Measuring the one-way speed of light

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Abstract

In this paper, we present a method for measuring the one-way speed of light. In order to achieve this, it is necessary to solve two problems. The first problem is how to synchronize the clocks at the source and the detector, and the second problem is to prove that the slow movement of one of the clocks does not significantly affect the already established synchronization between the two clocks.

Keywords: *one-way speed of light, clock synchronisation*

1. Introduction

In his famous paper [1], Einstein formulated a definition of two synchronized clocks. The author begins with a descriptive definition of *simultaneous events* :

If, for instance, I say, "That train arrives here at 7 o'clock," I mean something like this: "The pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events.

If the observer is at the railway station, then he could confirm that the train arrived at 7h, but if the observer is sitting in a city park, the fact that his watch shows 7h does not mean that the train really arrived at 7h.
The author further assumes that there are two clocks at points A and B, which we will denote by C_A and C_B, respectively.

If at the point A of space there is a clock, an observer at A can determine the time values of events in the immediate proximity of A by finding the positions of the hands which are simultaneous with these events. If there is at the point B of space another clock **in all respects resembling** the one at A.

What does it mean that the watch C_B **in all respects resembles** the watch C_A? We can understand that watches C_A and C_B are physically identical, but does that mean that they also measure identical time? In other words, how can an observer at a train station and an observer sitting in the city park know that their clocks measure the identical time? And finally, assuming that the speed of light is constant, the author formulates a definition of two synchronized clocks.

...we establish **by definition** that the "time" required by light to travel from A to B equals the "time" it requires to travel from B to A. Let a ray of light start at the "A time t_A" from A towards B, let it at the "B time" t_B be reflected at B in the direction of A, and arrive again at A at the "A time" t'_A. In accordance with definition the two clocks synchronize **if**

$$t_B - t_A = t'_A - t_B \tag{1}$$

We assume that this definition of synchronism is free from contradictions.

From Equation (1) it follows that:

$$t_B = \frac{t_A + t'_A}{2} \tag{2}$$

According to the definition, the clocks C_A and C_B are synchronized **if** Equation(2) holds. If the initial times on the clocks C_A and C_B have been set independently of each other, then Equation (2) can only be true by chance.

It is obvious that this definition has two major drawbacks. Firstly, it remains unclear how the initial times were set on the clocks and secondly, on the basis of which experiment the author concluded that the speed of light is constant.

Therefore, we cannot accept the assumption that 'the definition of synchronism is free from contradictions' so it is necessary to reformulate the definition of two synchronized clocks.

2. Synchronization of clocks without the assumption of a constant speed of light

Suppose that at the same altitude we have three collinear points that we will mark with A, S and B. At each point there is a clock, which we will mark with C_A,C_S and C_B. All three clocks measure time in the same time units and the times measured by clocks C_A, C_S and C_B will be denoted by t_A,t_S and t_B respectively.

Let us define a one-dimensional coordinate system whose origin is determined by the point O, Fig(1). Suppose we have chosen four points A,S,B and B' so that $SA = SB$.

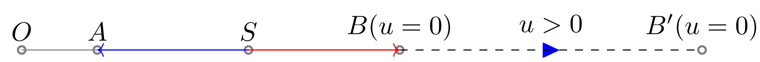

Figure 1: At points A, S and B there are three clocks marked with C_A, C_S and C_B respectively

From point S we will simultaneously emit two signals in the direction of points A and B. The time on clock C_S at that moment is equal to zero ($t_S = 0$). When the signal arrives at point A, the time on clock C_A will be reset ($t_A = 0$). The same applies to the C_B clock. If with c_B and c_A denote the speed of light in the direction SB and SA respectively, then we have the following equations:

$$c_B = c + v \tag{3}$$
$$c_A = c - v \tag{4}$$
$$l = SA = SB \tag{5}$$
$$\Delta t_A = \frac{l}{c_A} \tag{6}$$
$$\Delta t_B = \frac{l}{c_B} \tag{7}$$

Where c and v are unknowns that have yet to be determined. After the signals have arrived at points A and B, the following equations apply:

$$t_S = t_A + \Delta t_A \tag{8}$$
$$t_S = t_B + \Delta t_B \tag{9}$$
$$t_A + \Delta t_A = t_B + \Delta t_B \tag{10}$$
$$t_B = t_A + \Delta t_A - \Delta t_B \tag{11}$$
$$t_A = t_B + \Delta t_B - \Delta t_A \tag{12}$$

Definition 1 *We will say that the clocks C_A and C_B are synchronized if there exists a function $f()$ and its inverse function $f^{-1}()$ such that $t_B = f(t_A)$ and $t_A = f^{-1}(t_B)$, where t_A and t_B denote the times that were simultaneously measured by the clocks C_A and C_B respectively. We will say that the times t_A and t_B are equivalent and write this in the following way $t_A \sim t_B$. It is obvious that the relation $\sim$ is reflexive, symmetric, and transitive.*

We will define the functions f_A and f_B in the following way

$$t_A = f_A(t_S) = t_S - \Delta t_A \tag{13}$$
$$t_S = f_A^{-1}(t_A) = t_A + \Delta t_A \tag{14}$$
$$t_B = f_B(t_S) = t_S - \Delta t_B \tag{15}$$
$$t_S = f_B^{-1}(t_B) = t_B + \Delta t_B \tag{16}$$
$$(t_A \sim t_S) \wedge (t_S \sim t_B) \Rightarrow (t_A \sim t_B) \tag{17}$$
$$t_B = f_B(t_S) = f_B(f_A^{-1}(t_A)) = f_B \circ f_A^{-1}(t_A) = t_A + \Delta t_A - \Delta t_B \tag{18}$$
$$t_A = f_A(t_S) = f_A(f_B^{-1}(t_B)) = f_A \circ f_B^{-1}(t_B) = t_B + \Delta t_B - \Delta t_A \tag{19}$$

Now we will estimate the error in measuring the speed of light using two clocks in the case that one clock moves with a uniform velocity relative to the point O, Fig(1). Suppose clock C_B has been moved with velocity $\mathbf{u}$ from point B to point B'. Denote by $t_{BB'}$ the time measured by clock C_A that is required for clock C_B to be moved from point B to B'.

$$L = BB' \tag{20}$$
$$t_{BB'} = \frac{L}{u} \tag{21}$$

According to STR the clock C_B starts to measure time in the different time units compared to clock C_A. Denote by $\tau_{BB'}$ the time measured by clock C_B that is required for clock C_B to be moved from point B to B'. Then, the following relation holds between times $t_{BB'}$ and $\tau_{BB'}$ [1].

$$\tau_{BB'} = t_{BB'}\sqrt{1 - \frac{u^2}{c^2}} \tag{22}$$
$$\epsilon = t_{BB'} - \tau_{BB'} = t_{BB'} - t_{BB'}\sqrt{1 - \frac{u^2}{c^2}} = \Big| u << c \Big| \approx t_{BB'}\frac{u^2}{2c^2} = \frac{L}{u}\frac{u^2}{2c^2} = \frac{Lu}{2c^2} \tag{23}$$
$$\epsilon \approx \frac{Lu}{2c^2} \tag{24}$$

After arriving at point B', the clock C_B starts again to measure time in the same time units as the stationary clock C_A. This claim should be experimentally confirmed or rejected. This means that the following equality must hold:

$$t'_A - t_A = \tau'_{B'} - \tau_{B'} \tag{25}$$

Where t_A and t'_A are the times measured by the clock C_A and $\tau_{B'}$ and $\tau_{B'}$ are the corresponding times in relation to t_A and t'_A, measured by the clock C_B. We measure time t_A and send a signal in the direction of point B'. When the signal arrives at point B' with $\tau_{B'}$ we will mark the corresponding time on the clock C_B...

In addition to the relativistic interpretation of the reasons why a moving clock measures different time than a stationary clock, we will offer an alternative explanation. If we prove that the speed of light is not constant, then the STR is not correct either. Therefore, the movement of the clock C_B in relation to the clock C_A does not affect time, which is absolute, but the instrument which we use to measure time. This means that the actual "time dilation" should be determined empirically.

3. One way speed of light

In the previous section, we defined the method of synchronization between two clocks. In this section we will prove that the slow movement of one of the clocks does not significantly affect the already established synchronization between them.

Suppose that a signal is sent from point A in the direction of point B'. Let's denote by T_A the time shown by the clock C_A at that moment. Let's denote by $\tau_{B'}$ the time, measured by the clock C_B, when the signal arrived at point B' and was then reflected in the direction of point A. And finally, mark with T''_A the time, measured by the clock C_A, when the signal arrived at point A.

We will denote with T'_A the time measured by the clock C_A so that $T'_A \sim \tau_{B'}$

From the Equation(12) it follows:

$$T'_A = \tau_{B'} + \Delta t_B - \Delta t_A \tag{26}$$

We will define $\Delta T'$ and $\Delta T''$ as it follows:

$$\Delta T' = T'_A - T_A \tag{27}$$
$$\Delta T' = \tau_{B'} - T_A + \Delta t_B - \Delta t_A \tag{28}$$
$$\Delta T'' = T''_A - T'_A \tag{29}$$
$$\Delta T'' = T''_A - \tau_{B'} - \Delta t_B + \Delta t_A \tag{30}$$

Our task is to measure the time it takes for light to travel from point A to point B' (and vice versa). We will denote that time with $\Delta T_{AB'}$ and compare it with the $\Delta T'$. Due to the "time dilation" caused by the movement of the clock C_B it follows that:

$$\Delta T_{AB'} = \Delta T' + \epsilon \tag{31}$$
$$\Delta T' = \Delta T_{AB'} - \epsilon \tag{32}$$

$$c + v = \frac{2l + L}{\Delta T_{AB'}} \tag{33}$$
$$\Delta e = \frac{\epsilon}{\Delta T_{AB'}} = \frac{Lu}{2c^2}\frac{c+v}{2l+L} = \frac{L}{2l+L}\frac{c+v}{c}\frac{u}{2c} \tag{34}$$
$$\epsilon = \Delta T_{AB'} \Delta e \tag{35}$$

Unknown c nor v have not yet been determined, but we assume that $(v << c)$.

$$(v << c) \ \Rightarrow \ \left(\frac{c+v}{c} \ \rightarrow \ 1\right) \tag{36}$$
$$(u \ \rightarrow \ 0) \tag{37}$$
$$(36) \wedge (37) \Rightarrow (\Delta e \ \rightarrow \ 0) \tag{38}$$

$$\frac{2l+L}{\Delta T'} = \frac{2l+L}{\Delta T_{AB'} - \epsilon} = \frac{2l+L}{\Delta T_{AB'} - \Delta T_{AB'} \Delta e} = \frac{2l+L}{\Delta T_{AB'}} \frac{1}{1-\Delta e} \approx \frac{2l+L}{\Delta T_{AB'}} (1+\Delta e) \tag{39}$$
$$\frac{2l+L}{\Delta T'} = (c+v) + \frac{L}{2l+L} \left(\frac{c+v}{c}\right)^2 \frac{u}{2} \approx (c+v) \tag{40}$$

Making an error ϵ, arbitrarily small compared to $\Delta T_{AB'}$, it follows that:

$$\Delta T' = \frac{2l+L}{c+v} \tag{41}$$
$$\frac{2l+L}{c+v} = \tau_{B'} - T_A + \Delta t_B - \Delta t_A \tag{42}$$
$$\frac{2l+L}{c+v} - \Delta t_B + \Delta t_A = \tau_{B'} - T_A \tag{43}$$
$$\frac{2l+L}{c+v} - \frac{l}{c+v} + \frac{l}{c-v} = \tau_{B'} - T_A \tag{44}$$
$$\frac{l+L}{c+v} + \frac{l}{c-v} = \tau_{B'} - T_A \tag{45}$$

In a similar way, we will determine the speed of light in the direction BA.

$$\Delta T'' = \frac{2l+L}{c-v} \tag{46}$$
$$\frac{2l+L}{c-v} = T''_A - \tau_{B'} - \Delta t_B + \Delta t_A \tag{47}$$
$$\frac{2l+L}{c-v} + \Delta t_B - \Delta t_A = T''_A - \tau_{B'} \tag{48}$$
$$\frac{2l+L}{c-v} + \frac{l}{c+v} - \frac{l}{c-v} = -T''_A - \tau_{B'} \tag{49}$$
$$\frac{l}{c+v} + \frac{l+L}{c-v} = T''_A - \tau_{B'} \tag{50}$$

Thus, from Equations (45) and (50), we obtained a system of two linear equations with two unknowns $\frac{1}{c+v}$ and $\frac{1}{c-v}$.

$$(l+L)\frac{1}{c+v} + l\frac{1}{c-v} = (\tau_{B'} - T_A) \tag{51}$$
$$l\frac{1}{c+v} + (l+L)\frac{1}{c-v} = (T''_A - \tau_{B'}) \tag{52}$$

$$\Delta = \begin{vmatrix} l+L & l \\ l & l+L \end{vmatrix} = L^2 + 2Ll = L(L+2l)$$

In order to have non-trivial solutions, it is necessary that $L > 0$.

$$\Delta_+ = \begin{vmatrix} (\tau_{B'} - T_A) & l \\ (T''_A - \tau_{B'}) & l+L \end{vmatrix}$$

$$\Delta_- = \begin{vmatrix} l+L & (\tau_{B'} - T_A) \\ l & (T''_A - \tau_{B'}) \end{vmatrix}$$

$$c + v = \frac{\Delta}{\Delta_+} \tag{53}$$

$$c - v = \frac{\Delta}{\Delta_-} \tag{54}$$

It is obvious that from Equations (53) and (54) we can easily determine c and v. In this way, we can claim that we have fully established the synchronization between the clocks C_A and C_B according to the **Definition** 1.

Now we can make measurements in different directions taking into account that the clocks are at the same altitude, and compare the obtained values for c and v.

If there is at least one direction such that $(v > 0)$ then the assumption of a constant speed of light should be rejected.

References

[1] A. Einstein
(1905) On The Electrodynamics of moving bodies
https://www.fourmilab.ch/etexts/einstein/specrel/specrel.pdf