

Rebuilding Set Theory

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Abstract: This paper gives a new definition of set and introduces the concept of elastic set with variable extension. Without any unprovable or unreliable proposition, hypothesis or conjecture, it eliminates Russell's paradox, Cantor's paradox, Galileo's paradox and Infinite Hotel's paradox. Reveals the essence of infinity and gives the expression of the number of elements of infinite set, Corrects the error that an infinite set can correspond to its proper subset one by one. At the same time, the hypothesis that there is a unique natural numbers set that has included all natural numbers has been refuted. It is pointed out that the diagonal Argument and Cantor theorem have not proved that real numbers can not be one-to-one corresponded to natural numbers. Since the set is variable, we can also use the variable things such as variables or functions as the elements of the set.

There are so many mistakes in the most strict mathematics, let alone the humanities? It shows that the wisdom of modern Westerners, who lack the gene of cultural diversity, is far less than that of the ancient Greek era, or even degenerated to the barbaric era. No wonder a good world would fight for some differences that may not be meaningful, and even foolishly walked to the edge of nuclear war!

Key words: set theory; Elastic set; Paradox; The number of infinite set elements; Diagonal Argument; One-to-one correspondence

0 Introduction

There are many paradoxes and errors in traditional set theory.

The traditional set theory mentioned in this paper includes Cantor's naive set theory^[1] and axiomatic set theory^[2]. Because compatible set theory^[3] is different from them, they are collectively called traditional set theory.

The compatible set theory tries to eliminate the paradoxes and errors in the mathematical foundation by improving the traditional set theory.

1 The improvement of set definition by compatible set theory

Cantor defined the elements of a set as "objects that can be determined and distinguished by our intuition and thinking"^[1], and a set is the totality or whole of these elements (Ganzer).

The set definition of compatible set theory is as follows:

Definition 1 In the existing things, the things that researchers are interested in are called elements, and the symbols $\{ \}$ are used to separate the elements from other things, which is called a set. In particular, $\{ \}$ defines an empty set if something of interest cannot be found.

This definition stipulates that an element must already exist. This is because it is not necessarily meaningful to study things that may not exist, and it is easy to confuse them with

things that actually exist, thus causing various thinking confusion.

This seems to be a trivial difference, but actually it has brought many changes to set theory. It is shown in the following aspects.

1) Eliminates the paradoxes in set theory

According to definition 1, the elements of the set must be existing things. In this way, it is impossible to have a set that contains itself: when we plan to define set A, set A has not been defined yet at this time, as a result, A does not exist. Of course, it is impossible to have nonexistent things, that is, A, in the elements of A, so $A=\{A\}$, $A=\{A, B\}$ and so on do not conform to definition 1. The so-called Russell paradox, Cantor paradox and so on certainly do not exist.

In the history of set theory, axiomatic set theory was developed to eliminate Russell's paradox and Cantor's paradox. However, according to the razor principle, since these paradoxes can be eliminated by simply stipulating that elements must already exist, it is not necessarily necessary to have these more complex axiomatic systems.

2) The element must be the thing that is interested in researching or thinking

In Cantor's definition, there is no statement of interest. In this way, all determinable things in the world will be regarded as elements. The set defined by these elements is too complex to study.

Therefore, in definition 1, the elements of the specified set are the things that need to be concerned about, that is, the things of interest, in a certain research or thinking. For example, if you are only interested in 1 and π , $\{1, \pi\}$ can also be a set; If you are only interested in natural numbers, then the elements of the set are natural numbers. Of course, according to the needs of research, there can also be even number set, odd number set, rational number set, real number set, etc.

For the convenience of discussion, the concepts of connotation and extension of a set are sometimes used: if the elements of a set have one or some common properties P, then P is called the connotation of the element, and each specific element constitutes the extension of the set.

Be careful not to confuse the connotation and extension of sets with the connotation and extension of concepts: in any relatively mature concept system, each concept usually has a relatively definite connotation and extension, and the definition of sets is much more flexible: from the perspective of sets, there are only two kinds of things in the world, namely, interested and not interested. The former are called elements, while the latter are not called elements, Therefore, the connotation and extension of sets change with people's research needs

In addition, unless otherwise specified, other definitions in naive set theory, such as union, intersection, difference, inclusion, belonging, subset, proper subset and other definitions, are invariant in compatible set theory.

These definitions facilitate the study of the relationship between sets, for example, the union of even set and odd set is a natural number set.

2 Extensive variable set

In the traditional set theory^{[1][2]}, the extension of the set is fixed, so that the elements of the set are determined, which is convenient for research.

However, the number of actual things may change. For example, a new student comes to Class One (1), and we usually regard this class as Class One (1) still. For another example, due to the constant birth and death of people, the number of elements in a set of "Chinese people" is

changing every moment.

Generally speaking, a set with a fixed number of elements is easier to study than a set with an unfixed number of elements, however, when the number of elements of an actual problem is changing, if we insist on fixing the number of elements, it is not only impossible to describe the facts correctly, but will complicate the original very simple problem.

Taking the set of "Chinese" as an example, some people think that we can put the dead Chinese together with the living Chinese and all the Chinese who will be born in the future to form a whole with a fixed number of elements. This approach is obviously far fetched: who can know exactly how many dead and unborn Chinese people are? Moreover, how to ensure that these three types of people together become a set with a fixed number of elements? Is the reason sufficient?

In fact, for the census, at least it is never possible to turn the living Chinese into a set with a fixed number of elements.

Mathematical concepts are used to describe facts, so the definition of mathematical concepts must be as close as possible to the facts to be described rather than to let facts to adapt to mathematical concepts that may not be reasonable!

For this reason, in the compatible set theory, a set with variable number of elements of the set is introduced, which is called elastic set.

2.1 Definition and representation of elastic set

Definition 2 The set with variable number of elements is called elastic set.

According to definition 2, n in the finite set of natural numbers $\{1,2,3,\dots, n\}$ is no longer a constant, but a natural number variable, which can be expressed as

$$N(n)=\{1,2,3,\dots,n\} \quad (1)$$

The number of elements is the natural number variable n .

Natural number variables can also be represented by other symbols such as m .

In the limit definition of mathematical analysis, n is also a natural number variable strictly. Take $n \rightarrow \infty$ as an example. Since n is changing, it is not a constant, but a variable.

It should be noted that the constant n in the traditional set theory is regarded as a variable in mathematical analysis, which is an important difference between this paper and the traditional set theory. As will be seen below, this will unify the finite set and the infinite set, thus making the traditional set theory undergo earth shaking changes.

In order to strictly distinguish between constants and variables, in the following description of this paper, natural number constants are generally expressed in n_0 .

When the natural number variable $n=n_0$, (1) defines a set of natural numbers whose number of elements is n_0 .

That is to say, the finite set we usually talk about is just a special case of elastic set. Therefore, the elastic set defined in (1) should not be regarded as a series of finite sets, but the series of finite sets can be regarded as the special cases of the elastic set defined in (1).

Since the number of elements of an elastic set is specified to be variable, it cannot be considered that an elastic set is no longer the original elastic set based on the change in the number

of elements: it is still the original elastic set, but the number of elements has changed.

Therefore, judging whether two elastic sets are the same elastic set depends on two factors: ① whether they have the same element properties, that is, whether the connotation is the same. For example, if they are natural number sets, each element must be a natural number; ② Whether they have the same extension.

The set conforming to ① is called the elastic set with the same connotation, and the set conforming to ② is called the elastic set with the same extension. It can be seen from the definitions 1 and 2 that the same denotative elastic set must also be the same denotative elastic set, but the same denotative elastic set is not necessarily the same denotative elastic set.

Elastic sets of the same connotation with different number of elements can have inclusion relations, and one of them can be a proper subset of the other. For example, when $n < m$, $N(n)$ is a proper subset of $N(m)$. Elastic sets with the same extension can be represented by an equal sign, for example, when $n = m$, $N(m) = N(n)$.

When $N_1 = \{0\} \cup N(n)$, no matter how the number of elements of $N(n)$ changes, the number of elements of N_1 also changes synchronously: every time $N(n)$ increases or decreases one element, N_1 also increases or decreases one element, so $N(n)$ is always the proper subset of N_1 .

Of course, when the number of elements changes asynchronously, the relationship between elastic sets may change. For example, if n changes from $n > m$ to $n < m$, $N(n)$ changes from including $N(m)$ to being included in $N(m)$.

The union, intersection and difference operations in the traditional naive set theory are still valid in elastic sets. Taking $N(n) = \{1, 2, 3, \dots, n\}$ and $N(m) = \{1, 2, 3, \dots, m\}$ as examples, when $n > m$, $N(n) \cup N(m) = \{1, 2, 3, \dots, n\}$, $N(n) - N(m) = \{m+1, \dots, n\}$, $N(n) \cap N(m) = \{1, 2, 3, \dots, m\}$.

When $n = m$, $N(n) = N(m)$, $N(n) - N(m) = N(m) - N(n) = \Phi$, $N(n) \cup N(m) = N(n) \cap N(m) = N(n) = N(m)$.

Unlike real numbers, the define domain of natural number variables cannot be written in the form of real number intervals, but can be represented by sets. The define domain of n can be $\{1, 2, 3, \dots, M\}$, where M is the upper bound, that is, the maximum natural number, or the unbounded $\{1, 2, 3, \dots\}$, that is

$$n \in N = \{1, 2, 3, \dots\} \quad (2)$$

Where N is called an infinite set of natural numbers.

Definition 3 The set whose elements are natural number sequence $1, 2, 3, \dots$ without upper bound (without maximum natural number) is called infinite natural number set, which is referred to as natural number set, and is represented by N .

Among them, natural number sequence $1, 2, 3, \dots$ existed as early as before the birth of set theory, and can be traced back to the earliest human mathematical practice.

2.2 Application of elastic sets

After the introduction of the concepts of variables and functions in mathematics, mathematics has undergone tremendous changes: people can describe not only the static world, but also the moving world. Analytic geometry, calculus, and industrial revolution came into being, and artificial satellites have also gone up in the sky.

Similarly, the introduction of sets with variable extensions into set theory will not only

greatly expand the scope of application of set theory, but also avoid various academic traps due to improper treatment of sets with actually varying extensions as sets with fixed extensions. Give some examples.

2.2.1 Understanding of infinity

It is not difficult to prove that the extension of elastic set $N(n)$ can be the same as N , that is

$$N(n) = N \quad (3)$$

holds true.

Proof For any $n_0 \in N(n)$, clearly there is $n_0 \in N$. In other hand, because n is a variable, $n > n_0$ can holds for any $n_0 \in N$, that is $n_0 \in N(n)$ may hold. **Certificate completion**

Since the left end of (3) is an elastic set with n elements (n is an unbounded natural number variable), we can draw some important and seemingly unexpected properties about the definition 3.

Property 1: The number of elements of N can be represented by a natural number variable n without an upper bound.

This conclusion seems very simple, but it is significant. Because Cantor regards the infinite set as a completed set with fixed extension, which is seriously inconsistent with the fact that the natural number can continue to increase, it makes the original extremely simple problem extremely complex and full of errors (see examples 1, 2, 3 in this paper). Under the guidance of this wrong view, it is virtually impossible to obtain the infinite number of elements in a set. Many mathematicians have been studying the so-called cardinal number and one-to-one correspondence all their lives. In fact, they are studying the number of elements in an infinite set. However, no one is clear about the concept of the number of elements in an infinite set. The fundamental reason is that the starting point is wrong. Since this paper has clearly defined the concept of the number of elements of an infinite set at least for the infinite set of natural numbers, whether it is the cardinal number or the one-to-one correspondence, it has no academic significance in fact. Therefore, in new set theory, the concepts of cardinality and one-to-one correspondence are not specifically discussed.

Although n has no upper bound, that is, its values can increase infinitely, but any natural number it can take is finite,

Property 2: The number of elements of N is a finite value that can be infinitely increased.

Property 3: For the set of natural numbers, the essence of infinity is the finite value that can be increased infinitely.

Just starting from the definitions 2 and 3, we do not need any philosophical assumptions and conjectures, nor any axioms that can never be proved and whose reliability can never be guaranteed, we In fact have well answered the question that has puzzled mankind for a long time: what is infinity: at least for the infinite set of natural numbers, infinity is the finite value that can be infinitely increased.

For example, every natural number is finite, but the set of natural numbers is infinite, which has long been regarded as a contradiction. In fact, there is no contradiction: although every natural number is finite, and the number of elements of each set represented by natural number variables

is also finite, as long as the number can be increased infinitely, according to definition 3, it is an infinite set.

The concept of infinity in everyone's mind is actually different, and may not agree with each other, which is the reason why the issue of infinity has been debated for a long time. But in this paper, as long as the definitions 2 and 3 are recognized, at least for the infinite set of natural numbers, the implicit infinity view can be revealed. Since the definition itself does not need to be proved, there should be no controversy in fact.

In fact, the argument about infinity essentially confuses the two different concepts of the infinite of growth and the infinite of numerical value. For the set of natural numbers, infinity only refers to the infinity of growth, not the infinity of numerical value.

In addition,

Property 4: There is no unique N.

This is because when m is not equal to n, it can also be proved that $N(m)=N$ as well as $N(n)=N$ like in(3). Because $N(n) \neq N(m)$, therefore N cannot be unique. The following example 2 also describes this property in detail.

2.2.2 Avoid errors caused by the confusion of extension

In traditional set theory, infinite sets often end with an ellipsis. The contents of ellipsis are often not carefully examined. In elastic sets, since the infinite sets does not end with an ellipsis, the extension can be expressed relatively clearly, thus effectively avoiding various errors caused by the confusion of extensions.

For example, when $n < m$, the elastic set $N(m)$ and its infinite proper subset $N(n)$ can be expressed as $\{1,2,3,\dots\}$ in traditional set theory, which is difficult to distinguish, so it is easy to be confused. In the new set theory, of course, there is no case that sets are confused with their proper subsets.

Using the concept of elastic sets, we can also easily correct many previous mistakes.

Example 1: $N_1 = \{0\} \cup N$ cannot be one-to-one correspond to its proper subset N.

Cantor believes that N_1 can be one-to-one corresponded with its proper subset N through the "bijection" function $n_1 = n - 1$:

$$\begin{array}{l} N_1 \quad 0, 1, 2 \dots \\ N \quad 1, 2, 3 \dots \end{array}$$

In the history of mathematics, the so-called infinite hotel paradox, which is both full and not full, obviously anti logical (self contradictory), anti intuitive and anti common sense, is actually a fabricated story of the above deduction.

The above deduction and paradox, which seems reasonable, ingenious and has been widely accepted up to now, and even highly praised by the mainstream mathematical circles, actually cannot withstand the torture of the law of sufficient reason:

Similar to (3), it is not difficult to prove that when the natural number variable $n \in N = \{1,2,3,\dots\}$

$$N_1 = \{0\} \cup N = \{0,1,2,3,\dots,n\}, \tag{4}$$

Therefore, if N_1 can be one-to-one corresponded with N by $n_1 = n - 1$, then

$$\{n_1|n_1=n-1, n \in \mathbb{N}\} = N_1 = \{0,1,2,3,\dots,n\}$$

and

$$\{n|n=n_1+1, n_1 \in N_1\} = N = \{1,2,3,\dots, n\}$$

holds true. However, neither of the above two formulas holds:

$$\{n_1|n_1=n-1, n \in \mathbb{N}\} = \{0,1,2,3,\dots,n-1\} \neq \{0,1,2,3,\dots,n\} = N_1 \quad (5)$$

$$\{n|n=n_1+1, n_1 \in N_1\} = \{1,2,3,\dots,n+1\} \neq \{1,2,3,\dots,n\} = N \quad (6)$$

It is not difficult to find that the difference between the two ends of the inequality sign is obvious: because the n at both ends is the same variable, according to the identity, the n at both ends can only take the same value at any time, so the left end of the inequality sign minus the right end is always not equal to $\{ \}$, that is, the left end is always not equal to the right end. Take (5) as an example. No matter how much the variable n equals, the left end of the inequality sign is always one element less than the right end. Of course, both ends cannot be the same set.

It can be seen from (5) that it is the set $\{0,1,2,3,\dots, n-1\}$ that can one-to-one correspond with N , but the $N_1 = \{0,1,2,3,\dots, n\}$ can not one-to-one correspond with N . Cantor confused the differences between the two sets, and thus "ignored the tail" to draw an obvious anti logic anti common sense wrong conclusion that the infinite set N_1 can one-to-one correspond to its proper subset N .

Similar examples can also be found in the relationship between the set of natural numbers and the set of even and odd numbers. Easy to prove N can correspond one-to-one with the even set E or the odd set O respectively:

$$E = \{x|x=2n, n \in \mathbb{N}\} \quad (7-1)$$

$$O = \{x|x=2n-1, n \in \mathbb{N}\} \quad (7-2)$$

Let E_1 be an even proper subset of $N = \{1,2,3,\dots, n\}$, and O_1 be an odd proper subset of N , that is, $N = E_1 \cup O_1$. It is not difficult to prove that " $E_1 = E$ and $O_1 = O$ " are impossible with the method of contradiction: assuming " $E_1 = E$ and $O_1 = O$ " then

$$N = E_1 \cup O_1 = E \cup O = \{x|x=2n, n \in \mathbb{N}\} \cup \{x|x=2n-1, n \in \mathbb{N}\} = \{1,2,3,\dots, 2n-1, 2n\} \neq N \quad (8)$$

That is, $N \neq N$, contradiction!

Cantor obviously confused E with E_1 (or O with O_1), wrongly believed that since N can one-to-one correspond with E (or O), N can also one-to-one correspond with to E_1 (or O_1).

The so-called Galilean paradox, which has lasted for hundreds of years, has also been resolved here: although the number of elements in the even number set E is the same as that in the natural number set N , and the two can be one-to-one corresponded, the number of elements in the even number set E_1 , which is a proper subset of the N , is not the same as N (only half of N), and N

and E_1 cannot be one-to-one corresponded.

Many so-called paradoxes are actually caused by the confusion of various concepts, such as E and E_1 , O and O_1 . As long as people are careful enough, these paradoxes should not appear.

Example 2: There is no unique set N containing all natural numbers (property 4).

For example, if $N(n)=\{1,2,3,\dots, n\}=N$, then according to eqs.(7), we have another natural numbers set:

$$E \cup O = \{x|x=2n, n \in N\} \cup \{x|x=2n-1, n \in N\} = \{1, 2, 3, \dots, 2n-1, 2n\} = N(2n) \quad (9)$$

A similar example is that if one workshop produces one part per second and another workshop produces $m (>1)$ parts per second, the part number can be represented by two elastic sets $N(n)$ and $N(mn)$, no matter in finite time or infinite time, where $N(n)$ is the proper subset of $N(mn)$.

Whether n is finite or tends to be infinite, $(mn)/n=m$ is always true. Here, the number of elements in the set of natural numbers represented by two natural number variables n and mn (the infinite variable has no upper bound) is not equal.

The above examples have shown that there is no set that already contains all natural numbers: if $N(n)$ is such a set, then $N(2n)$ includes $N(n)$, that is, $N(n)$ is not a set that already contains all natural numbers. Similarly, if $N(2n)$ is such a set, then $N(4n)$ includes $N(2n)$, that is, $N(2n)$ is not a set that already contains all natural numbers.....

Therefore, the infinite axiom does not hold. For those who have been used to traditional set theory, the above results seem inconceivable, but in fact they are very simple:

It can be seen from the left end of (3) that the number of elements of the natural number set is a natural number variable without an upper bound, and it is an elastic set with increasing extension. A set that has included all natural numbers, of course, will no longer have other natural numbers, so the extension has been fixed, and the two cannot be the same set. Therefore, N at the right end of (3) cannot be a set that has included all natural numbers.

The logic and philosophical significance of Example 2 is also easy to understand: since natural numbers can be continuously increased, how can they be increased to the extent that they cannot be increased, so that a set of natural numbers can be formed that includes all natural numbers, that is, the extension can no longer be expanded, that is, the extension is unchanged? "Can be infinitely increased" and "can not be increased" are obviously contradictory and cannot be true at the same time. Therefore, the arbitrary infinite axiom is also anti logical (self contradictory), anti intuitive and anti common sense.

In fact, since natural numbers can be continuously increased, the extension of the set of natural numbers can certainly be continuously expanded. Therefore, in the set of natural numbers, only part of the natural numbers can be included, and it is impossible to include all the natural numbers.

It can be seen that the so-called whole concept(Ganzer) is sometimes just an idealized imagination and desire, which is not necessarily meaningful in fact. Therefore, the concept of the whole is abandoned in Definition 1, so that the set theory can not only study things that can form a whole, but also can study things that cannot form a whole, greatly expanding the scope of application of set theory.

From here, we can see again the necessity of introducing elastic sets: it is impossible to

expect to use set theory without elastic sets to describe infinite sets including natural number sets.

The above examples prove that the elastic set that is constantly changing is regarded as a fixed set. It will not really become a fixed set, but will only lead to various contradictions.

The above examples also prove that treating an ongoing infinite process as a completed infinite process does not really turn it into a completed infinite process, but only leads to various contradictions.

The above examples also prove the unreliability of axioms: it is meaningless to claim the existence of a set that does not actually exist in the form of axioms. For example, if we use the "ghost axiom" to claim that there is a set containing all ghosts and gods, this set will not exist.

In a word, science must be practical and realistic. We must not take wishes as facts, let alone distort facts with wishes. Otherwise, we will inevitably fall into various wrong academic traps.

Confusing various sets of natural numbers of different sizes and mistaking them for the same set of natural numbers is one of the main sources of Cantor's error.

After it is clear that there is no unique set of natural numbers, the so-called Diagonal Argument becomes very simple:

Example 3 Diagonal Argument lists the decimals in $[0,1)$

$$a_1, a_2, a_3, \dots \quad (10)$$

Where,

$$a_1 = 0.a_{11}a_{12}a_{13}a_{14} \dots$$

$$a_2 = 0.a_{21}a_{22}a_{23}a_{24} \dots$$

$$a_3 = 0.a_{31}a_{32}a_{33}a_{34} \dots$$

.....

Since both row and column labels can only be represented by natural numbers, there must be two sets of natural numbers, called row label set:

$$N(m) = \{1, 2, 3, \dots, m\} \quad (11)$$

and column label set:

$$N(n) = \{1, 2, 3, \dots, n\} \quad (12)$$

Because $m = 2^n > n$, the column label set representing the number of decimal places (digits) is only the proper subset of the row label set representing the number of decimals ,

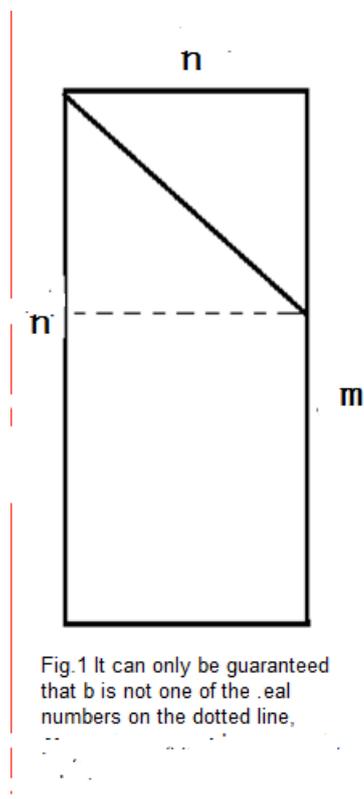
Real number found by Cantor according to diagonal

$$b = 0.b_1b_2 \dots, \quad b_k \neq a_{kk} (k=1, 2, 3 \dots) \quad (13)$$

whose digit k cannot be greater than n or m , because $n < m$, thus,

$$k \leq n \quad (13)$$

Therefore, the so-called Diagonal Argument only proves the existence of the decimal b , so the number of decimals can be greater than n . However, the number of decimals is m rather than n , and the value of m is inherently much greater than n . Therefore, nothing has been proven by Diagonal Argument. The so-called contradiction only comes from the confusion between $N(m)$ and its proper subset $N(n)$ (see Fig.1).



The abscissa in the figure represents the number of decimal places n , and the ordinate represents the number of decimals m . It can be seen from Fig. 1 that although b is not one of the n real numbers listed on the dotted line, it can be one of the m real numbers under the dotted line without any contradiction!

It should be noted here that neither n nor m is a natural number, but a natural number variable without an upper bound. As mentioned earlier, the so-called infinity, at least for the set of natural numbers, is a finite value that can be infinitely increased. Therefore, what is described here is not a finite phenomenon, but an infinite phenomenon, just because people did not know what the essence of infinity was, so they just mysterious and agnostic infinity.

Cantor theorem and interval nested method also have similar problems. Taking Cantor's theorem as an example, although subsets of the set of natural numbers $N(n)$ cannot correspond one-to-one with $N(n)$, it can correspond one-to-one with another set of natural numbers $N(m)$, where $m=2^n$.

Therefore, as long as we realize that the set of natural numbers is not unique, there is no contradiction.

2.3 Introduction of uncertain elements

In traditional set theory, in order to pursue the certainty of a set, not only the number of elements of the set is fixed, but also each element is unchanged. This also greatly limits the scope of application of set theory: since the elements of set theory refer to things that already exist in the world, are these things unchanged? Therefore, we can use changing things such as functions as elements of a set:

$$\{f_1(x), f_2(x), f_3(x), \dots\} \quad (14)$$

Therefore, in the set theory with variable extension, not only the number of elements of the set can be changed, but also the elements themselves can be changed. Obviously, this will also greatly expand the application scope of set theory.

3 Summary and discussion

Long before the birth of set theory, people knew the existence of natural number sequences. In fact, the most original mathematical achievement of human beings is to know the existence of natural numbers 1,2,3... and their related addition methods, such as $1+1=2$, from the counting practice. The initial expression method is probably a pile of stones.

Natural number and its related addition are the most reliable mathematical facts, and any further processing will not increase its reliability.

For example, the later Piano's axiom or the infinite axiom, in essence, are just wishful thinking after the fact, which will not increase its reliability in logic, but often self defeating. Take Piano's axiom as an example. Even if there is no definition of successor number, people begin to describe the axiom. Ambiguity is inevitable: different definitions of successor number lead to different "natural number sequences". In addition to the natural number sequences, even number sequences, odd number sequences and 1, 1.5, 2, 2.5, 3... all conform to Piano's five axioms. If there is no known about natural number sequence, how to know which of these sequences is the natural number sequence that conforms to human practice? The infinite axiom has the definition of successor numbers, but if we do not know the sequence of natural numbers in advance, how can we know that a series of sets defined thereby are natural numbers?

Magicians often deceive people with blinding tricks and cover up simple facts with dazzling phenomena. People know that he is a fake, but they are still very happy to be cheated: just for entertainment. It is meaningless to do this in science. People who are deceived will be angry: the purpose of science is to reveal the simple essence behind the phenomenon of light strange separation, not to do the opposite!

In addition to highly reliable natural number sequences, this paper does not need any axioms, assumptions and conjectures that cannot be proved, but gives a new set theory that can completely eliminate Russell, Cantor, Galileo, Hilbert and other paradoxes, and can greatly expand the application scope of set theory according to the definition that does not need to be proved.

At the same time, this paper corrects the errors caused by mistaking the infinite set for the deterministic set, and also corrects some major errors caused by unclear extension.

Although the traditional set theory has no problem in the aspect of finite sets, for the infinite problem, because the uncertain infinite set is wrongly treated as the deterministic set, the premise

is wrong, and the results are certainly wrong, we can say that nothing is left except for the error.

The confusion of certainty and uncertainty, and the confusion of various extensions, has caused a lot of errors in Cantor's theory.

Since the primitive man climbed out of the cave, he divided the outside world into two concepts: sky and earth. Since then, the continuous subdivision of the concept has become a sign of human progress. However, this process is too long. Frequent conceptual confusion makes people's progress very difficult. For example, confusing the sets at both ends of the inequality in (5) leads to the serious mistake that the infinite set can correspond to its proper subset one by one, which has delayed people for nearly a century and a half; For example, the confusion between the independent even number set and the even number proper subset of the natural number set caused the Galilean paradox that lasted for hundreds of years; For another example, the confusion between the set of natural numbers (11) and its proper subset (12) also leads to people's wrong understanding that real numbers are countable or uncountable

Greatly improving people's ability to distinguish and subdivide different concepts may be an important part of accelerating the development of human science.

Once book knowledge is full of mistakes, people's thinking ability will inevitably be greatly reduced: if wrong knowledge must be accepted as correct knowledge, how can people still have the view of right and wrong? Do people still have normal thinking ability?

The anti logic, anti common sense set theory has been greatly appreciated, which is evidence that the thinking ability of Westerners who lack the gene of cultural diversity in modern times has greatly degraded compared with ancient Greece.

Mathematics, which is considered to be the most strict, has so many invisible mistakes, not to mention the humanities? No wonder a good world would fight for some differences that may not be meaningful, and even foolishly walked to the edge of nuclear war!

Wake up, once brilliant Westerners!

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