A Relationship between Pythagorean Triples and Triangular Numbers

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In this paper, I share a discovery of a subset of Pythagorean Triples (X,Y,Z=Y+1), whose hypotenuses end in the digits 13 and begin in the digits of Triangular Numbers (T_n). Although this is a slightly curious relationship, it is of little mathematical significance, and I believe it to be previously unknown.

The following Pythagorean triples all hold to the Pythagorean Theorem which states that the square of the hypotenuse is equal to the sum of the squares of the other two sides:
(5, 12, 13)
(15, 112, 113)
(25, 312, 313)
(35, 612, 613)
(45, 1012, 1013)
(55, 1512, 1513)
(65, 2112, 2113)
(75, 2812, 2813)

The general formula for all integers, n, is easily deduced, as follows:

$$(10n + 5)^2 + (50n^2 + 50n + 12)^2 = (50n^2 + 50n + 13)^2. \tag{1}$$

A quick calculation shows this to hold true for every value of n.

The hypotenuses are also members of the OEIS entry A001844, such that:

$$Z_n = A001844(5n + 2). \tag{2}$$