can be written in the following form:

$[3 + 2z + 2 \times (q \times (3 + 2z) + r)] + [3 + 2z' + 2 \times (q' \times (3 + 2z') + r')] = 2 \times [3 + z + z' +(q \times (3 + 2z) + r) + (q' \times (3 + 2z') + r')]$

● Therefore, the sum of two primes greater than 3 is an even number great than 8, since the smallest prime greater than 3 is 5.

● And since prime numbers are infinite -according to Euclid- any even number greater than 8 can be written in the form of the sum of two prime numbers greater than 3, no matter how large it is, because if g is even greater than 8, then $\frac{g}{2}$ is a natural number greater than 4, as well as writing:

$3 + z + z' +(q \times (3 + 2z) + r) + (q' \times (3 + 2z') + r')$ -from the prime number formula- with z, q, r, z', q', r' natural numbers one of them at least greater than 0 (according to **).

V-Exceptions:

● The formula for the prime numbers mentioned above is for prime numbers greater than 3, which makes the prime numbers 2 and 3 exceptional, and this is expected because these two numbers are the basic stones for the rest of the prime numbers, and they are the only two consecutive prime numbers. It is also obvious that the rest of the even numbers less than 10 are formed by these two exceptions.

● The number 2 is the only even prime number, so it cannot form - when combined with an opposing prime number - an even number, so 2 constitutes an even number in one case: $2+2 = 2 \times 2 = 4$.

● As for the number 3, it can be combined with any other prime number greater than it, because it is odd, so it's present in the prime sum of the rest of the even numbers, which are 6 and 8, as: $6 = 3+3$ and $8 = 3+5$.

VI- Conclusion:

● According to the above, any even number greater than 3 (i.e., greater than 2) can be written in the form of the sum of two prime numbers, no matter how large the even number is. This is what Goldbach's conjecture states.