Collatz Conjecture Explored
Examination of Counter-Example Leads to a Complete Proof

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Section - Abstract

This is a mathematical analysis of everything Collatz. I've come up with a revolutionary way of representing the natural counting numbers as an infinite set of equations. From these I am able to make some provable connections that not only show that all counting numbers are used once in the Collatz Tree structure; but where additional loops originate; among other cool observations. I also show that there can only be one unbroken chain of continuous “3n+1 / 2” growing to infinite number sizes approaching infinity but never actually getting there. This would be the 'only' counter-example that is possible and as odds would have it, it does not pan out. That only possible counter example is not to be.

Using the induction method where we show that x = 1 is true (elementry, since it is part of the initial loop); from there we assume that x from 1 to k are also true building on the x=1 being true; then k+1 is also true. That is a complicated way of saying that if we know and assume all numbers from 1 to k are true, then the very next number k+1 is also true in as much as we apply the two rules correctly so the number reduces to one that is already in the proven set!

The first three equations of my infinite set of equations are easy to apply this induction to and cover 87.5% of the counting number set. I change things up a bit for the upper level equations. I am able to prove through the same induction method that any number that is not a multiple of 3 (falling in these levels/equations) is also provable. Stepping outside the usual method of this proof I investigate the multiples of three separately to prove they are all following a similar induction proof. And they do. All said and done I am able to prove that 100% are provable. (4x+1) is important in this proof as well the application of (3x+1)/2. Read on to find out what I mean.

I've covered off on the loop issue part of the proof by showing how additional loops come about in the Collatz Tree structure. There is only one loop in Collatz (positive counting numbers) and that is the trivial { 1 – 4 – 2 } loop. No others are possible no matter how close to infinity one gets and all numbers will reduce to this trivial loop.

The detailed discussion of how I arrived at these different conclusions is outlined below. I apologize if some sections are difficult to follow. I am not a mathematician by nature or profession. I do love mathematics though. I hope you enjoy my self awakening process on this subject as I continued to explore. As my expertise improved other intuitive aspects became readily useful in the proof.

This is an updated version of my original document with a new section near the end that gets into the details of the proof. The remainder of the original report remains intact for the most part but does have additional details and concepts introduced and dispersed therein.

This is the third version where I have solved the outstanding subset of multiples of 3. I believe you will

find that method eye-opening since it involves some under the sheets number manipulation by multiple applications of (3x+1)/2. I also introduce the 'duality' nature of some even numbers (if not all) that remain hidden in the Collatz tree structure... and that is that those even numbers can behave as if they were odd numbers; (Odd*3)+1=Even; (Even*3)+1=Odd; (4*Even)+1=Odd.

This detailed analysis has led me to a complete proof.

Section 1 - Introduction

The Collatz conjecture is a sequence of numbers generated by applying two rules; if the number is Odd multiply it by 3 and add 1 (3n+1); if the number is even then divide by 2 (n/2). So the Collatz sequence is { 3n+1 ; n/2 }.

The conjecture states that if you start at any number from 1 to infinity (positive natural counting numbers) you will eventually end up in a { 1 – 4 – 2 } loop.

Sounds simple enough. The concept is, but proving that this is infact true over the entire set of natural counting numbers is quite difficult. Apprently, folks have been searching for a proof for close to 100 years.

My attempt is to approach the proof from a slightly different angle and look at the natural counting numbers in a more confined fashion. This will allow for the observation that something fundamental is occuring. That will become clear in the following sections.

I am not a mathematician per say... but a computer scientist… and we all know computers are just large computational devices that rely on mathematics and logic. I do not have access to a mathematical addon for publishing in the correct format so I will make due with what I can get off the keyboard (symbol wise). My terminology may also be lacking, but I am confident you will comprehend it just fine.

I've created this report in a fashion where you can follow my maturation process as I studied the Collatz Conjecture. I ask myself questions and then go about determining if they are something I can use towards a proof.

Section 2 – Infinite Sequence of Equations to Create ALL Counting Numbers (Primes)

The basis of my observations and subsequent conclusion is the understanding that all the natural numbers (1 to infinity) can be represented by the following infinite set of equations.

- 0 + 2x { 0 + (2^1)x } { (((2^1) / 2) -1) + (2^1)x }
- 1 + 4x { 1 + (2^2)x } { (((2^2) / 2) -1) + (2^2)x }
- 3 + 8x { 3 + (2^3)x } { (((2^3) / 2) -1) + (2^3)x }
- 7 + 16x { 7 + (2^4)x } { (((2^4) / 2) -1) + (2^4)x }
- …
- (((2^y) / 2) -1) + (2^y)x
- …
- (((2^infinity) / 2) -1) + (2^infinity)x

As seen above this is an infinite sequence of equations and it will cover all the natural numbers (1 to infinity). Each individual counting number exists only ONCE in this set of equations. I've expanded out the first ten equations to show how they are formed. Note that 'powers of 2' play a very important role. Now, there is an unexpected reality to these equations in that 0 + 2x contains all the even numbers (a subset that contains exactly half (½) of the natural number set). For example { 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, … }. The next equation 1 + 4x spawns the following subset: { 1, 5, 9, 13, 17, 21, … } This subset contains exactly one quarter (¼) of the entire natural number set. So the first 2 equations account for (¾) of the natural number set. You will find that the next equation subset will contain only (1/8) of the natural numbers: { 3, 11, 19, 27, … }. And the following equation has (1/16) of the natural numbers { 7 + 16x } { 7, 23, 39, 55, … }. Do you see a pattern here? The subset for any equation contains (1/2^y): { (½) for 2^1; (¼) for 2^2; (1/8) for 2^3; … }. As we approach the infinity power of 2 we find that the subset contains only (1/infinity) elements...a very tiny number. So just for kicks, let's calculate how what proportion of the natural number set are included with the first 10 equations (½) + (¼) + (1/8) + (1/16) + (1/32) + (1/64) + (1/128) + (1/256) + (1/512) + (1/1024) = (1023/1024). Interesting, indeed. The vast majority of all the natural numbers can be created using only the first 10 equations. We will come back to this point later. Here's the above discussion in the form of a chart for easier visualization:

{ 0+2x }	1/2	50% of the entire natural counting number set
{ 1+4x }	1/4	25%
{ 3+8x }	1/8	12.5%
{ 7+16x }	1/16	6.25%
{ 15+32x }	1/32	3.125%
{ 31+64x }	1/64	1.5625%
{ 63+128x }	1/128	0.78125%
{ 127+256x }	1/256	0.390625%
{ 255+512x }	1/512	0.1953125%
{ 511+1024x }	1/1024	0.09765625%
{ 1023+2048x }	1/2048	0.048828125%

Just so we are all on the same page I've listed the first several equations with the numbers they create:

{ 0 + 2x }	→ 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, ...
{ 1 + 4x }	→ 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, 77, 81, 85, 89, ...
{ 3 + 8x }	→ 3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, 91, 99, 107, 115, 123, 131, 139, 147, 155, ...
{ 7 + 16x }	→ 7, 23, 39, 55, 71, 87, 103, 119, 135, 151, 167, 183, 199, 215, …
{ 15 + 32x }	→ 15, 47, 79, 111, 143, 175, 207, 239, 271, 303, 335, 367, ...
{ 31 + 64x }	→ 31, 95, 159, 223, 287, 351, 415, 479, ...
{ 63 + 128x }	→ 63, 191, 319, 447, 575, 703, 831, ...
{ 127 + 256x }	→ 127, 383, 639, 895, 1151, 1407, 1663, ...
{ 255 + 512x }	→ 255, 767, 1279, 1791, 2303, ...
{ 511 + 1024x }	→ 511, 1535, 2559, 3583, ...

This is likely as good a spot as any to show how primes work into my equations. The negative natural numbers shown is subsequant sections work in the same fashion. I'm going to list off the first 21 equations:

{ 0 + 2x }	→ 0	+ 2x
{ 1 + 4x }	→ 1	+ 4x
{ 3 + 8x }	→ **3**	+ 8x
{ 7 + 16x }	→ 7	+ 16x
{ 15 + 32x }	→ **5 * 3**	+ 32x
{ 31 + 64x }	→ 31	+ 64x
{ 63 + 128x }	→ **7 * 3 * 3**	+ 128x

{ 127 + 256x }	→	127	+ 256x
{ 255 + 512x }	→	**17 * 5 * 3**	+ 512x
{ 511 + 1024x }	→	73 * 7	+ 1024x
{ 1023 + 2048x }	→	**31 * 11 * 3**	+ 2048x
{ 2047 + 4096x }	→	89 * 23	+ 4096x
{ 4095 + 8192x }	→	**13 * 7 * 5 * 3 * 3**	+ 8192x
{ 8191 + 16384x }	→	8191	+ 16384x
{ 16383 + 32768x }	→	**127 * 43 * 3**	+ 32768x
{ 32767 + 65536x }	→	151 * 31 * 7	+ 65536x
{ 65535 + 131072x }	→	**257 * 17 * 5 * 3**	+ 131072x
{ 131071 + 262144x }	→	131071	+ 262144x
{ 262143 + 524288x }	→	**73 * 19 * 7 * 3 * 3 * 3**	+ 524288x
{ 524287 + 1048576x }	→	524287	+ 1048576x
{ 1048575 + 2097152x }	→	**41 * 31 * 11 * 5 * 5 * 3**	+ 2097152x

The important thing to notice here is that the first part of every equation is simply some {2^x – 1 } and that each of them in turn is formed by nothing but PRIME factors. The ultra important realization is that starting at 3 every second equation after that is comprised of factors that contain at least one factor of 3. All the other equations do not include that factor of 3. This makes every second equation a 'multiple of 3' equation? At the bare minimum those equations start with a multiple of 3. All of the equations contain multiples of 3. This observation likely plays into the process but at this point I'm not convinced it can be used to formulate a proof.

We will see that any odd number that is a multiple of 3 can not form further branches; it is a 'dead-end' row. I love how primes have made an appearance, but anyone involved with number theory knows that any number is created by nothing but prime factors. Later we will see the appearance of 3^x = 2^y +1 and how it can be used to explain the formation of additional loops. Again a connection with powers of 3 and powers of 2. Note there are only two cases where this is true; 3^1 = 2^1 + 1 and 3^2 + 2^3 +1. The above primes discussion play with 2^x – 1. Quite a coincidence, isn't it? Every second equation is the same as saying add 3 multiplied by ' 4 ' or ' 2^2 '. 0+(3*1)=3; 3+(3*4)=15; 15+(3*16)=63; 63+(3*64)=255;... Note that as we jump to the next equation we are multiplying by 4 more...3*4; 3*4*4; 3*4*4*4;... This is how we skip over every other equation and why we see branches separated by ' 4 ' or ' 2^2 '. You obviously see this is not the complete picture. The other subset of equations do something very similar. 1+(3*2)=7; 7+(3*8)=31; 31+(3*32)=127; 127+(3*128)=511. Again we are multiplying by 4 (2*2). This allows us to skip over every other equation. Combining the two cover all my equations.

Now, another item that may be important to explore here before going futher is the relationship between 3 and 2. This relationship fits in with how the Collatz tree propagates. If you multiply a number (say 1) by three and add one (3n+1) you are in effect doing 3+1=4. 4 is simply 2+2=4. 4 is an important transition point in the tree. Let's do another iteration of 3n+1 but not by multiplying but simply adding the effect. 3n+1+3n+1 = 3+3+2 = 8. Can we mirror this with 2? Yes, 2+2+2+2 or 4+4 =8. 3, 6 and 2, 4 are all an important numbers when building tables for Collatz:

Odd number	3n+1	n/2
1	4	2
3	10	5
5	16	8
7	22	11
9	28	14
11	34	17
13	40	20

15	46	23
17	52	26
19	58	29
21	64	32

See that the Odd number column is separated by 2 in each step up (+2). 3n+1 is (+6) in each step up. And just for kicks, n/2 is (+3) in each step up. Interesting INDEED! Not really... (+6)/2=(+3)! So there is a definite link between 3n+1 and n/2; that is 3 and 6.

What happens on the third iteration is very important to note. This is an important transition step. 3n+1+3n+1+3n+1 = 3+3+3+3 = 12. So the excess 1's give an even 3 after 3 iterations. That is important because it becomes evenly divisible by 3. And it's connection to 2 is 2+2+2+2+2+2 = 12 or 6+6. or 4+4+4. This is not needed for the proof I outline below. At least not in this fashion.

You are likely saying we can't use this and you are likely right but it was a stepping stone to show what I really intended. Again, suppose n=1 for ease of understanding. 3n+1 if n=1 is 4. Now apply 3n+1 to that and do it a second time ending up with 3(3(3n+1)+1)+1 or 27n+13. This is just three iterations of 3n+1. Lets rearrange 27n+13 to 27n+9+4 and factor out 9 giving 9(3n+1)+4 and since 4 is actually 3n+1, replace the 4 giving 9(3n+1)+(3n+1). This is the case so long as we keep n=1. You can now note that we actually have 10(3n+1). This means that after 3 consecutive iterations of 3n+1 we should be able to divide out an extra 2 (n/2). BUT, actually what is happening is (3n+1)/2. So to complicate things a tad bit what happens if we add in the n/2 each iteration. Should be nothing, really. First yields (3n+1)/2. Next yields (3((3n+1)/2)+1)/2. And the third gives (3((3((3n+1)/2)+1)/2)+1)/2. Multiplied through we get (27n+19)/8. If we try to do like above to factor out 9 we get (9(3n+1)+10)/8. Separate out a 4 from the 10 to give (9(3n+1)+(3n+1)+6)/8 or (10(3n+1)+6)/8. And we can still mathematically strip out a 2 as follows: 2(5(3n+1)+3)/8. In essence we continue to get an extra n/2 every three iterations. This observation must provide statistical advantage to increase the overall number of (n/2). Something similar must be happening when n is other than 1. I am unable to make that leap at this point.

I will come back to this connection later in this discussion when I formulate the proof. It is very useful in proving a subset of multiples of 3.

Why have I discussed any of this in the first place. It was to show that all natural counting numbers are included in the tree structure. None are missed. As well, it is to show how powers of 2 and 3 play an important role in the construction of this tree. Since all odd numbers are in the tree implies that all even numbers are as well (since any even number can be formed by multiplying an odd number by two or another even number by 2). Again, this is a multiple of 2 (2^1).

Section 3 – Cascading Effect

You are likely asking why I am about to point out the cascading effect. That's where this gets very interesting. The structure of the tree is dictated by the odd number at any of the nodes; a 'node' being designated by it's location in the tree – in this case anywhere where you can go right by multiplying by two and up by multiplying by three and adding one. There are only two paths. Other nodes with two paths contain only two multiply by 2. So I call them connector nodes. I also call all other nodes with 3 paths connector nodes; they have a 'minus one and divide by three' and a 'divide by two' and a 'multiply by two'.

"node"

```
{even number = node * 3 + 1}
         |
{      node      } – {    node * 2  }
```

"connector node"

{ connector node / 2 } – { connector node } – { connector node * 2 }

"connector node (all other nodes)"

```
{ connector node / 2 } – { connector node } – ( connector node * 2}
                                         |
                                   {     node      }
```

```
1 – 2 – 4 – 8 – 16 – …
                  |
                 05 – 10 – 20 – 40 – ...
                    |          |
                    |         13 – 26 – 52 – ...
                    |                   |
                    |                  17 – 34 – 68 – ...
                    |                         |
                   03 – 06 – 12 – …          11 – 22 – 44 – ...
                                                  |
                                                 07 – 14 – 28 – ...
                                                          |
                                                         09 – 18 – ...
```

1, 5, 3, 13, 17, 11 – nodes (1 is included as a node because it loops back to 4)
2, 4, 8, 16, 10, 20, 40, 6, 12, 26, 52, 34, 68, 22, 44, 14, 28, 18 – connector nodes

If a node contains an odd number from say { 7 + 16x } … the very next odd number will be (3n+1)/2 and will be a number contained in { 3 + 8x } … with the very next odd number a further (3n+1)/2 and it will fall in { 1 +4x } … till it finally falls into { 0 + 2x }. This is the case no matter what subset you were to start at. If you started at {511 + 1024x } it would cascade uninterupted through each prior equation one-by-one till it gets to { 0 + 2x }.

Note that any odd number that is a multiple of '3' is a starting node. No other node can migrate through it on it's way back to the { 1 – 4 – 2 } loop. 3 and 9 shown above in red are such nodes. Do you see why this is the case? Any multiple of 3 can not be arrived at by applying 3n+1 to another odd.

A cascade is in the form:

E – E

```
|
O – E – E
     |
     O – E – E
          |
          O – E – E
               |
               O – E – E
```

Using O=odd and E=even we see an unbroken O – E – O – E – O – E – O...
7 – 22 – 11 – 34 – 17 – 52 – 26 is such a chain... 7 is level 4; 11 is level 3; 17 is level 2; and 26 is level 1. If there are two evens side by side in the chain is not a cascade. 17 – 52 – 26 – 13 – 40 – 20 – 10... O – E – E – O – E – E – E. The following paragraphs point other important features of the tree structure.

This may be an appropriate place to point out another mathematical oddity that occurs in this tree. A node (all nodes are Odd) will also cover itself times 4 plus 1. 4*(Odd Node) + 1... and that new node will have the same applied to it and so forth all the way though the tree for all nodes with the exception of multiple of 3 nodes.

```
1 – 2 – 4 – 8 – 16 – 32 – 64 – 128
         |       |           |
         |       |          21 – 42 - 84
         |       |
         |      05 – 10 – 20 – 40 – ...
         |             |           |
         1             |          13 – 26 – 52 – ...
                       |                   |
                       |                  17 – 34 – 68 – ...
                       |                         |
                      03 – 06 – 12 – …          11 – 22 – 44 – ...
                                                      |
                                                     07 – 14 – 28 – ...
                                                                |
                                                               09 – 18 – …
```

Drawn slighty different with the '1' hanging where it should be you can see this... 1, 5, 21, … or 1*4+1 = 5; 5*4+1=21; etc. 3, 13, 53, … or 3*4+1 =13; 13*4+1 = 53; etc. All nodes display this feature. This occurs because of the way the tree is constructed and branches form...namely that after the first branch on any row is formed, 2^2 or multiply by 4 to get the next branch on the row. An example is 10 and 40 on that row. 10*2*2 = 40. The branch at 10 gives a node of 3. The branch at 40 gives a node of 13. And the next branch at 160 (40*2*2 = 160) will give a node of 53 which is 53*3+1 = 160! And 53 is 13*4+1. All rows that can have branches do this indefinately.

You will notice that this 4x+1 plays prominantly in the Collatz structure. Every backbone (except those that start with multiples of 3) spawn limbs that have this 4x+1 applied over and over...except of course those backbones that start with multiples of 3.

Looking a little deeper into this we see the following:

2 – 4 – 8 – 16 – 32 – 64 – 128 – 256 – 512 – 1024 - …

```
  |        |        |          |            |
  1        5        21         85           341
```

The above is a snippet of the '2' backbone. It starts with the only even node in the entire tree which is a special case because it is actually the '1' backbone which is odd too. I've drawn the 1 hanging off of 4 to make this 4x+1 easier to spot. All other nodes will follow this feature without question. Notice how each of 1, 5, 21, 85, 341, display this feature:

1 → 4(1)+1=5
5 → 16(1)+5=21
21 → 64(1)+21=85
85 → 256(1)+85=341
...

Now you can see how multiplying by 4 (2 followed by 2) gives rise to these. It's convenient that 1+4=5; 5+16=21; 21+64=85;... This is exactly the same as saying 4x+1... 4(1)+1=5; 4(5)+1=21; 4(21)+1=85; 4(85)+1=341...

This exact same thing occurs with all the non-multiple of 3 nodes. For example let's show '5'.

```
5 – 10 – 20 – 40 – 80 – 160 – 320 - 640 …
     |         |          |            |
     3         13         53           213
```

3 → 4(3)+1=13
13 → 16(3)+5=53
53 → 64(3)+21=213
…

Let's do one more to hammer this point home; let's do '11':

```
11 – 22 – 44 – 88 – 176 – 352 – 704 – 1408 …
      |        |          |           |
      7        29         117         469
```

7 → 4(7)+1=29
29 → 16(7)+5=117
117 → 64(7)+21=469

So all nodes with the exception of the multiples of 3 will do this. This is the 4x+1 rule and will prove invaluable in the following proof.

Something important to mention is that there are special occurances where an even number (all even numbers) will give the same result as an odd multiplied by 3 and then add one. They behave like; but instead of giving an even number one gets an odd number. But the 4x+1 rule stands. Example:

```
07 – 14 – 28 – 56 – 112 – 224 - 448 ...
           |         |          |
02         09        37         149
```

2 → 4(2)+1=9
9 → 16(2)+5=37
37 → 64(2)+21=149

That's very cool...but it is invisible when drawing the tree. And all even numbers display this feature. Let's do a couple more to show this:

```
13 – 26 – 52 – 104 – 208 – 416 – 832 – 1664 …
                |           |            |
4              17          69           277
```

4 → 4(4)1=17
17 → 16(4)+5=69
69 → 64(4)+21=277

And how about 6:

```
19 – 38 – 76 – 152 – 304 – 608 – 1216 – 2432 …
               |            |             |
6             25           101           405
```

6 → 4(6)+1= 25
25 → 16(6)+5=101
101 → 64(6)+21=405

This is the case for 'all' even numbers. They will make an invisible presense in the tree.

The first snippet from the Collatz structure with 2 placed in there does show the point. The even number when multiplied by 4 and add one gives the odd 9. (2*4)+1=9. Also note that that same even number when multiplied by 3 and add one gives another odd number very closely related to 9. (2*3)+1=7.

```
13 – 26 – 52 – ...
          |
04        17 – 34 – 68 – ...
```

This can only happen where you have that opening available. That appears to be whereever a level 2 (1+4x) starts. No other levels can do this because they collapse or cascade directly down to level 1 (0+2x). This is very important to remember. When we visit the prood later, we'll see situations where an odd number can be passed though reverse 4x+1 and give these evens. These do not mess up the Collatz structure and shows the inter-connectivity between the different backbones. These special even number play dual roles, not only can they have the n/2 rule for being even; they also fit into the structure (invisibly) where they are also 3x+1 and 4x+1 rules.

Section 4 – Validating the Cascade Mathematically

Now I will take a moment to show how this works. Let's start with { 7 + 16x }. Any number created from this equation will be odd so one must apply the 3n+1 followed by n/2.

(3 ({ 7 + 16x }) + 1) / 2
(21 + 48x + 1) / 2

(22 + 48x) / 2
11 + 24x
3 + 8 + 24x
3 + 8 (1 + 3x) or { 3 + 8x since 1+3x is actually an 'x' after applying 3n+1 }

So as you can see from the above the very next odd number will fall in the prior equation { 3 + 8x }. Since it falls in this subset it is automatically an odd and can't be further divided by 2. Replace 1+3x with the new x and run this new odd again:

(3 ({ 3 + 8x }) + 1) / 2
(9 + 24x + 1) / 2
(10 + 24x) / 2
5 + 12x
1 + 4 + 12x
1+ 4 (1 + 3x) or { 1 + 4x since 1+3x is actually an 'x' after applying 3n+1 }

And this continues uninterupted until you get to the very first equation, which are the even numbers:

(3 ({ 1 + 4x }) + 1) / 2
(3 + 12x + 1) / 2
(4 + 12x) / 2
2 + 6x
2 (1 + 3x)
2 (1 + 3x) or { 0 + 2x since 1+3x is actually an 'x' after applying 3n+1 }

Now this is an even number which can be divided at least once more by 2. Continully dividing by additional 2's will give us another odd number eventually. This odd number will fall into an upper equation but we have no way of knowing which one...we can not predetermine as far as I can tell. This will cause another uninterupted cascade down to the { 0 + 2x } equation. All cascades behave in this fashion and since the tree is nothing but cascades, the entire tree is one giant cascade.

Section 5 – Observations from Cascading

This a good place to point out an obvious fact. Starting at any level equation, it must then continually and directly cascade to the first level { 0 + 2n }. So for each number in a given level it cascades directly to level { 0 + 2n } through it's very own path. This implies that the same number of entries in the preceding cascade are acounted for. So if { 7 + 16x} has a finite number of say 8 entries; and the preceding level { 3 + 8x } has twice as many to start; 16; then 8 of those are automatically accounted for. If level { 1 + 4x } has double that again; 32; and 8 of those are accounted for; leaving 24. And so on and so forth. But remember that all entries in the { 3 + 8x } also cascade uninterupted to first level...so only half of the prior levels entries are left in play... meaning that at level { 0 + 2x } only half remain in play? (that means ½ of the entire natural counting numbers set). The rest fall on/within some predetermined path from higher levels? As seen the following chart level { 0+2x } behaves a bit differently in that only 1/3 of it's members are part of upper level cascading stacks. Right? That's because each level spreads out in multiples of 3...and it's only when you reach level (0+2x) that this becomes obvious

Let's see if I can show this concept in a chart:

{0+2x} 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52

{1+4x}	1	5	9	13	17	21	25	29	33
{3+8x}		3		**4**	11		**8**	19	
{7+16x}					7			**6**	
					2				

Duality	2 (2*3+1=7)	8 → 25	14 → 43	20 → 61	26 → 79
	4 (4*3+1=13)	10 → 31	16 → 49	22 → 67	28 → 85
	6 (6*3+1=19)	12 → 37	18 → 55	24 → 73	30 → 91

So you can now see how all the odd numbers are covered and consumed in a stack that leads/cascades back to level { 0+2x }. I've included dualities of even numbers to show that they do not impact our thoughts and only show up at the start of already existing cascade stacks. Only 1/3 of the even numbers are consumed. But remember the other rule n/2 allows us to consume any even that is double (2*odd); example 1*2=2; 3*2=6; 5*2=10; 7*2=14; 9*2=18; 11*2=22; 13*2=26... Shown in red below. Remember that the terminus of stacks accounts for 1/3 shown in blue. There is some overlap between the red and blue. You can begin to imagine how the entire tree is held together by those even numbers. The remainder of the even numbers are simply double some other even already covered (shown in green).

{0+2x}	**2**	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**	**22**	**24**	**26**
{1+4x}	1			5			9			13			17
{3+8x}				3						11			
{7+16x}										7			

{ 0 + 2x } **2**, 4, 6, **8**, 10, 12, 14, 16, 18, 20, 22, 24, **26**, …
{ 1 + 4x } **1**, **5**, 9, 13, **17**, 21, 25, 29, …
{ 3 + 8x } **3**, **11**, 19, 27, **35** …
{ 7 + 16x } **7**, **23**, 39, …

So for the above all 3 number shown in subset for { 7 + 16x } cascade through each prior level consuming one number in each of those levels. And there's a pattern that forms. Taking 7; it translates to (7*3+1)/2 = 11. 23 translates to 35 in the prior level. So the first entry (smallest) ends up tranlating to the second entry in the prior level. The next translates to the third item past 11 in the prior level – 35; and the next to three items past 35; as so on. If we start in the prior level with that first item 3; it translates to 5 in the prior level...11 translates to to three past 5 or 17...and so on. When jumping to the first (evens) level it does not translate to the second but the first...so 1 translates to 2 which is the first item in { 0 + 2x }. But each additional item hits 3 items higher after that; 5 translates to 8 – 9 translates to 14.

It may not be so obvious at this point but all the odd entries (all odd number in the natural number set) are accounted for. All the entries are already accounted for in all levels above { 0 + 2n }. That implies that any of the evens when divided by the appropriate number of 2s will spill to an odd number in a higher level that has already been acounted for. So without taking a leap of faith we can be confident that each and every natural number is included in the tree. Right?

Section 6 – Trivial Loop Jumps Out

This is a good place to point out the trivial loop and how it comes into being:

{ 0 + 2x } **2**, **4**, 6, 8, 10, …
{ 1 + 4x } **1**, 5, 9, 13, …

See how this happens? With these equations it jumps right out the page.

Section 7 – Putting it Together

The next leap comes when you accept that no matter how much bouncing around it does, this process will eventually lead down to the trivial loop { 1 – 4 – 2 }. But I don't expect you to accept this blindly. If every third item in { 0 + 2x } is accounted for; that is 2, 8, 14, 20, 26, … let's do some quick number crunching... 2 reduces to the trivial; 8 also reduces to the trivial loop; infact all powers of 2 which are included in this subset will do just that. I call these the 'backbone collapse to trivial'. This is the obvious part. The final not so obvious is in the tree structure itself as I have drawn it. The power of 2s backbone is across the very top and the only possible direction in that row is left to '1' by dividing by 2 over and over. The next level down is where any possible backbone entry less a 1 is divisible by 3...example (5*3)+1 = '16'. Now 5 can grow to the right by multiplying by 2 consecutively – 10, 20, 40, … or it can go up if multiplied by 3 and one added.

```
1 – 2 – 4 – 8 – 16 – …
                 |
                05 – 10 – 20 – 40 – ...
                   |         |
                   |        13 – 26 – 52 – ...
                   |                 |
                   |                17 – 34 – 68 – ...
                   |                   |
                  03 – 06 – 12 – …    11 – 22 – 44 – ...
                                         |
                                        07 – 14 – 28 – ...
                                                 |
                                                09 – 18 – …
```

Once the tree is built it should be obvious that you can only proceed left and up. Going left and up will eventually lead to the backbone. Right? So the rest of those evens that are not exact powers of 2 will be found somewhere else in this tree structure where it can only go up or left and approach the backbone. Maybe someone has a better way to explain this. I sure hope that made sense!

I'm not too worried about the rest of the structure because I am ultimately trying to show there is at least one case where this cascade will be infinite and hence unending (or continually growing). This is the only case where the tree can grow forever...it has to be an infinite cascade. So how does this play into it?

As one approaches infinity the ultimate number of steps in the cascade I discussed above approaches infinity as well. At infinity the process breaks. Infinity will enter an infinite number of steps in this cascade. So is this not a counter example? To disprove the conjecture?

I am thinking NOT. Since this happens at the very endpoint we can likely use this to show that the only case where it can grow infinitely is at that endpoint of infinity and since we can never get to the endpoint of infinity; there are no other situations where it is possible so long as { n < infinity }. All numbers from 1 up to but not including infinity will reduce to the ultimate loop { 1 – 4 – 2 }.

This is an aside that may useful to point out at this time. And it is likley to play an important role in an inductive proof. Notice how going right and down will allow us to realize a smaller ending number than the beginning number in most cases. If you include the duality concept I will introduce later all of these will be able to do just that. Put this on the back burner for now.

Section 8 – Exploring the Negative Numbers in the Sequence { 3n-1 ; n/2 }

I found it interesting in that if one uses the negative natural counting numbers from -1 to -infinity in the { 3n-1 ; n/2 } instead of the above Collatz { 3n+1 ; n/2 } one gets the exact same tree as outlined above...except it contains nothing but negative numbers; and instead of going left and up as seen in Collatz it goes right and up. It changes direction which is expected. The magnitude remains the same. The same trivial loop occur except it is { -1 – -4 – -2 }.

My special set of equations are slightly different but the same rules apply (Negatized).

- $-0 + 2x \quad \{ -0 + (2^1)x \} \quad \{ -(((2^1) / 2) -1) + (2^1)x \}$
- $-1 + 4x \quad \{ -1 + (2^2)x \} \quad \{ -(((2^2) / 2) -1) + (2^2)x \}$
- $-3 + 8x \quad \{ -3 + (2^3)x \} \quad \{ -(((2^3) / 2) -1) + (2^3)x \}$
- $-7 + 16x \quad \{ -7 + (2^4)x \} \quad \{ -(((2^4) / 2) -1) + (2^4)x \}$
- …
- $-(((2^y) / 2) -1) + (2^y)x$
- …
- $-(((2^{infinity}) / 2) -1) + (2^{infinity})x$

$\{ -0 + 2x \}$ -2, -4, -6, -8, -10, -12, -14, -16, -18, -20, -22, -24, -26, …
$\{ -1 + 4x \}$ -1, -5, -9, -13, -17, -21, -25, -29, …
$\{ -3 + 8x \}$ -3, -11, -19, -27, -35 …
$\{ -7 + 16x \}$ -7, -23, -39, …

See the same trivial loop { -1 – -4 – -2 } and it jumps out as well. The rest of the argument is exacly the same for the negative natural counting numbers in the sequence { 3n-1 ; n/2 }.

Do my formulas show a convergence as well:

$(3 (\{ -7 + 16x \}) - 1) / 2$
$(-21 + 48x - 1) / 2$
$(-22 + 48x) / 2$
$-11 + 24x$
$-3 - 8 + 24x$
$-3 + 8 (-1 + 3x)$ or { 3 + 8x since -1+3x is actually an 'x' after applying 3n-1 }

And this is the case for all these equations.

Let's try the cascade to { 0 + 2x }:

$(3 (\{ -1 + 4x \}) - 1) / 2$
$(-3 + 12x - 1) / 2$
$(-4 + 12x) / 2$
$-2 + 6x$

-0 - 2 + 6x
-0 + 2 (-1 + 3x) or { 0 + 2x since -1+3x is actually an 'x' after applying 3n-1 }

They behave exactly the same way as the positives. So I will not bore you by showing more of them in detail. Once was quite enough to prove the point.

Here is the only tree with negative number in {3n-1; n/2}

```
-1 – -2 – -4 – -8 – -16 – …
                     |
               -05 – -10 – -20 – -40 – ...
                     |             |
                     |           -13 – -26 – -52 – ...
                     |                         |
                     |                       -17 – -34 – -68 – ...
                     |                              |
                  -03 – -06 – -12 – …            -11 – -22 – -44 – ...
                                                       |
                                                     -07 – -14 – -28 – ...
                                                                |
                                                              -09 – -18 – …
```

But, lets consider if 4x+1 holds true in this tree. No it does not. Here it must be altered to 4x-1. (-1*4)-1 =-5. (-3*4)-1=-13. We will likely see the same thing occur in the following sections where 3 trees (loops) become possible.

<u>Section 9 – Exploring the Negative Numbers in the Collatz Sequence { 3n+1 ; n/2 }</u>

Placing the negative numbers in my original equations (they have been negatized) yeilds the following:

- -0 + 2x   { -0 + (2^1)x } { -(((2^1) / 2) -1) + (2^1)x }
- -1 + 4x   { -1 + (2^2)x } { -(((2^2) / 2) -1) + (2^2)x }
- -3 + 8x   { -3 + (2^3)x } { -(((2^3) / 2) -1) + (2^3)x }
- -7 + 16x { -7 + (2^4)x } { -(((2^4) / 2) -1) + (2^4)x }
- …
- -(((2^y) / 2) -1) + (2^y)x
- …
- -(((2^infinity) / 2) -1) + (2^infinity)x

{ -0 + 2x } -2, -4, -6, -8, -10, -12, -14, -16, -18, -20, -22, -24, -26, …
{ -1 + 4x } -1, -5, -9, -13, -17, -21, ...
{ -3 + 8x } -3, -11, -19, -27, -35 …
{ -7 + 16x } -7, -23, -39, …

I had to make a slight change to my equations to cover all the negative natural numbers but for all intents and purpose the same levels pop out valid.

Now what does the tree structure look like:

```
-1 – -2 – -4 – -8 – -16 – -32 – …
                |           |
                |          -11 – -22 – -44 – -88 – -176 – …
                |                        |            |
                |                        |           -59 – -118 – -236 – ...
                |                        |                          |
                |                        |                         -79 – -158 – ...
                |                        |
                |                       -15 – -30 – -60 – ...
               -3 – -6 – -12 – ...
```

This first tree has the loop at the very top left before any branching begins. The loop is { -1 – -2 }. Keep that in mind for the following two loops. Seems this tree does not include -5 so lets start a new tree with -5 as part of the loop:

```
-5 – -10 – -20 – -40 – -80 – -160 – …
            |            |
            |           -27 – -54 – -108 – ...
            |
            |
           -7 – -14 – -28 – -56 – ...
                 |
                -5
```

Seems this loop is { -5 – -14 – -7 – -20 – -10 }. Also note that this being a loop for the second tree does in fact start at the top left and works it way down the first possible branch

And finally there is yet a third tree with it's own loop that covers the remainder of (1/3) of the natural counting numbers. And I'm taking an educated guess that it is -16-1 = -17 because the last loop was -4-1 = -5 and the very first loop was just -1. So my thinking was -(0)-1 = -1 is the { -1 – -2 } loop; -(2^2)-1 = -5 is the { -5 – -14 – -7 – -20 – -10 } loop; -(2^4)-1 = -17 is the next loop. Interesting, ehh? Also note that this loop as well begins at the upper left and proceeds down the second possible branch. I have not been able to show why this is the case but an educated guess would indicate it definitely has something to do with the 0; 2^2 and 2^4. It's also interesting that all three start numbers for each of the trees originates from { -1 + 4x } = **-1**, **-5**, -9, -13, **-17**, -21, -25, …And it is not a coincidence. Another way to look at it is simply 1+0; 1+4; 1+16 or 1+0; 1+2^2; 1+2^4. Powers of 2 still play an important role. It's going to take more work to determine exactly what is happening... the joy of number theory!

The following discussion is a fitting guess on what is happening and how these powers of 2 play into it. Directly following this I get into how to divide the natural counting numbers into 3 sets because 3n in the 3n+1 dictates that much. It takes a little leap of faith to notice that in Collatz a power of three comes into play at two critical jump points (to new separate trees). Here is a table layout of the odd numbers applied to both 3n+1;n/2 & 3n-1;n/2:

Note that I have highlighted the odd numbers that can potentially jump off into their own tree which of course are given by 1; 1+2^2; 1+2^4 – 1, 5, 17. See above. And because we are dealing with multiples of 3 and three groupings/sets where we have { multiples of 3 }; { multiples of 3 + 1}; { multiples of 3 +2 }. Seems 3 plays a critical role.

So in Collatz we see what happens when we look at the three jump points 1, 5 and 17. 1 starts the natural loop { 1 – 4 – 2 }. At 5 we have the potential to jump off to a new tree but because 5 goes to 5 + 3 = 8 it stays in the original tree. It's also interesting that 8 = 2^3. Anything other than the addition of a power of 3 would have caused it to form it's own tree. Now with 17 we can see that again it goes to 17 + 3*3 = 26. Now again there was the potential of jumping off to a new tree had this number been created using a power of 3. The power of 3 kept it in the original loop. So in the case of Collatz and 1, 5, 17 all three stay in the same 1 – 4 – 2 loop.

Now see what happens when we look at the jump points 1, 5, 17 in the 3n-1;n/2 sequence. { 1 – 2 – 1 } is the natural first base loop. In the case of 5 it gives 5 + 2 = 7. This is adding a power of 2...not three. So 5 can break clean of the original loop because it has no way (needed to add a power of 3 to fall into the original loop) of entering the { 1 – 2 } loop.

The same thing happens with 17 in the 3n-1;n/2 sequence. Instead of adding a multiple of 3 to enable it access to the original loop it has a multiple of 2 (specifically 2 ^ 3 = 8). Note as well that 3 = 2+1 and 3*3 = 2*2*2+1. I point this out because we are actually dealing with 3n-1; so I would expect at these jump points to see a number that is one less than what it would've been in Collatz. Now I suspect that the jump points 5 and 17 are the only two points where we can have 3^? = 2^? +1. I've seen this at play elsewhere I think in the a^x = b^y + 1; where x <> y (not equal).

How's that for some obscure reasoning?

Odd number	3n+1	n/2	3n-1	n/2
1	4	2	2	1
3	10	5	8	4
5	16	**8 (5+'3')**	14	**7 (5+'2')**
7	22	11	20	10
9	28	14	26	13
11	34	17 (11+3*2')	32	16 (11+'5')
13	40	20	38	19
15	46	23	44	22
17	52	**26 (17+'3*3')**	50	**25 (17+'2*2*2')**
19	58	29	56	28
21	64	32	62	31

Another interesting observation is that the set of all natural counting numbers can be subdivided into three distinct groupings. This provides ammunition and goes hand in hand with what I was dicussing above regarding only three possible trees.

Lets look at the number line and logically break into three groups. This will make more sense as we look at it in detail.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, …

Starting at 0; add 3 consecutively to isolate all the multiples of 3. This is one third of the entire set:

0, 3, 6, 9, 12, 15, 18, 21, 24, … and leaves:

1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, …

Next, starting at 1, add 3 consecutively and strip out that third. This is the subset that is any multiple of 3 plus 1.

1, 4, 7, 10, 13, 16, 19, 22, … and leaves the final sub group:

2, 5, 8, 11, 14, 17, 20, 23, …

So starting at 2 and adding 3 consecutively gives us all the remaining numbers of the final sub-group. This final sub-group is simply a multiple of 3 plus 2! There are no more multiples of 3 plus anything that will result in a fourth sub-grouping.

The three sub-groups are:

{ 1, 4 ,7, 10, 13, 16, 19, 22, … }
{ 2, 5, 8, 11, 14, 17, 20, 23, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, … }

This shows the three evenly distributed groupings that contain exactly 1/3 of the original natural counting numbers set. It also shows that even deeper than that, half of each of these 3 sub-groupings is even numbers. These 3 sub-groupings are integral in the Colatz tree as well. **3n**(+1) dictates that. Right?

I wonder if there is a conection to my original group of equations:

{ 0 + 2x } 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, …
{ 1 + 4x } 1, 5, 9, 13, 17, 21, 25, 29, …
{ 3 + 8x } 3, 11, 19, 27, 35 …
{ 7 + 16x } 7, 23, 39, …

And there is! Let's start with { 0 + 2x }:

{ 1, **4** ,7, **10**, 13, **16**, 19, **22**, … }
{ **2**, 5, **8**, 11, **14**, 17, **20**, 23, … }
{ 3, **6**, 9, **12**, 15, **18**, 21, **24**, … }

What about { 1 + 4x }:

{ **1**, 4 ,7, 10, **13**, 16, 19, 22, … }
{ 2, **5**, 8, 11, 14, **17**, 20, 23, … }
{ 3, 6, **9**, 12, 15, 18, **21**, 24, … }

And { 3 + 8x }:

{ 1, 4 ,7, 10, 13, 16, **19**, 22, 25, 28, 31, 34, 37, … }
{ 2, 5, 8, **11**, 14, 17, 20, 23, 26, 29, 32, **35**, 38, … }
{ **3**, 6, 9, 12, 15, 18, 21, 24, **27**, 30, 33, 36, 39, … }

And last for us to make the point { 7 + 16x }:

{ 1, 4 ,**7**, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, … }
{ 2, 5, 8, 11, 14, 17, 20, **23**, 26, 29, 32, 35, 38, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, **39**, … }

My original equations hit each of these three sub-groupings evenly and in a defined pattern. Notice that in { 0 + 2x } each entry in the subgroup is separated by 6; 4 + 6 = 10 + 6 = 16, … In the next { 1 + 4x } it is a separation of 12; 1 + 12 = 13 + 12 = 25, … And if I was a betting man I would wage a guess that the next { 3 + 8x } is sepatated by 24; 11 + 24 = 35,... with each equation we multiply the difference by an additional 2 (double it)... 6, 12, 24, 48, 96, … The original 6 in that sequence is the result of 3 * 2. Hmmm, multiples of 3 and powers of 2! 3 * 2^1; 3 * 2^2; 3 * 2^3; ...

So the third loop looks like this:

```
-17 – -34 – -68 – -136 – -272
            |            |
            |          -91 – -182 – -364 – ...
            |                 |
            |               -61 – -122 – -244 – ...
            |                       |
          -23 – -46 – -92 – …     -41 – -82 – -164 – ...
                       |                   |
                     -31 – -62 – -124 – …  -55 – -110 – ...
                            |                     |
                          -21 – -42 – -84 – …   -37 – -74 – ...
                                                     |
                                                   -25 – -50 – -100 – ...
                                                          |
                                                         -17
```

This loop is a little more involved: { -17 – -50 – -25 – -74 – -37 – -110 – -55 – -164 – -82 – -41 – -122 – -61 – -182 – -91 – -272 – -136 – -68 – -34 }

Also of some interest is the length of these loops and how they appear to relate to the jump points they start from:

{ -1 - -2 } - two steps
{ -5 – -14 – -7 – -20 – -10 } - five steps
{ -17 – -50 – -25 – -74 – -37 – -110 – -55 – -164 – -82 – -41 – -122 – -61 – -182 – -91 – -272 – -136 – -68 – -34 } - eighteen steps

The first loop begins at -1; but you need at least two steps to form a loop so voila you have a two step loop. The second loop starting with -5 requires exactly five steps. And the third loop starting -17 requires exactly eighteen steps. Now remember the way these trees work, powers of 2 and branching. The first and the third loops require one more step than the starting numbers. The second loop only requires the original five steps. This seems very coincidental, doesn't it? Too convenient! Now if I consider that in this case we are dealing with negative numbers (treat the negative sign as direction only; the actual magnitude of the numbers are same no matter the sign) then instead of adding '1' to the step count for the first and third loops I should've indicated that we are actually adding '-1'. -1 + -1 = -2; -17 + -1 = -18.

Generally, I would say since 'three' is prominent in the way this sequence works, we will only find the three separate trees with their own single loop. And I would expect that the numbers are distributed evenly among the three; with half of that third evenly split between even and odd.

Someone else has already done the statistics that show this to be the case; there are only the three trees and they each contain a 1/3 of the entire natural number set. So I'm not going to rehash that here and simply accept it.

Do my formulas show a cascading convergence as well:

(3 ({ -7 + 16x }) + 1) / 2
(-21 + 48x + 1) / 2
(-20 + 48x) / 2
-10 + 24x
-3 + 1 - 8 + 24x
1 - 3 - 8 + 24x
1 - 3 + 8 (-1 + 3x)
-3 + 8 (-1 + 3x) +1

It does cascade to an odd number in the prior level but has 1 added to make it even (or it ultimately jumps to { 0 + 2x }).

It's a little difficult to explain. Suffice to say we do infact cascade back to the prior level but instead of the number remaining odd it has one added to make it even again and thus divisible once more by 2...but this actually brings us directly back to the first level { 0 + 2x }. This is holding true for all three of those loops. It does appear to be the case in other two trees with the other two loops? I'm going to have to investigate this further to see if I can determine what is happening there and explain it in mathematical terms. I will show in later sections how I was able to arrive at this conclusion which is true for all three loops.

So, No, they break down and can not show a step by step cascade! In the case of the first tree with the { -1 – -2 } loop the cascade is directly to level { -0 + 2x }. The other two trees do the same thing at least mathematically as we have shown by working these equations through 3n-1.

I Think we need to look specifically at what is happening at { -1 + 4x } level. It's likely buried but doing the same cascade to { 0 + 2n } level.

(3 ({ -1 + 4x }) + 1) / 2
(-3 + 12x + 1) / 2
(-2 + 12x) / 2
-1 + 6x
1 - 2 + 6x
1 + 2 (-1 + 3x)
-0 + 2 (-1 + 3x) +1

It is doing the same thing. There is a hidden cascade to the prior level but it gets lost in translation and is overidden to first even level { -0 + 2x }. So what this is ultimately saying is that all levels over { -0 + 2x } have all their elements cascade directly to level { -0 + 2x }. Luckily there are enough elements in { -0 + 2x} for a one-to-one match with all the elements combined from upper levels. Right?

So we can likely build on that fact like we did before. In this case all levels cascade directly to { -0 + 2n }. So yes, all odd numbers will be accounted for and as a result all evens. Likewise, if magically have three evenly (1/3) distributed trees; that is 1/3 of all the natural number set falls in each of trees. The same odd and even as shown above will hold in each of these three trees as well.

Needless to say it is much easier to show with these three smaller trees that as n approaches infinity it is not creating a multi-level cascade that could reach infinity in steps...but instead have only a single cascade directly to level { 0 +2n }. So, there is NO situation where this sequence can grow indefinately and no quasi-counter to use to prove by contradiction like we did above in earlier discussion. I don't think we need to.

It is easily shown after all this that there is one and only one loop for each of the three individual trees. The structure dictates that.

The Collatz trees each hold the 4x+1 rule we've seen in the above discussion. -3*4+1 = -11; -23*4+1 = -91.

Let's go back to these three subsets outlined above:

{ 1, 4 ,7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, … }
{ 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, … }

The three loops occur and contain only numbers from the first two subsets... the ones that are not a multiple of 3 (the third subset). So the first two subsets only. The { -1 – -2 } loop:

{ **1**, 4 ,7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, … }
{ **2,** 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, … }

And the { -5 – -14 – -7 – -20 – -10 } loop:

{ 1, 4 ,**7**, **10**, 13, 16, 19, 22, 25, 28, 31, 34, 37, … }
{ 2, **5**, 8, 11, **14**, 17, **20**, 23, 26, 29, 32, 35, 38, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, … }

And the { -17 – -50 – -25 – -74 – -37 – -110 – -55 – -164 – -82 – -41 – -122 – -61 – -182 – -91 – -272 – -136 – -68 – -34 } loop:

{ 1, 4 ,7, 10, 13, 16, 19, 22, **25**, 28, 31, **34**, **37**, 40, 43, 46, 49, 52, **55**, 58, **61**, 64, 67, 70, 73, … }
{ 2, 5, 8, 11, 14, **17**, 20, 23, 26, 29, 32, 35, 38, **41**, 44, 47, **50**, 53, 56, 59, 62, 65, **68**, 71, **74**, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, … }

This makes sense since any row in the Collatz tree that starts with a multiple of 3 is a dead end row that can't spawn new branches so the loop items must not venture into that subset.

I wonder if there's a pattern here that me might pick up on if we overlay the three loops each in a different color:

{ **1**, 4 ,7, 10, 13, 16, 19, 22, **25**, 28, 31, **34**, **37**, 40, 43, 46, 49, 52, **55**, 58, **61**, 64, 67, 70, 73, … }
{ **2**, 5, 8, 11, 14, **17**, 20, 23, 26, 29, 32, 35, 38, **41**, 44, 47, **50**, 53, 56, 59, 62, 65, **68**, 71, **74**, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, … }

I wonder what these loops look like in my equations:

{ 0 + 2x }	– **2**, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, **34**, 36, 38, 40, 42, 44, 46, ...
{ 1 + 4x }	– **1**, 5, 9, 13, **17**, 21, **25**, 29, 33, **37**, **41**, 45, 49, 53, 57, **61**, 65, 69, 73, 77, 81, 85, 89, ...
{ 3 + 8x }	– 3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, **91**, 99, 107, 115, 123, 131, 139, 147, 155, ...
{ 7 + 16x }	– 7, 23, 39, **55**, 71, 87, 103, 119, 135, 151, 167, 183, 199, 215, …
{ 15 + 32x }	– 15, 47, 79, 111, 143, 175, 207, 239, 271, 303, 335, 367, ...
{ 31 + 64x }	– 31, 95, 159, 223, 287, 351, 415, 479, ...
{ 63 + 128x }	– 63, 191, 319, 447, 575, 703, 831, ...
{ 127 + 256x }	– 127, 383, 639, 895, 1151, 1407, 1663, ...
{ 255 + 512x }	– 255, 767, 1279, 1791, 2303, ...
{ 511 + 1024x }	– 511, 1535, 2559, 3583, ...

That's interesting but doesn't tell us much except that the loops are confined to elements from { 0 + 2x }, { 1 + 4x }, { 3 + 8x } and { 7 + 16x } only; with each loop starting on an element in { 1 + 4x } ONLY.

If I display the above observation in a slightly different fashion I'll be able to point out more easily some items I mentioned above.

-2	-8	-14	-20	-26	-32	-38	-44	-50	-56	-62	-68	-74	-80
-1		-5		-9		-13		-17		-21		-25	
	-3				-11				-19				-27
			-7								-23		
							-15						

I would like you to note right here that level 2 (1+4x) equation items are not divisible by 4 after subtracting 1 (none of them); however all upper levels members are divisible by 4 after subtracting 1. This is the complete opposite of what I'll show you later for the positive numbers in Collatz where only the members of level 2 (1+4x) are 4x+1 rule (all of them) with no other upper levels having such members.

You can see from the above table that all upper levels (levels 2 and up) immediately jump to level 1 (being even and all). No cascading appears in these trees.

All three trees (loops) can be built using the jump points identified and the 4x+1 rule to glue the backbones together. I'm not going to go into any further detail on how all that works. It does though. Be sure to use the dual even numbers as explained above for cohension. They are invisible in the structures presented but can be drawn in for connectivity.

Section 10 – Exploring the Positive Numbers in the Sequence { 3n-1 ; n/2 }

Much like the previous section, placing the positive numbers in the { 3n-1 ; n/2 } sequence will generate the exact same three loops only in this case all the numbers are positive and the direction of travel is left and up instead of right and up.

We would use the original set of equation that have not been negatized.

- 0 + 2x { 0 + (2^1)x } { (((2^1) / 2) -1) + (2^1)x }
- 1 + 4x { 1 + (2^2)x } { (((2^2) / 2) -1) + (2^2)x }
- 3 + 8x { 3 + (2^3)x } { (((2^3) / 2) -1) + (2^3)x }
- 7 + 16x { 7 + (2^4)x } { (((2^4) / 2) -1) + (2^4)x }
- …
- (((2^y) / 2) -1) + (2^y)x
- …
- (((2^infinity) / 2) -1) + (2^infinity)x

{ 0 + 2x } 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, …
{ 1 + 4x } 1, 5, 9, 13, 17, 21, ...
{ 3 + 8x } 3, 11, 19, 27, 35 …
{ 7 + 16x } 7, 23, 39, …

Now what does the tree structure look like:

```
1 – 2 – 4 – 8 – 16 – 32 – …
            |         |
            |         11 – 22 – 44 – 88 – 176 – …
            |                   |          |
            |                   |          59 – 118 – 236 – ...
            |                   |                      |
            |                   |                     79 – 158 – ...
            |                   |
            |                  15 – 30 – 60 – ...
            3 – 6 – 12 – ...
```

This first tree has the loop at the very top left before any branching begins. Keep that in mind for the following two loops. Seems this tree does not include 5 so lets start a new tree with 5 as part of the loop:

```
5 – 10 – 20 – 40 – 80 – 160 – …
         |         |
         |        27 – 54 – 108 – ...
         |
         |
         7 – 14 – 28 – 56 – ...
             |
             5
```

Seems this loop is { 5 – 14 – 7 – 20 – 10 }. Also note that this being a loop for the second tree does in fact start at the top left and works it way down the first possible branch

And finally there is yet a third tree with it's own loop that covers the remainder of (1/3) of the natural counting numbers. And I'm taking an educated guess that it is 16+1 = 17 because the last loop was 4+1 = 5 and the very first loop was just 1. So my thinking was 0+1 = 1 is the { 1 – 2 } loop; 2^2+1 = 5 is the { 5 – 14 – 7 – 20 – 10 } loop; 2^4+1 = 17 is the next loop. Interesting, ehh? Also note that this loop as well begins at the upper left and proceeds down the second possible branch. I have not been able to show why this is the case but an educated guess would indicate it definitely has something to do with the 0; 2^2 and 2^4. See above for further observations on this coincidence.

So the third loop looks like this:

```
17 – 34 – 68 – 136 – 272
          |           |
          |          91 – 182 – 364 – ...
          |                |
          |               61 – 122 – 244 – ...
          |                     |
         23 – 46 – 92 – …      41 – 82 – 164 – ...
                   |                     |
                  31 – 62 – 124 – …     55 – 110 – ...
                        |                     |
                       21 – 42 – 84 – …      37 – 74 – ...
                                                   |
                                                  25 – 50 – 100 – ...
                                                        |
                                                       17
```

This loop is a little more involved: { 17 – 50 – 25 – 74 – 37 – 110 – 55 – 164 – 82 – 41 – 122 – 61 – 182 – 91 – 272 – 136 – 68 – 34 }

Generally, I would say since 'three' is prominent in the way this sequence works, we will only find the three separate trees with their own single loop. And I would expect that the numbers are distributed evenly among the three; with half of that third evenly split between even and odd.

Again, someone else has already done the statistics that show this to be the case; there are only the three trees and they each contain a 1/3 of the entire natural number set. So I'm not going to rehash that here and accept it.

All of the exact same discussion remain true here for a proof as we have shown above in earlier sections.

Lets try a couple of the equations to make sure:

$(3 (\{ 7 + 16x \}) - 1) / 2$
$(21 + 48x - 1) / 2$
$(20 + 48x) / 2$
$10 + 24x$
$3 - 1 + 8 + 24x$
$-1 + 3 + 8 + 24x$
$-1 + 3 + 8 (1 + 3x)$
$3 + 8 (1 + 3x) -1$

and:

(3 ({ 1 + 4x }) - 1) / 2
(3 + 12x - 1) / 2
(2 + 12x) / 2
1 + 6x
-1 + 2 + 6x
-1 + 2 (1 + 3x)
0 + 2 (1 + 3x) -1

As expected, instead of adding one to get even we subtract 1 to get even and back to level { 0 + 2x }. The mechanics are the same.

Also, the 4x+1 rule does NOT hold here as expected. This is the other situation where we need to use 4x-1. 3*4-1 = 11; 15*4-1 = 59. Mirror images. Think about that.

```
1 – 2 – 4 – 8 – 16 – 32 – …
          |         |
          |         11 – 22 – 44 – 88 – 176 – …
          |                   |         |
          |                   |         59 – 118 – 236 – ...
          |                   |                     |
          |                   |         20          79 – 158 – ...
          |                   |
          |         04        15 – 30 – 60 – ...
          3 – 6 – 12 – ...
```

Remember how I pointed out even numbers could play a dual role in these trees. I've shown two examples above. Only in this case it makes use of 4x-1 and 3x-1 rules. 15+1=16/4=4; 4*3=12-1=11. Again, this is the mollasses that holds the tree together.

Section 11 – Understanding the 'NOT so Random Jumps' Within the Collatz Tree

What appears to be random jumps is actually constrained. Let's explore what is happening at each of my equations starting with { 0 + 2x }.

{ 0 + 2x } 2, **4**, 6, **8**, 10, **12**, 14, **16**, 18, **20**, 22, **24**, 26, **28**, 30, **32**, 34, **36**, 38, **40**, 42, **44**, 46, **48**, 50, ...

In the above illustration I have highlighted in different colors the sequences where you take the first x=2 and multiply by 2 successively.. 2, 4, 8, 16, 32,... I left the very first number in this sequence un-hilighted which will come in play later. The next available number is x=6 giving 6, 12, 24, 48, 96, … The next available number is x=10 giving 10, 20, 40, 80, 160,... And the next is x=14 giving 14, 28, 56, … Then it's x=18 giving 18, 36, 72, … Obviously there is a distinct pattern here and that is after rooting out all numbers that are multiples of '2' of a prior lower number we end up having every second number starting at 6 available for this operation... 6, 10, 14, 18, 22, …. So obviously, every number in this equation will end up in the Collatz Tree. Where it is in that tree is unimportant. Half of this set is divisible by at least 4. The other half is only divisible by 2 leading to an odd number that will fall somewhere else in the tree. I hope you can accept that.

Let me show the next few equations expanded out:

{ 1 + 4x } **1**, **5**, **9**, 13, 17, 21, 25, 29, 33, 37, …
{ 3 + 8x } **3**, **11**, 19, 27, 35, 43, 51, 59, 67, 75, …
{ 7 + 16x } **7**, 23, 39, 55, …
{ 15 + 32x } 15, 47, 79, …
{ 31 + 64x } 31, 95, ...

There is a pattern to how every second base even number in { 0 + 2x } jumps to upper level equations. So for the sequence 2, 6, 10, 14, 18, 22, 26, 30, 34, 38, 42, … do the division by 2 and you get 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, … Obviously 1, 5, 9, 13, 17, 21, … of this list all fall in the { 1 + 4x } equation. Note that this list is formed by adding 4 consecutively; 1+4=5+4=0+4=13... I'll be willing to bet that starting at 3 and adding 8 consecutively will give us a list that in the { 3 + 8x }... 3+8=11+8=19+8=27... 3, 11, 19, 27, … Then if we take 7 which is the next available starting sequence you would add 16 consecutively giving 7, 23, 39, 55, … which is the { 7 + 16x } equation. The pattern should now be obvious.

Let's explore the cascading level effect starting with the { 3 + 8x } equation. If you pick 3 you will pass through to the prior level { 1 + 4x } and that is so. 3*3+1=10/2=5. The same happens to 11...3*11+1=34/2=17. And the next 19 does it as well 3*19+1=58/2=29. And it just so happens 5, 17, 29, … are separated by 12 (3 * 4 or 3 * 2 * 2). This covers every number in { 3 + 8x }. The exact same thing happens if we investigate { 7 + 16x }... 3*7+1=22/2=11; 3*23+1=70/2=35; 3*39+1=118/2=59; or 11, 35, 59, … separated by 24 (3 * 8 or 3* 2 * 2 * 2). Looking at { 15 + 32x } we see similar 3*15+1=46/2=23; 3*47+1=142/2=71; 3*79+1=238/2=119; 23, 71, 119 are separated by 48 (3 * 16 or 3 * 2 * 2 * 2 * 2). Pattern has been established. Finally let's look at what happens with level { 1 + 4x }. We can see from the above that only 5, 17, 29, .. are pass through from upper levels. All other points in this equation remain untouched from upper levels leaving 1, 9, 13, 21, 25, 33, 37, … Note that all those that are passed through from upper levels reduce to an odd number that is smaller than it started at. 5 reduces to 1; 17 reduces to 13; 29 reduces to 11; 41 reduces to31; 53 reduces to 5; 65 reduces to 49, and so on. This is good because we can prove that given all numbers up to k are proven, then k+1 = 5 ends in a number that is less than 5 (actually 1) and this is the case for all of these.

Let's continue on with this trend of thought. 1, 13, 25, 37, … is another sequence separated by 12 in { 1 + 4x } that has not been touched from pass through from upper levels. These behave the same way as the pass throughs seen above. They all reduce to a number smaller than the starting number; 1 reduces to 1 (trivial); 13 reduces to 5; 25 reduces to 19; 37 reduces to 7; 49 reduces to 37; 61 reduces to 23. So with the same assumption that for k all lower assume true; k+1 = some number from this list results in a number smaller than k that has already been proven.

This leaves the final multiple of 3 sequence (again separated by 12) 9, 21, 33, 45, 57, 69, … And once again for the same agruement above all these reduce to numbers smaller than the original. 9 reduces to 7; 21 reduces to 1; 33 reduces to 25; 45 reduces to 17; 57 reduces to 43; 69 reduces to 13; so if up to k assumed true; it is obvious that k+1 ends up smaller than k so it is true as well.

This may be an ackward way to prove all numbers are included and reduce to the trivial loop in the Collatz tree. It does seem to work though. That will all become apparent in the below discussion when I start to use these building blocks to formulate the proof.

Section 12 – Putting It All Together (Formulation of a Proof)

Expanding upon the first few sections in this report, I will show where my set of equations originated and this is an important observation in showing that all the natural numbers are contained in the union of these subsets.

	2	4	6	8	10	12	14	16	18	20	22	24	26
1		5		9		13		17		21		25	
	3				11				19				
			7								23		
							15						

Let me draw the above in a format that is a little cleaner to follow, noting that I will skip over all the even number not formed by successively adding 6 to 2...

2	8	14	20	26	32	38	44	50	56	62	68	74	80
1	5	9	13	17	21	25	29	33	37	41	45	49	53
	3			11			19			27			35
				7									23
													15

There is a very unique pattern that makes it very easy to see that 'ALL' the natural numbers will be included. I've taken away any even number that does not grow 'stacks' back to smaller upper level members. This shows the cascading effect I've tried to explain above in other sections. Note how each upper level injects it's first member onto the stack resulting from the second member of the previous level. It's next member is is injected onto the thrid stack from the previous level...so each new member skips two prior level stacks before being injected. That is why I dropped two even numbers before creating stacks. It really jumps out now! Once you accept this you can see where my sequence of equations originated. And just in case you don't realize it, the lowest number in a stack multiplied by 3 and add 1 then divide by 2 give the next up... continue (*3+1)/2 to next level up and so on. These are the basic rules for odd/even numbers in Collatz conjecture.

This realization also brought me to the idea that if this goes on toward infinity there should be '1' stack approaching infinity! Right? The farther right one goes the longer the stacks can grow. But no prior stack less than infinity can be in the same state...the next closest one is one level smaller half way back from infinity (infinity/2). Think about that for a moment. Remember that each upper level equation has half the members the previous one did... hence my halving infinity. This should be enough to show all numbers are infact included; it's a complete set.

The very first row of even numbers is 50% of the total natural number set. The second row is an additional 25% (¼) of the natural number set... the third is 12.5% (1/8) and so on and so forth. You can also see several patterns when written in this fashion. Each set contains only half as many members as the previous set. You will also note that starting at row 1; the first available odd numbers missed in prior levels (even numbers row) start those sets. So the second row uses 1 as its starting number with successive members formed by adding 4 over and over. The third row would begin with 3 since it was not already used in the two prior rows...and it's members are given by adding 8 successively over and over. The next row begins with 7 and it's members are separated by 16. And this continues on. As you can see every number will be used and only ONCE. I'm also going to point out that if you pick the first member of any row greater than 1 (the even numbers row) and apply the (3n+1)/2 rules you will go up one row and to the right! For example (1*3+1)/2=2. The next row is (3*3+1)/2 =5. The next row starting with 7... (7*3+1)/2=11; (11*3+1)/2=17. The next is (15*3+1)/2=23; (23*3+1)/2=35; (35*3+1)/2=53. That was the important stuff to take forward...

I've shown above that only 3 loops can occur in the negative counting numbers under 3x+1; x/2 and only 1 loop using the positive counting numbers under 3x+1; x/2. So the existence of a second loop is not possible if following the original conjecture using only positive counting numbers under 3x+1; x/2. Two additional loops become possible only when using the negative counting numbers under 3x+1; x/2. There are two breakaway points, one at -5 and an additonal one at -17. The reasonaing as shown above plays with the -3+1=-2 & -3*3+1=-2*2*2 observation. The original loop as unstated would be -1*3+1=-2. As you can obviously see this gives rise to -1*3+1=-1*2 & -1*3*3+1=-1*2*2*2. I probably did a better job of showing this above. Needless to say the 3 jumping points (or three loops) start at -1; -5; and -17. You'll also note that -1+-2*2=-5 and -1+-2*2*2*2=-17 or -1+-2*2=-3*2+1 and -1+-2*2*2*2=-3*3*2+1. This special state can not occur in the positive counting numbers so there is only one loop starting at 1. No other loops can exist. So part one of the proof is confirmed...only the main loop exists.

Now I can build the other part of the proof from above observations. I noted that these counting numbers can be created using an infinite set of sequences; 0+2x; 1+4x; 3+8x; 7+16x; … The first sequence forms all the even numbers. The second sequence has half as many members all of which are odd and seperated by 4. The third sequence has half as many members as the second sequence with these being separated by 8...and so on and so forth.

I also noted that any number you start at would fit in one of the sequences and that as you apply the rules you end in the previous sequence stepping through each all the way back to the first. So if you started in the 7th sequence you end up in the 6th, then the 5th, 4th, 3rd, 2nd, and finally1st. But the 1st may not and usually does not end there and this brings up to another sequence greater than or equal to 1! And that process continues until one reaches the main loop 4-2-1. And this observation is VERY important. No matter what the starting number it will cascade down through the second sequence (1+4x) a number of times on it's way to the first sequence (0 +2x) where it'll make another jump wherever.

Lets take a closer look at just the even numbers. We know that there's a pattern here too. Check out the following:

2 – 1
4 – 2 – 1
6 – 3
8 – 4 – 2 – 1
10 – 5
12 – 6 – 3
14 – 7
16 – 8 – 4 – 2 – 1
18 – 9
20 – 10 – 5
22 – 11
24 – 12 – 6 – 3
26 – 13
28 – 14 – 7
30 – 15
32 – 16 – 8 – 4 – 2 – 1
34 – 17
36 – 18 – 9
38 – 19
40 – 20 – 10 – 5
42 – 21

44 – 22 – 11
46 – 23
48 – 24 – 12 – 6 – 3
50 – 25
52 – 26 – 13
54 – 27
56 – 28 – 14 – 7
58 – 29
60 – 30 – 15
62 – 31
64 – 32 – 16 – 8 – 4 – 2 – 1
66 – 33
68 – 34 – 17
70 – 35
72 – 36 – 18 – 9
74 – 37
76 – 38 – 19
78 – 39
80 – 40 – 20 – 10 – 5
82 – 41
84 – 42 – 21
86 – 43
88 – 44 – 22 – 11
90 – 45
92 – 46 – 23
94 – 47
96 – 48 – 24 – 12 – 6 – 3
98 – 49
100 – 50 – 25

As clearly seen above only powers of 2 'even' numbers can reduce directly to 1. example 2^1, 2^2; 2^3;... or 2, 4, 8, 16, 32, 64, … Now looking at the remainder of this may be critical if you are playing the stats game. As you can see from the way I have it drawn... half the even numbers are divisible by 2 only once. That's 50% of them. Of the 50% that remain a further 50% of them are divisible by an additional 2. So 25% of the total natural even numbers are divisible by 4 (2*2). And if you take the remaining 25%, half of them are divisble by another 2... 12.5% are divisible by 8 (2*2*2). 6.25% are divisible by 16; and so on and so forth. I do not need any of this even number stuff for my proof though.

Because of the way the the Collatz Tree forms I've noted that the starting odds on the successive limbs of any backbone branch are formed by applying (odd*4)+1 to each upper limb. Example 1, 5, 21, … and another 3, 13, 53,...

```
1 – 2 – 4 – 8 – 16 – 32 – 64 – 128
          |        |          |
          |        |          21 – 42 - 84
          |        |
          |       05 – 10 – 20 – 40 – ...
          |              |          |
         1               |         13 – 26 – 52 – ...
                         |                  |
```

```
|                          17 – 34 – 68 – ...
|                               |
03 – 06 – 12 – …          11 – 22 – 44 – ...
                                     |
                                07 – 14 – 28 – ...
                                           |
                                      09 – 18 – …
```

So this leads to the obvious next step that I overlooked in my original report and that is that any odd number where you can substract 1 and have it evenly divisible by 4 is automatically collapsable to the 4-2-1 loop. For example; 21 – 1 = 20 /4 =5. Note that you may be able to continue subtracting another 1 and still have it divisible by a further 4. But this is not the norm. So if we know 1 to x (assumed) are true, then x+1 being an odd number where x+1 subtract 1 is evenly divisible by 4 is also true. So the 25% of natural numbers in the 1+4x series are all true as well. Like the even numbers; if we know 1 to x (are assumed to be true) then x+1 as long as it is even is also true because (x+1)/2 is in the set we already assumed true...that's 1 to x.

So, we can easily show that all even numbers can reduced to the main 4-2-1 loop knowing that if you have already proven 1 to x; then x+1 if it happens to be even has the rule x/2 applied and the result is a proven x! We can now bring the above discussion (for odd numbers) about what happens if you can subtract 1 and have it divisible by 4...and that this will result in a number that falls in the 1 to x already proven. And this is good because the original sequences I used to create the counting number sets has a special feature. The second sequence 1+4x has all of it's elements being evenly divisible by 4 after subtracting 1. For example ((1+4x)-1)/4 =x That is the set 1, 5, 9, 13, 17, 21, … None of the other sequences will ever have an element that can do this. So the fact that we cascade through all sequences on the way down to the first sequence means we will go through the 1+4x sequence...and all elements in that set will automatically bring one to a number that is in the proven 1 to x! But this is only true if you start in 1+4x sequence. If you cascade from a higher level through 1+4x you are by no means proven. In some cases you may have a number that is smaller than the starting number and in the assume 1 to x true set, but this is not the norm.

Now, any odd number that falls in (is a member of) 1+4x sequence means that it starts proven. So we have been able to prove all even numbers (50%) & all odd numbers where x-1 is evenly divisible by 4 (25%) are Proven. That's 75% total.

If we take the third sequence 3+8x we can show that when it cascades into 1+4x it is close enough that it will be automatically proven.

(3(3+8x)+1)/2
(10+24x)/2
(4+6+24x)/2
(4+6(1+4x))/2
2+3(1+4x) now see if it is evenly divible by 4 after subtracting 1...
(2+3(1+4x)-1)/4
(1+3(1+4x))/4
(4+12x)/4
1+3x.

So any odd number that falls in 3+8x sequence will automatically be smaller or in the 1 to x assumed. 1+3x is smaller than the original 3+8x.

So as seen above any number that falls in 3+8x sequence (level 3) will cascade directly to level 2

(1+4x) where it automatically becomes true! The resulting number is smaller than the starting odd and in the 1 to x assumed. So that's an additional 12.5% which gives us a 87.5% of natural numbers proven.

Let's try doing the same thing to the next two sequences to see if they are close enough as well. That's the 7+16x and 15+32x. I'm going to try 31+64x as well because I know that's where it begins to fail. The math shows they both are... however 31+64x is not! Nor are any above that.

$(3(7+16x)+1)/2$	$(3(15+32x)+1)/2$	$(3(31+64x)+1)/2$
$(22+48x)/2$	$(46+96x)/2$	$(94+192x)/2$
$11+24x$	$23+48x$	$47+96x$
$(3(11+24x)+1)/2$	$(3(23+48x)+1)/2$	$(3(47+96x)+1)/2$
$(34+72x)/2$	$(70+144x)/2$	$(142+288x)/2$
$17+36x$	$35+72x$	$71+144x$
$(17+36x-1)/4$	$(3(35+72x)+1)/2$	$(3(71+144x)+1)/2$
$(16+36x)/4$	$(106+216x)/2$	$(214+432x)/2$
$4+9x$	$53+108x$	$107+216x$
$4+9x<7+16x$!	$(53+108x-1)/4$	$(3(107+216x)+1)/2$
	$(52+108x)/4$	$(322+648x)/2$
	$13+27x$	$161+324x$
	$13+27x<15+32x$!	$(161+324x-1)/4$
		$(160+324x)/4$
		$40+81x$
		$40+81x>31+64x$!

I'm going to apply a twist to all levels greater than the third (3+8x). Let's go in the opposite direction. First let's look at something special that occurs with a number of the upper level sequences...

Level 1 (0+2x) starts with 2 (even numbers)
Level 2 (1+4x) starts with 1
Level 3 (3+8x) starts with 3 (starts with a multiple of 3!)
Level 4 (7+16x) starts with 7
Level 5 (15+32x) starts with 15 (starts with amultiple of 3!)
Level 6 (31+64x) starts with 31
Level 7 (63+128x) starts with 63 (starts with a multiple of 3!)
Level 8 (127+256x) starts with 127
Level 9 (255+512x) starts with 255 (starts with a multiple of 3!)
Level 10 (511+1024x) starts with 511

Any backbone row starting with an odd number that is divisible by 3 (multiple of 3) can not spawn new backbones. That exactly half of the remaining levels which is clearly the case as seen above. However, just because the first member in that level is a multiple of 3 does not mean the others are multiples of 3 too; quite the opposite. As we'll see below all upper level equations display the same properties.

Let's start with an odd number from the sequence 7+16x...say 23! Now let's multiply it by 2 and see if the result subtract 1 is evenly divisible by 3. If it is, the number is proven because it falls in the 1 to x assumed proven and is smaller than the original.

So the sequence starting with 7 has an even division of members into three groups; one where after you multiply by 2 you can subtract 1 and have it evenly divisble by 3; one where you must multiply by 4 then subtract 1 and it will be evenly divisible by 3 (but the resulting number is not smaller than the starting! It is however evenly divisible by 4 after subtracting 1! This then makes it smaller than the starting.); and a final group that is evenly divisible by 3 (a multiple of 3 – dead end backbone) which I can not handle at this time. So each group is exacly 1/3 (33%). I can prove 2 of these subgroups meaning 66% are provable.

7 – 14 – 28	(28 – 1 = 27 / 3 = 9) 9 – 1 = 8 / 4 = 2!
	or directly using even number 'duality' 7-1 = 6 / 3 = **2**!
23 – 46	(46 – 1 = 45 / 3 = **15**!)
39 – 78 – 156	(**Multiple of 3**; I can't do anything with this yet)
55 – 110 – 220	(220 – 1 =219 / 3 = 73) 73 – 1 = 72 / 4 = 18!
	or duality again 55-1 = 54 / 3 = **18**!
71 – 142	(142 – 1 = 141 / 3 = **47**!)
87 – 174 – 348	(**Multiple of 3**!)
103	103-1=102 / 3 = **34** (duality)
119 - 238	238-1=237 / 3 = **79**
135	(**Multiple of 3**)
151	151-1=150 / 3 = **50** (duality)
167 – 334	334-1=333 / 3 = **111**
183	(**Multiple of 3**)

And this pattern in the above listing continues to infinity. I've highlighted the ones I consider 'duality' evens and you will note that they increase by exactly 16. The next subset are separated by exactly 32. As you can see this goes on toward infinity. This sequence (7+16x) is where 16 and 32 come from. In the first subset our node is the key and being the first it reflects exacly 16. The second subset must have the node multiplied by 2 exactly once to give an even number and those are separated by exactly 32 (16*2). All other levels display these same features.

Let's see if the next two levels (15+32x) and (31+64x) do the same thing:

15	(**Multiple of 3**)
47 – 94	94-1=93 / 3 = **31**
79	79-1=78 / 3 = **26** (Duality)
111	(**Multiple of 3**)
143 – 286	286-1 = 285 / 3 = **95**
175	175-1 = 174 / 3 = **58** (Duality)
207	(**Multiple of 3**)
239 – 478	478-1= 477 / 3 = **159**
271	271-1 = 270 / 3 = **90** (Duality)
303	(**Multiple of 3**)
335 – 670	670-1= 669 / 3 = **223**
367	367-1= 366 / 3 = **122** (Duality)

31	31-1 = 30 / 3 = **10** (Duality)
95 – 190	190-1= 189 / 3 = **63**
159	(**Multiple of 3**)
223	223-1= 222 / 3 = **74** (Duality)
287 – 574	574-1 = 573 / 3 = **191**
351	(**Multiple of 3**)
415	415-1 = 414 / 3 = **138** (Duality)
479 – 958	958-1 = 957 / 3 = **319**

543	(**Multiple of 3**)
607	607-1= 606 / 3 = **202** (Duality)
671 – 1342	1342-1 = 1341 / 3 = **447**
735	(**Multiple of 3**)

They both do and display the exact same attributes. So it is safe to assume that all the other levels whether or not they start with multiples of 3, behave in the same fashion. Let's look at the level starting with 127 (127+256x).

127 – 254 – 508	(508 – 1) / 3 = 169 (169 – 1) / 4 = **42**!
	or duality 127-1 = 126 / 3 = 42.
383 – 766	(766 – 1) / 3 = **255**!
639 – 1278 – 2556	(**Multiple of 3**)
895 – 1790 – 3580	(3580 – 1) / 3 = 1139 (1139 – 1) / 4 = **298**!
	duality 895-1 = 894 / 3 = 298.
1151 – 2302	(2302 – 1) / 3 = **767**!
1407 – 2814 – 5628	(**Multiple of 3**)
1663	1663-1=1662 / 3 = **554** (Duality)
1919 – 3838	3838-1=3837 / 3 = **1279**
2175	(**Multiple of 3**)
2431	2431-1=2430 / 3 = **810** (Duality)
2687 – 5374	5374-1=5373 / 3 = **1791**
2943	(**Multiple of 3**)

In this sequence/level (127+256x) we see the first subset separated by exactly 256 with the second set separated by exactly 512 (2*256). Cool.

Just for kicks, let's look at another multiple of 3 level (63+ 128x) to see if it displays the same properties.

63 – 126 – 252	(**Multiple of 3**)
191 – 382	(382 – 1) / 3 = **127**!
319 – 638 – 1276	(1276 – 1) / 3 = 425 (425-1) / 4 = 106!
	duality 319-1 = 318 / 3 = **106**.
447 – 894 – 1788	(**Multiple of 3**)
575 – 1150	(1150 – 1) /3 = **383**!
703 – 1406 – 2812	(2812 – 1) /3 = 937 (937-1) / 4 = 234!
	duality 703-1 = 702 / 3 = **234**.
831	(**Multiple of 3**)
959 – 1918	1918-1= 1917 / 3 = **639**
1087	1087-1=1086 / 3 = **362** (Duality)
1215	(**Multiple of 3**)
1343 – 2686	2686-1=2685 / 3 = **895**
1471	1471-1=1470 / 3 = **490** (Duality)

A multiple of 3 equation behaves in exactly the same fashion...they are simply ordered otherwise. 66% are easily provable by the same techniques. Immediately above is level (63+128x) with one subset separated by exactly 128 and the other by exacly 256. Not a coincidence!

So like I mentioned above 7+16x and 15+32x can be proven in the same fashion as 3+8x because they are within a distance that will allow for it. I do however use both those sequences above to show what happens in all upper levels and how three distinct groupings/sets become possible. The numbers are smaller to deal with to show this point. Looking at sequence 127+256x you can see how quickly the numbers grow.

So as stated above we've shown that 66% of the members in each upper level sequences (the ones that start with multiples of 3 are simply ordered differently) by simply applying the rules as shown above; one third are simply multiplied by 2 then divisible by by 3 after subtracting 1; another third by multiplying by 4 then divisble by 3 after subtracting 1...but can be further reduced by subtracting 1 and have it divisible by 4; the remaining third are multiples of 3 and no proof yet.

I now realize that the approach I'm taking by backward traversing to prove by induction can be used to prove all numbers that are not multiples of 3; example is multiply by 2 and/or subtract 1 and then divisible by 3. You will notice that all odd numbers (except multiples of 3) display this feature. We can use this as a second method that compliments my first method. As to speak they work hand in hand and prop up one another as an even stronger proof concept. Using duality makes this doable and easier to spot.

Snippet one...

```
5 – 10 – 20 – 40 …
    |         |
    |        13 – 26 ...
    |
   3 – 6 ...
```

Snippet two...

```
31 – 62 – 124 – 248 ...
 |          |
10         41 – 82 …
               |
              27 – 54 ...
```

The above two snippets show this concept clearly. By working backwards we have a result number smaller than the beginning number. Induction! 5 can easily be reduced to 3. 13 easily reduces to 4 (duality). 31 easily reduces to 10. I can't believe this has been staring me in the face all this time. My discussion on duality made it a reality for me.

Doing the math we have 12.5% remaining to cover off the upper levels but remember that as we go up levels the members included are halved. So the levels have the following associated percentages:

Level 1 – 50%	(100% provable)
Level 2 – 25%	(100% provable)
Level 3 – 12.5%	(100% provable)
Level 4 – 6.25%	(100% provable)
Level 5 – 3.125%	(100% provable)
Level 6 – 1.5625%	(66% provable = 1.0417%)
Level 7 – 0.78125%	(66% provable = 0.516%)
Level 8 – 0.390625%	(66% provable = 0.2604167%)
Level 9 – 0.1903125%	(66% provable = 0.1256%)
Level 10 – 0.09515625%	(66% provable = 0.0628%)

and so on ...

So continuing on with the math we can prove 50% + 25% + 12.5% + 6.25%+ 3.125% + 1.0417% + 0.516% + 0.2604167% + 0.1256% + 0.0628 % giving a grand total of 98.88% . So I am able to prove slightly more than 98% of all the natural counting numbers set are provable.

My quandry now is that I can not fashion a method to handle those multiples of 3 instances (the remainder and only case yet to be proven) which account for less than 2%. Wow, that's close. I wonder if anyone else has come this close?

I haven't abandoned hope of solving the 'multiple of 3' issue and wish to share what I do know so far. The following table contains all the multiples of 3 up to 207. Take note of how I reduce them to provable. I have not included any even multiples of 3...examples 6, 12, ...:

3	**(3*3+1)/2 = 5**	**(5-1)/4 = 1**	**Provable**		
9	**(9-1)/4 = 2**	**Provable**			
15	**(3*15+1)/2 = 23**	**(3*23+1)/2 = 35**	**(3*35+1)/2 = 53**	**(53-1)/4 = 13**	**Provable**
21	**(21-1)/4 = 5**	**Provable**			
27	**(3*27+1)/2 = 41**	**(41-1)/4 = 10**	**Provable**		
33	**(33-1)/4 = 8**	**Provable**			
39	**(3*39+1)/2 = 59**	**(3*59+1)/2 = 89**	**(81-1)/4 = 20**	**Provable**	
45	**(45-1)/4 = 11**	**Provable**			
51	**(3*51+1)/2 = 77**	**(77-1)/4 = 19**	**Provable**		
57	**(57-1)/4 = 14**	**Provable**			
63	Not Provable				
69	**(69-1)/4 = 17**	**Provable**			
75	**(3*75+1)/2 = 113**	**(113-1)/4 = 28**	**Provable**		
81	**(81-1)/4 = 20**	**Provable**			
87	**(3*87+1)/2 = 131**	**(3*131+1)/2 = 197**	**(197-1)/4 = 49**	**Provable**	
93	**(93-1)/4 = 23**	**Provable**			
99	**(3*99+1)/2 = 149**	**(149-1)/4 = 37**	**Provable**		
105	**(105-1)/4 = 26**	**Provable**			
111	**(3*111+1)/2 = 167**	**(3*167+1)/2 = 251**	**(3*251+1)/2 = 377**	**(377-1)/4 = 94**	**Provable**
117	**(117-1)/4 = 29**	**Provable**			
123	**(3*123-1)/2 = 185**	**(185-1)/4 = 46**	**Provable**		
129	**(129-1)/4 = 32**	**Provable**			
135	**(3*135+1)/2 = 203**	**(3*203+1)/2 = 305**	**(305-1)/4 = 76**	**Provable**	
141	**(141-1)/4 = 35**	**Provable**			
147	**(3*147+1)/2 = 221**	**(221-1)/4 = 55**	**Provable**		
153	**(153-1)/4 = 38**	**Provable**			
159	Not Provable				
165	**(165-1)/4 = 41**	**Provable**			
171	**(3*171+1)/2 = 257**	**(257-1)/4 = 64**	**Provable**		
177	**(177-1)/4 = 44**	**Provable**			
183	**(3*183+1)/2 = 275**	**(3*275+1)/2 = 413**	**(413-1)/4 = 103**	**Provable**	
189	**(189-1)/4 = 47**	**Provable**			
195	**(3*195+1)/2 = 293**	**(293-1)/4 = 73**	**Provable**		
201	**(201-1)/4 = 50**	**Provable**			
207	**(3*207+1)/2 = 311**	**(3*311+1)/2 = 467**	**(3*467+1)/2 = 701**	**(701-1)/4 = 175**	**Provable**

As seen in table above (green rows) which account for 50% of the multiples of 3 are immediately provable (x-1) / 4. You should come to realize that this 50% are contained in my level 2 equation. All the even multiples of 3 which I excluded from the above list are with the level 1 equation. Another 25% (yellow) of the multiples of 3 first have to go through 1 iteration of 3x+1 which will immedaitely be reducable because less 1 is divisible by 4. These coincide with my level 3 equation. Another 12.5% (blue) must run through two iterations of 3x+1 before becoming candidates for less 1 divisble by 4 evenly. This is my level 4 equation. And finally another 6.25% after 3 iterations of 3x+1 become evenly divisible 4 after subtracting 1 (purple). These are my level 5 equation. Now that totals to 93.75% of the odd multiples of 3 are provable. If we include the even multiples in the overall calculation it turns out to be 50% + 25% + 12.5% + 6.25% + 3.125% for a total of 96.88% are easily provable by the techniques already outlined above.

Now I list the non-provables in a table where one can note they are separated by 96:

63
159
255
351
447
543
639
735
831
927
1023
1119

What's important to note here are the items I marked red. These are the very first members of my equations for those levels that are multiples of 3. Imagine that. You can see that 127 and 511 which are not multiples of 3 are not included in this list.

Taking a look at my equation that starts with 31 will proceed with the following members 95, **159**, 223, 287, **351**, 415, 479, **543**, 607, 671, **735**, 799, 863, **927**, 991, 1055, **1119.** If we look at another equation starting with 127 we have the following sequence of members 383, **639**, 895, 1151, **1407** ... Notice how the multiples of 3 entries found in the non-multiple of 3 equations in upper levels are found in this above list. This is also the case for those equations that start with multiples of 3...they are ordered differently but each multiple of 3 appears in this list too! Example **63**, 191, 319, **447**, 575, 703, **831**, 959, 1087.

We can safely concluded that all the easily provable multiples of 3 fall in those levels 1 to 5 equations. And that the remainder of those multiples of 3 are found in upper levels and not easily provable; not proven so far.

I've also found another connection that I will point out here (remember the duality of even numbers):

Level starting with 31:

31	95	159	223	287	351
(31-1)/3	(95*2-1)/3	Mult 3	(223-1)/3	(287*2-1)/3	Mult 3
10	**63**		74	**191**	

Level starting with 63. This clearly brings us back to the cascading effect.

63	**191**	**319**	**447**	575	703

Mult 3	(191*2-1)/3	(319-1)/3	Mult 3	(575*2-1)/3	(703-1)/3
	127	106		**383**	234

Level starting with 127:

127	**383**	639	895	1151	1407
(127-1)/3	(383*2-1)/3	Mult 3	(895-1)/3	(1151*2-1)/3	Mult 3
42	**255**		298	767	

Does the same thing as above 3 levels with the second member pointing to the first item of the next level up. There are other observations that I don't think will play a role in the proof. The fifth item points to the second of the next level. I wonder if the eigth item will poin to the 3^{rd} in the next level. Lets check:

Level starting with 255:

415	479	543	607	671	735
(415-1)/3	(479*2-1)/3	Mult 3	(607-1)/3	(671*2-1)/3	Mult 3
138	**319**		202	**447**	

It does and if one continues this the pattern becomes obvious and holds true in all upper levels. I find that very interesting, indeed. You can likely see other connections as I do but nothing that will help me with the multiple of 3 delima I have.

I wouldn't be surprised if we see this same pattern all the way from the level that starts with 3 (3+8x). I'll leave that to you to investigate. I do not believe I need it to prove those lower levels since I already have a method to do just that. And my quick inquiry does indicate it is! There's all kinds of patterns and connectivty.

With some further pondering, I've decided to reconsider the multiples of 3 in a their own light. The do cover $1/3^{rd}$ of the entire natural counting number set. First, if I look at just the multiples of 3 the following chart becomes obvious. These multiples of 3 account for 1/3 of the entire counting number set. Right?

3	6	9	12	15	18	21	24	27	30	33	36	39	(+3)
	3		6		9		12		15		18		(x/2)
		2				5				8			(x-1)/4

Dividing by 2 will eliminating 50% half of these as automatically provable. These coincidentally coincide with my level 1 equation (0+2x). You'll also note that all these are separated by exactly 6; 6+6=12+6=18+6=24. Half of the remaining are divisible by 4 after subtracting 1. That's another 25%. This is my level 2 equation (1+4x). These are separated by 12; 9+12=21+12=33. I might also point out that results all seem to be spaced out by exactly 3. For example, after dividing through by 2 we get 3+3=6+3=9+3=12+3=15+3=18... After doing (x-1)/4 we get 2+3=5+3=8+3=11...

For easier viewing I'm going to elimate that 75% from my next chart as provable.

3	15	27	39	51	63	75	87	99	111	123	135	(+12)
5		41		77		113		149		185		(3x+1)/1
1		10		19		28		37		46		(x-1)/4

You can clearly see that 50% of the remainder are level 3 equation (3+8x) and are separated by 12. As seen elsewhere these can be reduced to provable after one iteration of (3x+1)/2 then apply (x-1)/4. The end row is separated by 9; 1+9=10+9=19+9=28...

Let's redraw the remainder:

15	39	63	87	111	135	159	183	207	231	255	279	(+24)
	59		131		203		275		347		419	(3x+1)/2
	89		197		305		413		521		629	(3x+1)/2
	22		49		76		103		130		157	(x-1)/4

These are the level 4 equation (7+16x) and make another 50% of the remaining provable after running through two cycles of (3x+1)/2 and one cycles of (x-1)/4. The end row is separated by 27. Now 27 may be an interesting coincidence in that starting at that number produces a very long chain. This might prove useful if one wishes to try to determine the length of the chains. I'm not interested in that here.

Redrawing the remainder we get:

15	63	111	159	207	255	303	351	399	447	495	543	(+48)
23		167		311		455		599		743		(3x+1)/2
35		251		467		683		899		1115		(3x+1)/2
53		377		701		1025		1349		1673		(3x+1)/2
13		94		175		256		337		418		(x-1)/4

These are level 5 equation (15+32x) and proves a further 50% of the remainder. This is after 3 cycles of (3x+1)/2 and one of (x-1)/4. Our list is getting pretty small with these first 5 levels removed. Note that the end row is separated by 81. Are you beginning to see a pattern with this spacing as we go higher in my equations to upper levels. Level 2 (1+4x) has them separated by 3; Level 3 (3+8x) has them separated by 9 = (3 * 3). Level 4 (7 + 16x) separated by 27 = (3 * 3 * 3). And level 5 needless to say will be 81 as shown above (3 * 3 * 3 * 3). Cool.

Now we are beginning to step into my realization that is we apply (3x+1)/2 over and over we will reach a point where we hit level 2 after a specific number of iterations... and the final step in that at level 2 we can do the (x-1)/4. Right? That's the cascade I've been pointing out. What I didn't consider is that the resulting number may still be 'even' and further divisible by another (x-1)/4 or simply by 2 or a combination and number of these which results in the final number being smaller than the starting number. So what I am saying is if we end up with a number that is still larger than the starting number and cannot reduce it further with (x-1)/4 or (x/2)...then continue to apply (3x+1)/2 until you can start reducing again. My belief is that no matter the number (multiple of 3) it can be manipulated into provable in short fashion. Let's take the remainder and start a new chart:

63	159	255	351	447	543	639	735	831	927	1023	1119	(+96)
	^		^		^		^		^		^	
	159		351		543		735		927		1119	(+192)

As seen above the same pattern exists for pulling out those entries that are related to the current level we are investigating. In this case they are separated by 192. So let's start a new chart with just those expanded out:

159	351	543	735	927	1119	1311	1503	1695	1887	2079	2271	(+192)
809	1781	2753	3725	4697	5669	6641	7613	8585	9557	10529	11501	4{(3x+1)/2}
202	445	688	**931**	1174	1417	1660	**1903**	2146	2389	2632	**2875**	(x-1)/4
101		344		587		830		1073		1316		(x/2)
25	111				354			268	597			(x-1)/4

		172			177	415		134		658		(x/2)

As can be seen in the above chart every 4th column is not reducable. The other columns through a decernable pattern are reducable well below the starting number. See if you can pick out that pattern yourself... So at this point we have shown that ¾ of the multiples of 3 are provable. Let's pull out those that were not and start yet another sub-chart:

735	1503	2271	3039	3807	4575	5343	6111	6879	7647	8415	9183	(+768)
931	1903	2875	3847	4819	5791	6763	7735	8707	9679	10651	11623	(prelims)
1397	2855	4313	5771	7229	8687	10145	11603	13061	14519	15997	17435	(3x+1)/2
	4283		8657		13031		17505		21779		26153	(3x+1)/2
	6425				19547				32669			(3x+1)/2
349	1606	**1078**	**2164**	**1807**		**2536**	**4376**	**3265**	8167	**3999**	**6538**	(x-1)/4

Continuation of the above chart:

9951	10719	11487	12255	13023	13791	14559	15327	16095	16863	17631	18399	(+768)
12595	13567	14539	15511	16483	17455	18427	19399	20371	21343	22315	23287	(prelims)
18893	20351	21809	23267	24725	26183	27641	29099	30557	32015	33473	34931	(3x+1)/2
	30527		34901		39275		43649		48023		52397	(3x+1)/2
	45791				58913				72035			(3x+1)/2
4723		**5452**	**8725**	**6181**	14728	**6910**	**10912**	**7639**		**8368**	**13099**	(x-1)/4

Again, it appears that every 4th column do not reduce to provable. Let's pick off the remaining that did not reduce to a provable level into a new chart:

1503	4575	7647	10719	13791	16863	19935	23007	26079	29151	32223	35295	(+3072)
6425	19547	32669	45791	58913	72035	85157	98279	111401	124523	137645	150767	(+13122)
1606		8167		14728		21289		27850		34411		(x-1)/4
803				**7364**				**13925**				(x/2)
	29321	12251	68687		108053	31934	147419		186785	51617	226151	(3x+1)/2
						15967						(x/2)
	7330				27013				46696			(x-1)/4
	3665								**23348**			(x/2)
					6753					**12904**		(x-1)/4
		18377	103031				221129				339277	(3x+1)/2
		4594					55282				84819	(x-1)/4
							27641					(x/2)
							6910					(x-1)/4
			154547								127229	(3x+1)/2
											31807	(x-1)/4

Continuation of the above chart:

38367	41439	44511	47583	50655	53727	56799	59871	62943	66015	69087	(+3072)
163889	177011	190133	203255	216377	229499	242621	255743	268865	281987	295109	(+13122)
40972		47533		54094		60655		67216		73777	(x-1)/4
20486				**27047**				**33608**			(x/2)
	265517	71300	304833		344249	90983	383615		422981	110666	(3x+1)/2
		35650								**55333**	(x/2)

66379		86062		105745	(x-1)/4
		43031			(x/2)
	76208			**23436**	(x-1)/4
	38104				(x/2)
99569			136475 575423		(3x+1)/2
24892					(x-1)/4
12446					(x/2)
			204713 863135		(3x+1)/2
			51178		(x-1)/4

Numbers really start reducing in these cycles of (3x+1)/2. We only have every 16^{th} column left unproven. I am pretty certain all the above numbers are correct. Hmmm, there seems to be spreading out, 4 in the other chart; 16 in this chart. 16 = 4*4. Now if I were to place a bet I would safely assume that if I pulled out the leftovers and expanded into a new chart we would find that we would have leftover columns not reducable after every 64^{th} column. And the folowing chart would see leftovers after every 256^{th} column; and the next after every 1024^{th} column; etc. Wow! Interesting indeed. So we see ¼ leftover after first chart; $1/16^{th}$ leftover after the second chart; $1/64^{th}$ after the third; $1/256^{th}$ in the fourth and $1/1024^{th}$ in the 5^{th}. This process continues and shows that with deductive reasoning, an unproven multiple of 3 simply run through another 3 iterations of (3x+1)/2, will likely become provable. Right? Maybe not. So I worked outside this report in a spreadsheet to prove that is the case. That was an involved process indeed and placing the results in this report would make it far to long. What I found out is that the very next chart with the leftovers expanded will result in a mimimum of $1/64^{th}$ but it could be much less like $1/128^{th}$; I decided against trying to find the exact number; the minimum of $1/64^{th}$ fits my theory but I had to run the process through 3 iterations of 3 iterations to reach only $1/64^{th}$ remaining. That's running through 9 iterations of (3x+1)/2. I wonder if this has something to do with 2*2 and 3*3. I believe that '3' is important in understanding what is going on but I don't think I need it for the proof. I was able to show that we approach 100% reducable with more interations of (3x+1)/2. Anyways, I'll archive that spreadsheet or find a way to place it as an appendix to this report.

It would appear that each time I do a set of (3x+1)/2, I reduce the remaining set by ¾ leaving only a quarter. In the next set of (3x+1)/2 I reduce the remaining set to $1/16^{th}$. And the next the remaining is reduced to just $1/64^{th}$;... So we have a situation where as we approach an infinite number of (3x+1)/2 iterations we reduce the set to very, very, very, very, very tiny. For all intents and purpose we have proven all these multiples of 3? We would have to map out many more numbers in the above chart to show this clearly; that is why I am clearly pointing out this observation. At this state of the charting it appears to be what is going on.

So my idea almost played out in that we could apply further (x-1)/4 or x/2 to reduce to make provable in ¾ of the cases. As I've shown, if we apply that last quarter (¼) through multiple iterations of (3x+1)/2 it then becomes divisible by 4 after subtracting 1. That's another ¾ easily proven. That leaves a quarter of a quarter to prove. It appears that if given enough iterations of (3x+1)/2 one can reduce any multiple of three to an inductive state! Some of these multiples of 3 are going to consume a very large number of iterations as you can imagine; almost enough to consider it a runaway growth cycle. But, if you will notice there is again a decernable pattern to all this madness. So even if you do not want to take that last step to having them all provable...you can accept that ¾ of with an additional $15/16^{th}$ of that final ¼ are easily provable. 75% + 23.4375% = 98.4375% total. For level 6 (31+64x) we easily show that 66% are provable leaving only multiples of 3. Above we have shown that 98.4375% of those multiples of 3 are also easily proven. The remainder are a little questionable. So that works out to 66.66666% + 98.4375% of 33.3333333% = 66.66667% + 32.81% = 99.48%. So level 6 has 1.5625% of the natural numbers...with 99.48% of them easily provable... 1.554%. I'll do the next level 7 immediately to show this concept holds in upper levels. If you follow my above reasoning and agreee with the mathematics displayed you will notice that we can prove ¾ leaving ¼; of that ¼ we can prove $15/16^{th}$ of that leaving just 1/16 to prove; one more itteration set and we prove 63/64 leaving $1/64^{th}$ unproven. Can you see that this is approaching 100% provable after a finite number of steps? Now, a little more statistics (with just shows:

Level 1 (0+2x) is 100% provable for 50% of natural counting number set		50%
Level 2 (1+4x) is 100% provable for 25% of natural counting number set		25%
Level 3 (3+8x) is 100% provable for 12.5% of natural counting number set		12.5%
Level 4 (7+16x) is 100% provable for 6.25% of natural counting number set		6.25%
Level 5 (15+32x) is 100% provable for 3.125% of natural counting number set		3.125%
Level 6 (31+64x) is 99.48% provable for 1.5625% of natural counting number set		1.554%
Level 7 (63+128x) is 99.48% provable for 0.78125% of natural counting number set		0.777%
Level 8 (127+256x) is 99.48%	0.3906%	0.3886%
Level 9 (255+512x) is 99.48%	0.1953%	0.1943%
Level 10 (511+1024x) is 99.48%	0.0977%	0.0972%

For a grand total of 99.886% easily provable! That's 0.11% not so easily provable but I do believe I was able to show that they are as well. Do you agree that if I put the full observable from above this number approaches 100% provable multiples of 3.

63	255	447	639	831	1023	1215	1407	1599	1791	1983	2175	(+192)
95		671		1247		1823		2399		2975		(3x+1)/2
143		1007		1871		2735		3599		4463		(3x+1)/2
215		1511		2807		4103		5399		6695		(3x+1)/2
323		2267		4211		6155		8099		10043		(3x+1)/2
485		3401		6317		9233		12149		15065		(3x+1)/2
121		850		1579		2308		3037		3766		(x-1)/4
		425				**1154**				**1883**		(x/2)
30		106						**759**				(x-1)/4
15		53				577						(x/2)
				2369								(3x+1)/2
				592		144						(x-1)/4
												(3x+1)/2
												(3x+1)/2
		13										(x-1)/4
		3										(x-1)/4
						72						(x/2)

Continuation of above chart to show same patterns...

2367	2559	2751	2943	3135	3327	3519	3711	3903	4095	4287		(+192)
3551		4127		4703		5279		5855		6431		(3x+1)/2
5327		6191		7055		7919		8783		9647		(3x+1)/2
7991		9287		10583		11879		13175		14471		(3x+1)/2
11987		13931		15875		17819		19763		21707		(3x+1)/2
17981		20897		23813		26729		29645		32561		(3x+1)/2
4495		5224		5953		6682		7411		8140		(x-1)/4
		2612				**3341**				**4070**		(x/2)
				1488		835						(x-1)/4
		1306		744						2035		(x/2)
6743								11117				(3x+1)/2
								2779				(x-1)/4

10115		(3x+1)/2
15173		(3x+1)/2
3793		(x-1)/4
948		(x-1)/4
464	372	(x/2)

This is getting somewhat invloved. I hope you can appreciate that if you end up with unprovables simply pass them through (x-1)/4 and (x/2) as many times as needed to reduce to odd and if the number is still larger than the start number apply (3x+1)/2 however many times to get it reducable once again using (x-1)/4 and (x/2). As seen in the above detailed work with level 6 the exact same trends hold in this level. I didn't go into as great detail; just enough to show this was the case. It is. So 75% are easily provable with 93.75% of the remaing quarter also easily provable, and so on...with 98.4375% of that remaining 1/16th also provable...

I spoke about this aspect in an upper section where I believed that if you apply (3x+1)/2 three times in a row you make it possible to extract (x/2) and/or (x-1)/4 a number of times. At that time I wasn't clear how it worked in Collatz...but now it is becoming very clear. You can see it is a little involved but the basic premis is there.

After having done all the work above I made a discovery that really simplifies proving all multiples of 3, whether they be even or odd. I think you're going to enjoy this piece since it is so obvious after having done all the other research. I'm going to start by putting together several charts I mastered last night:

3 → ((3*3)+1)/2=5; ((3*5)+1)/2=8; 8/2=4; (4-1)/3=1 (sequence starting 3; separation 24)
6 → 6/2=3 (sequence starting 0; separation 3)
9 → ((3*9)+1)/2=14; 14/2=7 (sequence starting 9; separation 12)
12 → 12/2=6
15
18 → 18/2=9
21 → ((3*21)+1)/2=32; 32/2=16
24 → 24/2=12
27 → ((3*27)+1)/2=41; ((3*41)+1)/2=62; 62/2=31; (31-1)/3=10
30 → 30/2=15
33 → ((3*33)+1)/2=50; 50/2=25
36 → 36/2=18
39
42 → 42/2=21
45 → ((3*45)+1)/2=68; 68/2=34
48 → 48/2=24
51 → ((3*51)+1)/2=77; ((3*77)+1)/2=116; 116/2=58; (58-1)/3=19
54 → 54/2=27
57 → ((3*57)+1)/2=86; 86/2=43
60 → 60/2=30
63
66 → 66/2=33
69 → ((3*69)+1)/2=104; 104/2=52
72 → 72/2=36
75 → ((3*75)+1)/2=113; ((3*113)+1)/2=170; 170/2=85; (85-1)/3=28
78 → 78/2=39
81 → ((3*81)+1)/2=122; 122/2=61
84 → 84/2=42

87
90 → 90/2=45
93 → ((3*93)+1)/2=140; 140/2=70
96 → 96/2=48
99 → ((3*99)+1)/2=149; ((3*149)+1)/2=224; 224/2=112; (112-1)/3=37
102 → 102/2=51
105 → ((3*105)+1)/2=158; 158/2=79
108 → 108/2=54
111
114 → 114/2=57
117 → ((3*117)+1)/2=176; 176/2=88

Starting another chart with the left overs from above chart:

15 → ((3*15)+1)/2=23; ((3*23)+1)/2=35; ((3*35)+1)/2=53; ((3*53)+1)/2=80; 80/2=40; (40-1)/3=13
39 → ((3*39)+1)/2=59; 89; 134; 134/2=67; (67-1)/3=22 (2nd column chart 1)
63 →
87 → ((3*87)+1)/2=131; 197; 296; 296/2=148; (148-1)/3=49
111 → ((3*111)+1)/2=167; 251; 377; 566; 566/2=283; (283-1)/3=94
135 → ((3*135)+1)/2=203; 305; 458; 458/2=229; (229-1)/3=76
159 →
183 → ((3*183)+1)/2=275; 413; 620; 620/2=310; (310-1)/3=103
207 → ((3*207)+1)/2=311; 467; 701; 1052; 1052/2=526; (526-1)/3=175
231 → ((3*231)+1)/2=347; 521; 782; 782/2=391; (391-1)/3=130
255 →
279 → ((3*279)+1)/2=419; 629; 944; 944/2=472; (472-1)/3=157
303 → ((3*303)+1)/2=455; 683; 1025; 1538; 1538/2=769; (769-1)/3=256
327 → ((3*327)+1)/2=491; 737; 1106; 1106/2=553; (553-1)/3=184
351 →
375 → ((3*375)+1)/2=563; 845; 1268; 1268/2=634; (634-1)/3=211
399 → ((3*399)+1)/2=599; 899; 1349; 2024; 2024/2=1012; (1012-1)/3=337
423 → ((3*423)+1)/2=635; 953; 1430; 1430/2=715; (715-1)/3=238
447 →
471 → ((3*471)+1)/2=707; 1061; 1592; 1592/2=796; (796-1)/3=265
495 → ((3*495)+1)/2=743; 1115; 1673; 2510; 2510/2=1255; (1255-1)/3=418
519 → ((3*519)+1)/2=779; 1169; 1754; 1754/2=877; (877-1)/3=292
543 →

As you can see from the above chart reflected in the two charts below; the next sequence in each will immediately reduce through existing columns.And all the remaining upper sequences do the exact same thing so I will not bore you with more charting. The two following charts puts it in a compact easy to understand package.

From the above chart you can see the multiples of 3 form two distinct charts below. I've simplified that in the quick table right below:

2-1	3*1=3	(2nd chart)	→	0 ←	(first starting sequence for 2nd chart)
1-1	3*4=12	(1st chart)	→	9 ← 10	(first starting sequence for 1st chart)
2-2	3*8=24	(2nd chart)	→	3	(2nd starting sequence for 2nd chart)
1-2	3*16=48	(1st chart)	→	39 ← 40	(2nd starting sequence for 1st chart)

2-3	3*32=96	(2^{nd} chart)	→	15	(3^{rd} starting sequence for 2^{nd} chart)
1-3	3*64=192	(1^{st} chart)	→	159 ← 160	(3^{rd} starting sequence for 1^{st} chart)
2-4	3*128=384	(2^{nd} chart)	→	63	(4^{th} starting sequence for 2^{nd} chart)
1-4	3*256=768	(1^{st} chart)	→	639 ← 640	(4^{th} starting sequence for 1^{st} chart)
2-5	3*512=1536	(2^{nd} chart)	→	255	(5^{th} starting sequence for 2^{nd} chart)
1-5	3*1024=3072	(1^{st} chart)	→	2559 ← 2560	(5^{th} starting sequence for 1^{st} chart)

It may not be obvious from the first sequence(s) in each chart but they have items/members that are automatically provable. In the case of the second chart the first two sequences fit that bill. I've highlighted one in red, one in green and the other in blue (above). You can also see that the even multiples of 3 are being accounted for in the first column of the second chart – that's because they are also easily proven by simply dividing by 2. Right? Just in case that doesn't work for you you'll find that all the even multiples of 3 are found in the second chart in the A column. The double lettered columns in each chart are simply the sequences (first half) with the last half being the result of multiple (3x+1)/2 and x/2 until final (x-1)/3 possible. For example take 3; ((3*3)+1)/2)=5; ((3*5)+1)/2=8; 8/2=4; (4-1)/3=1. Using this feature you'll notice that there is a cascade of all multiples of 3 through to the first columns which are provable so they are all provable too. Right?

A quick explanation of the following charts in case you didn't immediately see it. The single letter columns are arranged so that each column up (to the right) is simply 3x+1

A	B	C	D	E	F	G	H/I	J/K	L/M	N/O
3	9	27	81	243	729	2187	12/3	48/27	192/243	768/2187
2	7	**22**	67	**202**	607	**1822**	9/**2**	39/**22**	159/**202**	639/**1822**
5	16	49	148	445	1336	4009	21/5	87/49	351/445	1407/4009
8	25	76	229	688	2065	6196	33/8	135/76	543/688	2175/6196
11	34	103	310	931	2794	8383	45/11	183/103	735/931	2943/8383
14	43	130	391	1174	3523	10570	57/14	231/130	927/1174	3711/10570
17	52	157	472	1417	4252	12757	69/17	279/157	1119/1417	4479/12757
20	61	184	553	1660	4981	14944	81/20	327/184	1311/1660	5247/14944
23	70	211	634	1903	5710	17131	93/23	375/211	1503/1903	6015/17131
26	79	238	715	2146	6439	19318	105/26	423/238	1695/2146	6783/19318
29	88	265	796	2389	7168	21505	117/29	471/265	1887/2389	7551/21505
32	97	292	877	2632	7897	23692	129/32	519/292	2079/2632	8319/23692
35	106	319	958	2875	8626	25879	141/35	567/319	2271/2875	9087/25879
38	115	346	1039	3118	9355	28066	153/38	615/346	2463/3118	9855/28066
41	124	373	1120	3361	10084	30253	165/41	663/373	2655/3361	10623/30253
44	133	400	1201	3604	10813	32440	177/44	711/400	2847/3604	11391/32440
47	142	427	1282	3847	11542	34627	189/47	759/427	3039/3847	12159/34627
50	151	454	1363	4090	12271	36814	201/50	807/454	3231/4090	12927/36814
53	160	481	1444	4333	13000	39001	213/53	855/481	3423/4333	13695/39001
56	169	508	1525	4576	13729	41188	225/56	903/508	3615/4576	14463/41188
59	178	535	1606	4819	14458	43375	237/59	951/535	3807/4819	15231/43375
62	187	562	1687	5062	15187	45562	249/62	999/562	3999/5062	15999/45562
65	196	589	1768	5305	15916	47749	261/65	1047/589	4191/5305	16767/47749
68	205	616	1849	5548	16645	49936	273/68	1095/616	4383/5548	17535/49936
71	214	643	1930	5791	17374	52123	285/71	1143/643	4575/5791	18303/52123
74	223	670	2011	6034	18103	54310	297/74	1191/670	4767/6034	19071/54310
77	232	697	2092	6277	18832	56497	309/77	1239/697	4959/6277	19839/56497

A	B	C	D	E	F	G	H	I/J	K/L	M/N	O/P
3	9	27	81	243	729	2187	6561	24/9	96/81	384/729	1536/6561

0	**1**	4	**13**	40	**121**	364	**1093**	3/**1**	15/**13**	63/**121**	255/**1093**
3	10	31	94	283	850	2551	7654	27/10	111/94	447/850	1791/7654
6	19	58	175	526	1579	4738	14215	51/19	207/175	831/1579	3327/14215
9	28	85	256	769	2308	6925	20776	75/28	303/256	1215/2308	4863/20776
12	37	112	337	1012	3037	9112	27337	99/37	399/337	1599/3037	6399/27337
15	46	139	418	1255	3766	11299	33898	123/46	495/418	1983/3766	7935/33898
18	55	166	499	1498	4495	13486	40459	147/55	591/499	2367/4495	9471/40459
21	64	193	580	1741	5224	15673	47020	171/64	687/580	2751/5224	11007/47020
24	73	220	661	1984	5953	17860	53581	195/73	783/661	3135/5953	12543/53581
27	82	247	742	2227	6682	20047	60142	219/82	879/742	3519/6682	14079/60142
30	91	274	823	2470	7411	22234	66703	243/91	975/823	3903/7411	15615/66703
33	100	301	904	2713	8140	24421	73264	267/100	1071/904	4287/8140	17151/73264
36	109	328	985	2956	8869	26608	79825	291/109	1167/985	4671/8869	18687/79825
39	118	355	1066	3199	9598	28795	86386	315/118	1263/1066	5055/9598	20223/86386
42	127	382	1147	3442	10327	30982	92947	339/127	1359/1147	5439/10327	21759/92947
45	136	409	1228	3685	11056	33169	99508	363/136	1455/1228	5823/11056	23295/99508
48	145	436	1309	3928	11785	35356	106069	387/145	1551/1309	6207/11785	24831/106069
51	154	463	1390	4171	12514	37543	112630	411/154	1647/1390	6591/12514	26367/112630
54	163	490	1471	4414	13243	39730	119191	435/163	1743/1471	6975/13243	27903/119191
57	172	517	1552	4657	13972	41917	125752	459/172	1839/1552	7359/13972	29439/125752
60	181	544	1633	4900	14701	44104	132313	483/181	1935/1633	7743/14701	30975/132313
63	190	571	1714	5143	15430	46291	138874	507/190	2031/1714	8127/15430	32511/138874
66	199	598	1795	5386	16159	48478	145435	531/199	2127/1795	8511/16159	34047/145435
69	208	625	1876	5629	16888	50665	151996	555/208	2223/1876	8895/16888	35583/151996
72	217	652	1957	5872	17617	52852	158557	579/217	2319/1957	9279/17617	37119/158557
75	226	679	2038	6115	18346	55039	165118	603/226	2415/2038	9663/18346	38655/165118

Note that duality plays an important for even numbers that show up in the above charts and make this all possible. I should also mention that we can likely create a another two sets of equations something like my original ones that may be useful in expanding upon this proof. Those new equations would look very similar to mine. I, without realizing it in previous work had stumbled across this without realizing it's full potential. We may even be able to make similar charts for the other two subsets... multiples of 3 minus 1; and multiples of 3 minus two and use those in a proof. I'll leave that up to the reader to explore.

Let's do a visual to fix this idea in place so that you can easily agree with the concept as all encompassing for these multiples of 3.

```
607 – 1214 – 2428 …
 |              |
202     ←      809 – 1618 …
 |                    |
 67                  539 – 1078 …
 |                          |
 22                        359 – 718 …
 |                               |
 7                              239 – 478 …
 |                                     |
 2                                    159 – 318 ...
```

I believe you see it clearly now. This is the case and concept for all multiples of 3. 159 quickly reduces to a much smaller number than the starting 159.

So, we were left with a subset of multiples of 3 we could not easily prove with other explored methods previously explored and the route I orininally took became far too cumbersome to use for this proof. I left it there as a precursor to why I went this route. The above discussion exclusively dedicated to multiples of 3 shows that ALL are provable through simple induction because of the cascades through the two charts. The column headers have numbers immediately below them which indicate the separation of the sequence elements following in those columns. There are very nice patterns there. So having said that the remainder of outstanding multiples of 3 are previously proven if we consider the charts above and hence the proof is COMPLETE.

Could this be the elusive proof for the remainder of the multiples of 3 I could not prove with the other above methods? The end result is once again induction where the end number is less than the starting number and thus in 1 to k.

I am not going to go any deeper with the above levels because the numbers are going to get scary large quickly. I just wanted to get the concept across. With each additional level we halve the number of elements remain and achieve an amazing 100% provable. Actually we were 'approaching' 100% provable. I did not believe I could get any closer to proving this conjecture, but as you saw above, once I reconsidered the multiples of 3 in it's own subset the proof became obvious and 100% achievable.

It is not worth investigating here but I do wonder if what I last did to prove that small subset of multiples of 3 can also be used for any number as a more complicated way to a proof. I have a feeling it can. It may be worth investigating at some future date.

I believe this is the best part of what is required for a proof! Now to put it in a more 'proofy' format. Or can this be considered the proof?

<u>Section 13 - Conclusion</u>

With all the above discussion I have concluded that the original conjecture holds true for all positive counting numbers; 100% of them proven.

I am not a mathematician so my technical terminology leaves a lot to be desired. But I hope I have successfully made my case.

I believe it is possible for others to simplify or improve upon my concepts, but please do give me the due credit for my research. It may now be possible to compute the length and largest number reached in the chains. This would be something worth looking into at some future date.

It has been a joy working on this 'unsolvable' problem. It's not so unsolvable anymore!

One of my biggest issues worth mentioning is the disemination or dispersal of information into the real world. I have these insights but no real way to share them. I now have what I consider to be a complete proof with no one to show it to. I must resort to pre-print sites where my information will likely only be shared with the world at an expense to me. I have never expected to make any money from this and will never sign over my rights to this discovery nor spend my own money to have it published in a journal. That belittles the scholarly nature of the work and makes it appear that I bought that recognition. I leave it to my piers to review my work voluntarily. There should be a lineup. I believe it is worth their effort since it puts an end to nearly 100 years of searching for a proof. This can now be called the 'Collatz Theorem' with my humble contribution to mathematics. All I seek is the recognition and the fame; maybe a Nobel Prize in Mathematics...I can hope...