Just two primary sets of whole numbers: ultimates and non-ultimates

[Jean-Yves BOULAY]

Abstract. A mathematical definition integrating, for any number, the notion of inferiority of divisor makes it possible to classify the number zero, the number one and all the primes in a unique set. Also, a complementary definition makes it possible to classify all the other whole numbers into a second unique set. Thus the set ℕ is considered to consist of two sets of whole numbers at absolute properties.

1 Introduction

This study invests the whole number* set (ℕ) and proposes a double mathematical definition classifying all of these numbers into two unique sets. A first definition generates a set of numbers which is constituted by the totality of the primes and the two particular numbers which are zero (0) and one (1). This set is called the set of ultimate numbers. A second definition, connected to the first, generates a set of numbers which is constituted by the totality of non-primes and which does not contain the particular numbers zero (0) and one (1). This other set is called the set of non-ultimate numbers.

* In statements, when this is not specified, the term "number" always implies a "whole number". Also, It is agreed that the number zero (0) is well integrated into the set of whole numbers.

2 Prime numbers definition

In mathematical literature the definition of primes looks like this:

"Prime numbers are numbers greater than 1. They only have two factors, 1 and the number itself. This means these numbers cannot be divided by any number other than 1 and the number itself without leaving a remainder."

This is often supplemented by:

"Numbers that have more than 2 factors are known as composite numbers."

These definitions supposed to describe and classify the whole numbers immediately present two great ambiguities since they are embarrassed by the two singular numbers that are zero and one.

3 New definition approach

The conventional definition applied to define whether a number is prime or not does not specify the character of inferiority of the divisors. This seems obvious, trivial. However, with regard to the particular numbers that are zero and one, this notion has all its importance. Indeed, the number zero, the first of the whole numbers, has many divisors. But these dividers are all superior to it in value. Also, number one, second whole number, has not divisor inferior to it because it cannot be divided by zero.

This novel approach to the concept of divisibility of numbers which includes the notion of inferiority (and consequently the notion of superiority) of divisors allows the creation of two unique sets where all whole numbers can be referenced.

4 Absolute definitions

Considering the set of whole numbers (ℕ), these are organized into two sets: ultimate numbers and non-ultimate numbers.

Ultimate numbers definition:

An ultimate number admits at most one divisor being inferior to it in value.

Non-ultimate numbers definition:

A non-ultimate number admits more than one divisor being inferior to it in value.

5 Conventional designations

As "primes" designates prime numbers, it is agree that appellation "ultimates" designates ultimate numbers. Also it is agree that appellation "non-ultimates" designates non-ultimate numbers. So, the concept introduced here is therefore called that of ultimity of whole numbers.

6. Development

This new concept of ultimity, which may seem intriguing, is now explained in detail with, more concretely, the first of the whole numbers as representative examples, including the exotic numbers zero and one.

6.1 Expended definitions

Let n be a whole number (belonging to $\mathbb{N}$), this one is ultimate if **at most one divisor being inferior to it in value** divides it.

Let n be a whole number (belonging to $\mathbb{N}$), this one is non-ultimate if **more than one divisor being inferior to it in value** divides it.

6.2 Development

Below are listed, to illustration of definition, some of the first ultimate or non-ultimate numbers defined above, especially particular numbers zero (0) and one (1).

- 0 is ultimate: although it admits an infinite number of divisors superior to it, **since it is the first whole number**, number 0 does not admit any divisor **being inferior to it**.

- 1 is ultimate: since the division by 0 has no defined result, number 1 does not admit any divisor (whole number) being less than it.

- 2 is ultimate: since the division by 0 has no defined result, number 2 admit just one divisor (1) being less than it.

- 4 is non-ultimate: number 4 admits 1 and 2 as divisors being less than it, so more than one divisor.

- 6 is non-ultimate: number 6 admits numbers 1, 2 and 3 as divisors being less than it, so more than one divisor.

- 7 is ultimate: since the division by 0 has no defined result, number 7 admit just one divisor (1) being less than it.

- 12 is non-ultimate: number 12 admits numbers 1, 2, 3, 4 and 6 as divisors being less than it, so more than one divisor.

Thus, by these previous definitions and demonstrations, the set of whole numbers is organized just into these two entities:

- the set of ultimate numbers, which is the fusion of the prime numbers sequence with the numbers 0 and 1.

- the set of non-ultimate numbers identifying to the non-prime numbers sequence, deduced from the numbers 0 and 1.

6.3 Abbreviated definition

It is therefore possible to classify very clearly and unequivocally all whole numbers according to ultimity concept. Figure 1 summarizes the process of identifying any whole number that can only be either ultimate or non-ultimate.

Let *n* be a whole number

↙ ↘

if this one admits at most one divisor being inferior to it… | **if this one admits more than one divisor being inferior to it…**

↙ ↘

this one is a ultimate | **this one is a non-ultimate**

Fig 1 Process of identifying any whole number according to ultimity concept.

This ultimity or non-ultimity identification mechanism is universal for all the sequence of whole numbers starting with the number zero.

6.4 The first ten ultimate numbers and the first ten non-ultimate numbers

Considering the previous double definition, the sequence of ultimate numbers is initialized by these ten numbers:

0 1 2 3 5 7 11 13 17 19

Considering the previous double definition, the sequence of non-ultimate numbers is initialized by these ten numbers:

4 6 8 9 10 12 14 15 16 18

A large amount of singular arithmetic demonstrations are presented in the article "The ultimate numbers and the 3/2 ratio" [1]. This is not the subject of this present paper. We just draw attention here to the fact that among the first twenty whole numbers are ten ultimates and ten non-ultimates of which, in a reversible ratio of value 3/2, six ultimates versus four non-ultimates in the first ten numbers and four ultimates versus six non-ultimates in the next ten.

6.5 First twenty whole numbers classification

Figure 2 is a complete illustration summarizing the concept of ultimity applied to the set of all whole numbers with the first twenty numbers as example.

Let *n* be a whole number

0 1 2 3 4 5 16 7 8 9 10 11 12 13 14 15 16 17 18 19

if this one admits at most one divisor being inferior to it…	**if this one admits more than one divisor being inferior to it…**
this one is a ultimate	**this one is a non-ultimate**
0 1 2 3 5 7	***4 6 8 9***
11 13 17 19	***10 12 14 15 16 18***

Fig. 2 Clear and unequivocal classification of the first twenty whole numbers according to ultimity concept.

7 Conclusion

Until now, the set of whole numbers was scattered in four entities: prime numbers, non-prime numbers, but also ambiguous numbers zero and one at exotic arithmetic characteristics. Concept of ultimity that is deducted by the double definition of ultimate and non-ultimate numbers proposed here makes it possible to properly divide the set of whole numbers into two primary groups of numbers with unequivocal and absolute characteristics: a whole number is either ultimate or non-ultimate.

Also, and very importantly, the idea of specifying the numerically lower nature of a divisor to any envisaged number effectively allows that there is no difference in status between the ultimate numbers zero (0), one (1) and any other number identified as ultimate. Because by their singular nature, these two mathematical exotic entities have no divisor which is less than their value. From this point of view, these two numbers are totally opposed since one (1) has absolutely no divisor being lower than it while zero (0) has an infinite number of divisors being higher than it.

The study of these two new sets of whole numbers opens the way to interesting investigations like the one introduced in referencing. We just draw attention to the odd arithmetic arrangements that unfold when applying Sophie Germain's concept to the exotic numbers zero and one.

References

1. Jean-Yves Boulay. The ultimate numbers and the 3/2 ratio. 2020. ⟨hal-02508414v2⟩ DOI: 10.13140/RG.2.2.32843.34081/3

2. Jean-Yves Boulay. New Whole Numbers Classification. 2020. ⟨hal-02867134⟩ DOI: 10.13140/RG.2.2.26139.90402/2

Jean-Yves BOULAY independent researcher (without affiliation) – FRANCE –
jean-yvesboulay@orange.fr ORCID: 0000-0001-5636-2375