About the strong EULER-GOLDBACH conjecture.

PHILIPPE SAINTY

University Pierre et Marie Curie Paris, France.

July 2022

Abstract.

In this article, we define a " recursive local" algorithm in order to construct two reccurent numerical sequences of positive prime numbers (U_{2n}) and (V_{2n}), $((U_{2n})$ function of $(V_{2n}))$, such that for any integer n≥ 2, their sum is 2n. To build these , we use a third sequence of prime numbers (W_{2n}) defined for any integer n≥ 3 by : $W_{2n} = \text{Sup}(p \in IP : p \leq 2n\text{-}3)$, where IP is the infinite set of positive prime numbers. The Goldbach conjecture has been verified for all even integers 2n between 4 and $4.10^{18.}$. In the Table of Goldbach sequence terms given in paragraph § 10, we reach values of the order of 2n= 10^{1000}. Thus, thanks to this algorithm of "ascent and descent", we can validate the strong Euler-Goldbach conjecture.

Keywords.
Prime numbers, Prime Number Theorem, weak and strong Goldbach's conjectures, Dirichlet's theorem, Bertrand-Tchebychev theorem, gaps between prime numbers.

CONTENTS.

§ 1 Preliminaries and history.

Number theory, "the Queen of Mathematics" deals with structures and properties defined over integers (see Euclide [5], Hardy & Wright [7] and Landau [10]). This theory is built from the notion of prime number. Many problems have been raised and multiple conjectures with statements that are often simple but very difficult to demonstrate have been made. These main components include:

a)   Elementary arithmetic.
     Determination and properties of prime numbers.

1

Operations on integers (basic operations, congruence, gcd, lcm, and so on…….).

Representations and numbers bases of integers.

Decomposition of integers into product or sum of prime numbers (Fundamental theorem of arithmetic, decomposition of large numbers, cryptography and Goldbach's conjecture).

b) Analytic number theory.

(Riemann Hypothesis). Distribution of prime numbers (Prime Number Theorem, see Hadamard [6], De la Vallée-Poussin [18], Littlewood [11] and Erdos [4]), gaps between prime numbers (see Bombieri [1], Cramer [3], Iwaniec & Pintz [9] and Tchebychev [17]).

c) Algebraic, probabilistic, combinatorics and algorithmic theories of numbers.

d) Modular arithmetic, diophantine approximations, equations and geometry. Arithmetic sequences, functions, and algebraic geometry.

As part of research on the Goldbach conjecture, Chen [2], Hegfollt [8], Ramaré [12], Tchebychev [17], Vinogradov [19], Tao [16] and Zhang [20] have taken essential steps and obtained very promising results for its demonstration. In effect, Helfgott & Platt [8] proved in 2013 Goldbach's weak conjecture. Silva, Herzog & Pardi [15] held the record for calculating the terms of Goldbach sequences after having determined, pairs of positive prime numbers $(p_n; q_n)$ verifying : $(p_n + q_n = 2n)$ for any integer n such that: $(4 \leq 2n \leq 4.10^{18})$. In the scientific literature, one does not find explicit constructions of reccurent sequences of positive Goldbach primes $(G_{2n}) = (U_{2n}; V_{2n})$, satisfying for any integer n ≥ 2 the equality: $(U_{2n} + V_{2n} = 2n)$. In this article, we define two sequences of prime numbers, using a "local recursive" algorithm simple and efficient allowing to calculate for any integer n≥ 3, by successive iterations any term U_{2n} and V_{2n} of a Goldbach sequence. Using the scientific calculation software Maxima on a personal computer, one easily exceed Silva's record [15] set for $2n = 4.10^{18}$ and we can reach $2n = 10^{500}$ and even $2n = 10^{1000}$.

§ 2 Definitions and notations.

a) The integers n, k, p, q, r,…..and so on…. are always positive.

b) We denote by IP the infinite set of positive prime numbers (called simply prime numbers):

$IP = \{ p_k, (k \in IN^*) : p_k$ is the kth-positive prime number; $(p_k < p_{k+1}$ and $\lim p_n = +\infty \}$

$(p_1 = 2 ; p_2 = 3 ; p_3 = 5 ; p_4 = 7 ; p5 = 11 ; p_6 = 13; ……….).$

To simplify the writing of large numbers, we adopt the following notations .

c) $M = 10^9$

d) $R = 4.10^{18}$

e) $G = 10^{100}$

f) $S = 10^{500}$

g) $T = 10^{1000}$

h) log(x) denotes the decimal logarithm of the strictly positive real x : (x> 0).

i) We call Goldbach sequence any numerical sequence denoted by $(G_{2n}) = (U_{2n}; V_{2n})$ verifying:

For any integer n≥ 2, U_{2n} and V_{2n} are prime numbers and $U_{2n} + V_{2n} = 2n$.

§ 3 Introduction.

Let (W_{2n}) be the sequence of prime numbers defined by :

For any integer n ≥ 3,

(1) $W_{2n} = Sup(p \in IP : p \leq 2n-3)$

Using case disjunction reasoning, we construct two recurrent sequences of prime numbers (V_{2n}) and (U_{2n}) as a function of the sequence (W_{2n}) by the following process.

(2) $$(V_4=2; U_4=2)$$

Let n be an integer : $(n \geq 3)$.

1) Either,

$(2n - W_{2n})$ is a prime number and V_{2n} and U_{2n} are defined directly in terms of W_{2n}.

2) Either,

$(2n - W_{2n})$ is a composite number and V_{2n} and U_{2n} are defined from previous terms of the sequence (G_{2n}).

§ 4 Theorem.

4.1 There exists a recurrent numerical Goldbach sequence $(G_{2n}) = (U_{2n}, V_{2n})$ such that for any integer $n \geq 2$, we have : U_{2n} and V_{2n} are prime numbers and their sum is 2n.

(3) $$(\ U_{2n}, V_{2n} \ \epsilon \ \text{IP} \ \text{ and } \ U_{2n} + V_{2n} = 2n \ )$$

4.2 An algorithm makes it possible to explicitly calculate any term U_{2n} and V_{2n}.

§ 5 Methodology.

A method similar to that of the sieve is used, (see Selberg [14]). For this purpose, we build a positive sequence of prime numbers (V_{2n}) satisfying: $\lim V_{2n} = +\infty$, as a function of $W_{2n} = \text{Sup}(p \in \text{IP} : p \leq 2n-3)$, ((W_{2n}) is an increasing sequence who contains all prime numbers except two, and verifies: $\lim W_{2n} = +\infty$), and a complementary sequence of prime numbers (U_{2n}) negligible with respect to (2n) for very large integers n, i.e for any large enough integer n , we have :

(4) $$\text{Sup}(U_{2k} : (k \in \text{IN} : 3 \leq k \leq n)) \leq K_1(n).\log(2n).$$
$$(K_1(n) > 0; K_1(n) \text{ quasi-constant and hyper slowly increasing function of n}).$$

For any integer $n \geq 3$,

If $(2n - W_{2n})$ is a prime number "particular case", then we put:

(5) $$V_{2n} = W_{2n} \quad \text{and} \ U_{2n} = 2n - W_{2n}$$

Else, if $(2n - W_{2n})$ is a composite number "general case" ,

we return to the sequence (G_{2n}) of previous terms, already calculated. Therefore, we are looking for an integer k in order to obtain two terms $U_{2(n-k)}$ and $V_{2(n-k)}$ satisfying the following conditions :

(6) $$U_{2(n-k)}, V_{2(n-k)} \text{ and } U_{2(n-k)} + 2k \text{ are prime numbers}$$
$$U_{2(n-k)} + V_{2(n=k)} = 2(n-k)$$

(which is always possible ; see proof in paragraph § 6). Thus , by putting :

(7) $$V_{2n} = V_{2(n-k)} \ and \ U_{2n} = U_{2(n-k)} + 2k$$

we obtain two new prime numbers satisfying :

(8) $$U_{2n} + V_{2n} = 2n .$$

We can thus reiterate this process by incrementing n by one unit : $(n \rightarrow n+1)$.

§ 6 Proof.

For any integer q such that : $(1 \leq q \leq n-3)$, we have : $3 \leq U_{2(n-q)} \leq n$.

For any integer k such that $(2 \leq 2k \leq (n-1)/2)$, there exists two prime numbers p_m and p_r, $(m>r)$ in the interval $[2;n]$ such that :

$$(9) \qquad\qquad p_m - p_r = 2k$$

(see Bombieri [1], Cramer [3], Iwaniec & Pintz [9] and Tchebychev [17]).
Then, there exists an integer k , $(2 \leq 2k \leq n-3)$ such that :

$$(10) \qquad\qquad R_{2n} = U_{2(n-k)} + 2k \quad \text{is a prime number}$$

We choose the smallest integer k denoted by k_n such that R_{2n} is a prime number. Then we put :

$$(11) \qquad\qquad U_{2n} = U_{2(n-k_n)} + 2k_n \text{ and } V_{2n} = V_{2(n-k_n)}$$

$$\text{(These two terms are prime numbers)}$$

We have constructed in the previous steps two prime numbers , $U_{2(n-k_n)}$ and $V_{2(n-k_n)}$ whose sum is equal to $2(n-k_n)$:

$$(12) \qquad\qquad U_{2(n-k_n)} + V_{2(n-k_n)} = 2(n-k_n)$$

Thus , by adding the term k_n on each member of the equality (12), we obtain :

$$(13) \qquad\qquad U_{2(n-k_n)} + 2k_n + V_{2(n-k_n)} = 2(n-k_n) + 2k_n$$

$$(14) \quad \Leftrightarrow \qquad\qquad \{ U_{2(n-k_n)} + 2k_n\} + V_{2(n-k_n)} = 2n$$

$$(15) \quad \Leftrightarrow \qquad\qquad U_{2n} + V_{2n} = 2n$$

Finally, this algorithm constructs for any integer $n \geq 3$ two sequences of prime numbers (U_{2n}) and (V_{2n}) verifying Goldbach's conjecture .

§ 7 Corollary.

For any integer $n \geq 3$, we easily verify :

7.1 (W_{2n}) is a positive increasing sequence of prime numbers.

7.2 $\lim W_{2n} = +\infty$

7.3 $n < W_{2n} \leq 2n-3$

7.4 $3 \leq 2n - W_{2n} \leq U_{2n} \leq n$

7.5 $n \leq V_{2n} \leq W_{2n}$

7.6 $\lim V_{2n} = +\infty$

7.7 $\{ W_{2n} : n \in \text{IN}^* \} \cup \{2\} = \text{IP}$

§ 8 Remarks.

4

8.1   There is an infinity of integers n such that U_{2n} = 3 , 5 , 7 or 11.

8.2 $V_{2n} \sim$ (2n), for (n → +oo).

8.3   For any integer n large enough, $U_{2n} \ll V_{2n}$ and $\lim(\frac{U_{2n}}{V_{2n}})$=0.

8.4 $Sup(U_{2k} : (2 \leq k \leq n)) \leq$ K1(n).log(2n) :   (K1(n)>0; K1(n) $\sim cte$).

8.5   The first integer n such that : $U_{2n} \neq$ 2n -W_{2n} is obtained for n=49 and G_{98}= (79;19).
       (This type of term "densifies" in the Goldbach sequence (G_{2n}) when n increases, in the sense
       Of Schnirelmann [13] and there is an infinity of them; we can calculate their proportion
       by interval).

8.6   Let q$\geq$ 5 be an odd integer, then we could generalize this process with sequences
       (W'_{2n}) = Sup(p∈IP : p $\leq$ 2n-q) and we obtain other sequences (G'_{2n}) of Goldbach sequences independent
       of (G_{2n}).

8.7   The sequence (G_{2n}) is extremal in the sense that for any given integer n$\geq$ 2 , V_{2n} and U_{2n} are the greatest
and the smallest possible prime numbers such that: $U_{2n} + V_{2n}$ = 2n.

§ 9 Algorithm.

9.1 Algorithm written in natural language.

Input three variables integers: N, n and P.

Input : p_1=2 , p_2=3 , p3=5 , p_4 = 7 , ..., p_N , the first N prime numbers

      : n=3.

      : P=M, R, G, S, or T given in paragraph § 2.

Start :

Algorithm body :

A) Calculate : W_{2n} = Sup(p∈ IP : p $\leq$ 2n-3)

If T_{2n}= (2n − W_{2n}) is a prime number, then we put:

    (16) $U_{2n} = T_{2n}$ and $V_{2n} = W_{2n}$

else ,

B) If T_{2n} is a composite number, then let k be an integer ; we put : k=1.
      B.1) while : $U_{2(n-k)}$ + 2k is a composite number,
                      assign to k the value k+1
                      return to B1)

end while .

We put : k_n= k and

(17) $U_{2n} = U_{2(n-k_n)} + 2k_n$ and $V_{2n} = V_{2(n-k_n)}$

Assign to n the value n+1 , (n→ $n + 1$ and returh to A)

End

Output for integers less than 10^4: Print (2n = … ; 2n-3 = … ; W_{2n} = … ; T_{2n} = … ; V_{2n} = … ; U_{2n} = …).

Output for large integers: Print (2n-P = … ; 2n-3-P= … ; W_{2n}-P = … ; T_{2n} = …. ; $V_{2n} − P$ = ⋯ ; U_{2n} = …).

9.2 Algorithm written with the language of the Maxima scientific computing software.

r:0 ; n1:10**500 ; for n:5*10**499+10000 thru 5*10**499+10010 do
(k:1 , a:2*n , c:a-3 , test:0 , b:prev_prime(a-1) ,
if primep(a-b)

5

then print(a-n1,c-n1,b-n1,a-b,b-n1,a-b)
else (r:r+1 ,
 while test=0 do
 (if (primep(c) and primep(a-c))
then (test:1 , print(a-n1,a-n1-3,b-n1,a-b,c-n1,a-c," Ret ",r))
else (test:0 , c:c-2*k))));

§ 10 Appendices .
Application of the algorithm (paragraph § 9) : Table of Goldbach's sequences (U_{2n}) and (V_{2n}) calculated by the Maxima program written in paragraph § 9.2.
The abbreviation Ret) listed in the Table below indicates the results given by the algorithm in the case
B) of return to the previous terms of the sequence (G_{2n}). WATCH OUT ! For large integers n,
$(2n > 10^9$, for instance), in order to simplify the writing of big numbers, we write the results in the
form: $2n$-P , $(2n-3)$-P , W_{2n}-P , T_{2n} , V_{2n}-P and U_{2n} with P=M, R, G, S, or T given in paragraph § 2.

2n	I	2n-3	W_{2n}	$T_{2n} = 2n - W_{2n}$	V_{2n}	U_{2n}
4		1			2	2
6		3	3	3	3	3
8		5	5	3	5	3
10		7	7	3	7	3
12		9	7	5	7	5
14		11	11	3	11	3
16		13	13	3	13	3
18		15	13	5	13	5
20		17	17	3	17	3
22		19	19	3	19	3
24		21	19	5	19	5
26		23	23	3	23	3
28		25	23	5	23	5
30		27	23	7	23	7
32		29	29	3	29	3
34		31	31	3	31	3
36		33	31	5	31	5
38		35	31	7	31	7
40		37	37	3	37	3
And so on		....	....	....	....	....

70	67	67	3	67	3
72	69	67	5	67	5
74	71	71	3	71	3
76	73	73	3	73	3
78	75	73	5	73	5
80	77	73	7	73	7
82	79	79	3	79	3
84	81	79	5	79	5
86	83	83	3	83	3
88	85	83	5	83	5
90	87	83	7	83	7
92	89	89	3	89	3
94	91	89	5	89	5
96	93	89	7	89	7
R1)					
98	95	89	9	79	19
100	97	97	3	97	3
And so on..........	....	....	....	....	....

120	117	113	7	113	7
R2)					
122	119	113	9	109	13
124	121	113	11	113	11
126	123	113	13	113	13
R3)					
128	125	113	15	109	19
130	127	127	3	127	3
132	129	127	5	127	5
134	131	131	3	131	3
136	133	131	5	131	5
138	135	131	7	131	7
140	137	137	3	137	3

142	139	139	3	139	3
144	141	139	5	139	5
146	143	139	7	139	7
R4)					
148	145	139	9	137	11
150	147	139	11	139	11
And so on......................	....	....	....	....	....
180	177	173	7	173	7
182	179	179	3	179	3
184	181	181	3	181	3
186	181	181	5	181	5
188	181	181	7	181	7
Ret)					
190	187	181	9	179	11
192	189	181	11	181	11
194	191	191	3	191	3
196	193	193	3	193	3
198	195	193	5	193	5
200	197	197	3	197	3
And so on......................	....	....	....	....	....
Ret)					
500	497	491	9	487	13
502	499	499	3	499	3
504	501	499	5	499	5
506	503	503	3	503	3
508	505	503	5	503	5
510	507	503	7	503	7
And so on......................	....	....	....	....	....
1000	997	997	3	997	3
1002	999	997	5	997	5
1004	1001	997	7	997	7
Ret)					

1006	1003	997	9	983	23
1008	1005	997	11	997	11
1010	1007	997	13	997	13
1012	1009	1009	3	1009	3
1014	1011	1009	5	1009	5
1016	1013	1013	3	1013	3
1018	1015	1013	5	1013	5
1020	1017	1013	7	1013	7
And so on......................	....	....	....	....
5000	4997	4993	7	4993	7
5002	4999	4999	3	4999	3
5004	5001	4999	5	4999	5
5006	5003	5003	3	5003	3
5008	5005	5003	5	5003	5
5010	5007	5003	7	5003	7
5012	5009	5009	3	5009	3
5014	5011	5011	3	5011	3
5016	5013	5011	5	5011	5
5018	5015	5011	7	5011	7
Ret)					
5020	5017	5011	9	5009	11
And so on......................	....	....	....	....
	.				
Ret)					
10000	9997	9973	27	9941	59
10002	9999	9973	29	9973	29
10004	10001	9973	31	9973	31
Ret)					
10006	10003	9973	33	9923	83
Ret)					
10008	10005	9973	35	9967	41
10010	10007	10007	3	10007	3
10012	10009	10009	3	10009	3
10014	10011	10009	5	10009	5

2n-M	(2n-3)-M	$W_{2n} - M$	T_{2n}	$V_{2n} - M$	U_{2n}
10016	10013	10009	7	10009	7
Ret)					
10018	10015	10009	9	10007	11
10020	10017	10009	11	10009	11
And so on	....	....	....	....
2n-M	(2n-3)-M	$W_{2n} - M$	T_{2n}	$V_{2n} - M$	U_{2n}
+1000	+997	+993	7	+993	7
Rt)					
+1002	+999	+993	9	+931	71
+1004	+1001	+993	11	+993	11
+1006	+1003	+993	13	+993	13
Rt)					
+1008	+1005	+993	15	+919	89
+1010	+1007	+993	17	+993	17
+1012	+1009	+993	19	+993	19
+1014	+1011	+1011	3	+1011	3
+1016	+1013	+1011	5	+1011	5
+1018	+1015	+1011	7	+1011	7
Ret)					
+1020	+1017	+1011	9	+931	89
And so on	....	....	....	....
2n-R	(2n-3)-R	$W_{2n} - R$	T_{2n}	$V_{2n} - R$	U_{2n}
Ret)					
+1000	+997	+979	21	+903	97
+1002	+999	+979	23	+979	23
Ret)					
+1004	+1001	+979	25	+951	53
Ret)					
+1006	+1003	+979	27	+903	103
+1008	+1005	+979	29	+979	29

2n-G	(2n-3)-G	W_{2n}- G	T_{2n}	V_{2n} - G	U_{2n}
+1010	+1007	+979	31	+979	31
Ret)					
+1012	+1009	+979	33	+951	61
Ret)					
+1014	+1011	+979	35	+ 781	233
+1016	+1013	+979	37	+979	37
Ret)					
+1018	+1015	+979	39	+951	67
+1020	+1017	+1017	3	+1017	3
And so on......................	....	....	....	....
2n-G	(2n-3)-G	W_{2n}- G	T_{2n}	V_{2n} - G	U_{2n}
Ret)					
+10000	+9997	+9631	369	+7443	2557
Ret)					
+10002	+9999	+9631	371	+9259	743
+10004	+10001	+9631	373	+9631	373
Ret)					
+10006	+10003	+9631	375	+8583	1423
Ret)					
+10008	+ 10005	+9631	377	+6637	3371
+10010	+10007	+9631	379	+9631	379
Ret)					
+10012	+10009	+9631	381	+8583	1429
+10014	+10011	+9631	383	+9631	383
Ret)					
+10016	+10013	+9631	385	+9259	757
Ret)					
+10018	+10015	+9631	387	+4491	5527
+10020	+10017	+9631	389	+9631	389
And so on......................	....	....	....	....
2n-S	(2n-3)-S	W_{2n}- S	T_{2n}	V_{2n}- S	U_{2n}

Ret)					
+20000	+19997	+18031	1969	+17409	2591
Ret)					
+20002	+19999	+18031	1971	+ 17409	2593
+20004	+20001	+18031	1973	+18031	1973
Ret)					
+20006	+20003	+18031	1975	+16663	3343
Ret)					
+20008	+20005	+18031	1977	+16941	3067
+20010	+20007	+18031	1979	+18031	1979
Ret)					
+20012	+20009	+18031	1981	+5671	14341
Ret)					
+20014	+20011	+18031	1983	+4101	15913
Ret)					
+20016	+20013	+18031	1985	+3229	16787
+20018	+20015	+18031	1987	+18031	1987
Ret)					
+20020	+20017	+18031	1989	+16941	3079
And so on....................	....	....	....	....	....
$2n$-T	$(2n-3)$-T	W_{2n}-T	T_{2n}	$V_{2n}-T$	U_{2n}
Ret)					
+40000	+39997	+29737	10263	+21567	18433
Ret)					
+40002	+39999	+29737	10265	+22273	17729
+40004	+40001	+29737	10267	+29737	10267
Ret)					
+40006	+40003	+29737	10269	+21567	18439
+40008	+40005	+29737	10271	+29737	10271
+40010	+40007	+29737	10273	+29737	10273
Ret)					
+40012	+40009	+29737	10275	+10401	29611

Ret)					
+40014	+40011	+29737	10277	-56003	96017
Ret)					
+40016	+40013	+29737	10279	+27057	12959
Ret)					
+40018	+40015	+29737	10281	+25947	14071
Ret)					
+40020	+40017	+29737	10283	+24493	15527

§ 11 Perspectives and generalizations.

11.1 We can consider studying other Goldbach sequences (G'_{2n}) and (G''_{2n}) independent of (G_{2n}) using the increasing sequences of prime numbers (W'_{2n}), $(see\ paragraph\ \S\ 8)$ and (W''_{2n}) defined by :

For any integer $n \geq 3$, $W''_{2n} = \mathrm{Sup}(\ p\epsilon IP : p \leq f(n))$, f being a function defined on the interval I=[3;+∞[and satisfying the following conditions:

f is strictly increasing on the interval I, $\lim\limits_{x \to +\infty} f(x) = +\infty$, f(3)=3 and $\forall x \epsilon I, f(x) \leq 2x - 3$.

Thus, we can choose for example, one of the following functions defined on I.

a) f: x→ax+3-3a ; (a∈ IR : 0<a≤ 2).

b) g: x→ $[2\sqrt{x} - 2.\sqrt{6} + 3]$, ([.] designating the integer part of the real number x) .

c) h : x→ $ln\ (x)$+3-ln(3).

11.2 Using this method, it would be interesting to study the Schnirelmann density [13] of certain prime numbers like 3 , 5 , 7, 11 , …….. in the sequence (U_{2n}) and in arithmetic sequences (Dirichlet's theorem) for $n \in [K_N ; P_N]$ as a function of N .

11.3 It is possible to exceed the values indicated in the Table ($2n=10^{1000}$) by optimizing this algorithm, $by\ using\ supercomputers\ and\ more\ efficient\ scientific\ calculation\ software\ as$ Maple or PARI/GP .

11.4 Diophantine equations and conjectures of the same nature (Lagrange-Lemoine-Levy's conjecture, for example) can be addressed with similar processes and algorithms . To validate the Lagrange conjecture, we can study the following sequences of prime numbers (WL_{2n}), (VL_{2n}) and (UL_{2n}) $defined\ by: For\ any\ integer\ n \geq 3, WL_{2n}= \mathrm{Sup}(p\epsilon IP: p \leq n - 1)\ and,$

1) If $TL_{2n}= (2n+1 - 2.WL_{2n})$ is a prime number, then we put: $VL_{2n}= WL_{2n}$ and $UL_{2n}= TL_{2n}$.

2) If TL_{2n} is a composite number, then there exists an integer k , $(1\leq k \leq n - 3)$ such that: $UL_{2(n-k)} + 2k$ is a prime number and we put: $VL_{2n}=VL_{2(n-k)}$ and $UL_{2n}=UL_{2(n-k)}+ 2k$.

3) With the same type of process, one can validate generalized Bezout-Goldbach's conjectures of the form:
a) Given two integers K and Q odd and coprime, for any integer n such that : $(2n \geq 3(K+Q))$, there exist two prime numbers U'''_{2n} and V'''_{2n} verifying : $K.U'''_{2n} + Q.V'''_{2n} = 2n$.
b) Given two integers K and Q of different parity and coprime, for any integer n such that : $(2n\geq 3 (K+Q))$, there exist two prime numbers U'''_{2n} and V'''_{2n} verifying : $K.U'''_{2n} + Q.V'''_{2n} = 2n + 1$.

§ 12 Conclusion.

12.1 We have explicitly constructed a unique Goldbach sequence $(G_{2n}) = (U_{2n};V_{2n})$ such that U_{2n} and V_{2n} are prime numbers satisfying : $U_{2n}+V_{2n}= 2n$ for any integer n≥ 2 , using a simple and very efficient unconventional "local recursive" algorithm.

12.2 We broke the record held by Silva [15] on a personal computer, and we were able to reach values of the order of $2n=10^{1000}$ with a reasonable calculation time (less than three hours for the evaluation of ten values of U_{2n} and V_{2n}) .

13

12.3 For a given integer n, the evaluation of the terms U_{2n} and V_{2n} does not require the calculation of all the previous terms U_{2k} and V_{2k} , (1≤k<n-1). You just need to know the prime numbers p_l and the terms V_{2r} in a neighborhood of 2n , (on the interval J = [2n-K1.log(2n);2n]). This property allows a fast calculation of U_{2n} and V_{2n} even for values of 2n of the order of 10^{1000} .

12.4 Therefore, the strong EULER-GOLDBACH conjecture is true.

References.

[1] Bombieri and Davenport, "Small differences between prime numbers", Proc. Roy. Soc. Ser. A293 (1966), p. 1-18 .

[2] Jing Run Chen, "On the representation of a large even integer as the sum of a prime and the product of at most two primes". Kexue Tongbao 17 (1966), p. 385-386 (Chinese) .

[3] Harald Cramer, " On the order of magnitude of the difference between consecutive prime numbers", Acta Arithmetica vol. 2 , (1986), p.23-46 .

[4] Erdos, "On a new method in elementary number theory which leads to an elementary proof of the prime number theorem", Proc. Natl. Acad. Sci. USA 36, p. 374-384 (1949).

[5] Euclide, (trad. Bernard Vitrac), « Euclid's Elements », Ed. PUF Paris, vol.2, p. 444-446 and p. 339-341, (1994).

[6] J. Hadamard, « On the zeros of the function ζ(s) of Riemann ». C. R. 122, p.1470-1473 (1896), and « On the distribution of zeros of the function ζ(s) and its arithmetical consequences ». S. M. F. Bull. 24, p. 199-220 (1896).

[7] Hardy & Wright , "An introduction to the Theory of numbers", Oxford: Oxford University Press 621 p. (2008).

[8] Helfgott & Platt , "The ternary Goldbach conjecture", Gaz. Math. Soc. Math. Fr. 140, p. 5-18 (2014). "The weak Goldbach conjecture", Gac. R. Soc. Mat. Esp. 16, No. 4, 709-726 (2013). "Numerical verification of the ternary Goldbach conjecture up to 8,875. 10^{30}.", Exp. Math. 22, No. 4, 406-409 (2013).

[9] Iwaniec & Pintz, "Primes in short intervals" , Monatsh. Math. 98, p. 115-143 (1984).

[10] E. Landau, "Handbuch der Lehre von der Verteilung der Primzahlen", vol. 1 und vol. 2 (1909), published by Chelsea Publishing Company (1953).

[11] J. Littlewood , "Sur la distribution des nombres premiers", CRAS Paris, vol. 158, (1914), p. 1869-1875.

[12] Ramaré & Saouter, " Short effective intervals containing primes", J. Number theory, 98, No. 1, p..10-33 (2003).

[13] Schnirelmann & Mann, "Schnirelmann density ", Wikipédia, (on line, internet) and "A proof of the fundamental theorem on the density of sums of sets of positive integers", Annals of Math., 2ème série, vol. 43, n03, (1942), p. 523-527.

[14] Selberg , "Lectures on sieves", in Collected papers II, Springer-Verlag, Berlin, (1991), p. 65-247.

[15] Silva, Herzog & Pardi, " Empirical verification of the even Goldbach conjecture and computation of prime gaps up to 4.10^{18}. Math. Comput. 83, No. 288, p. 2033-2060 (2014).

[16] T. Tao, " Every odd number greater than 1 is the sum of at most five primes", Math. Comput. 83, No. 286, p.997-1038 (2014).

[17] Tchebychev, "Mémoire sur les nombres premiers" J. math. pures et appliquées, 1ère série, t.17, (1852), p. 366-390 and p. 381-382 .

[18] Jean Charles de La Vallée-Poussin, «Analytical research on the theory of prime numbers.», Brux. S. sc. 21 B, (1896), p. 183-256, 281-362, 363-397, vol.21 B, p. 351-368

[19] Vinogradov, " Representation of an odd number as a sum of three primes". Dokl. Akad.Nauk. SSR, 15:291-294, (1937).

[20] Zhang, "Bounded gaps between primes", Ann. Math. (2) 179, No. 3, p.1121-1174 (2014).

Adress : PHILIPPE SAINTY , 9 Allée Robert de Wey , 93190 LIVRY-GARGAN,. (FRANCE).

Email : psainty.math@gmail.com