The Case for Score Voting

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Abstract: Score voting (also called range voting) uses a *ratings ballot*. Each voter assigns (to as many candidates as she wishes) a *number* in a specified range. Greatest average score wins. Two main lines of evidence show score voting is a good decision-making method: biology and computer simulation. Honeybees achieved evolutionary success by annually deciding their new hive location via score voting. Some ants also use score voting. Computer simulations have been used to compare score versus other election methods by the criterion of *Bayesian regret* (BR), i.e. the expected value of the avoidable human unhappiness caused by an action—in this case the use of an election method. Score voting consistently outperforms other election methods measured by BR. Score voting also is among the simplest voting methods.

Keywords Score Voting \cdot Range Voting \cdot Bayesian Regret \cdot NESD \cdot 2-party domination **JEL Classification** D71 D72

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1 Plan

There are two lines of experimental evidence that score voting (SV) is a good decision-making method: biology (§2)—bee and ant elections use SV—and (§4) computer simulations using the "Bayesian regret" (BR) yardstick to compare the quality of election methods. Another approach (§7) is to prove theorems about SV. Evidence (§3) from surveys (and speed and error-rate measurements) of human voters shows SV is the simplest and most comprehensible method. Which voting methods strangle democracy into "2-party domination" (2PD)? That's mysterious, but §5 collects historical evidence and proposes a theory. Both suggest SV is less likely to yield 2PD than all common rival methods. Although (§6) I criticize the usual political-science approach of listing logical criteria and seeing which voting methods obey which criteria (that approach is made obsolete by BR), SV nevertheless does very well judged that way. **Definition of Score Voting:** SV uses a *ratings ballot*; each voter numerically *scores* each candidate within a specified range. The candidate with greatest average score wins.

Variants: I advocate allowing voters to leave any subset of candidates unscored or "blank." $SV\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is likely best for human use (<u>/RateScaleResearch</u>)^L, but the real interval [0,1] is best for analytic math discussion.

Some have wondered what scores "mean." Perhaps "6" doesn't mean the same to me as you. Answer: the only "meaning" of scores is their mathematical effect on who wins. That effect, and hence their meaning, are mathematically precise, unambiguous, and exactly the same for all. I turn now to the sections comprising the stated components of the Plan. As I do, please keep in mind that experimental results trump theory and intuition.

2 Biology

Two known classes of nonhuman animals have lives that are affected in a major way by voting in elections with three or more candidates: bees and ants. Honeybees annually vote democratically on a new hive location. Certain ants also conduct such a vote if their old nest is destroyed. Regard those elections as independent Darwinian experiments, for bees 10⁸ years long comprised of 10¹⁶ individual elections. Even today, honeybees exceed 200 times human population and run more than ten times more elections each year, with those elections more closely linked to each bee-voter's survival chances, than human elections are linked to the survival of almost all human

voters. That makes bees an indicator *at least* 10⁶ times more sensitive than humans. Democrats: Ignore bees at your peril. Similar remarks apply to ants, since they are 140-168 million years old, outnumbering bees more than 10,000 to one (/BeeAntPops).

Both bees and ants use SV (with blanks allowed; <u>ApisMExec</u>). They could have chosen many other voting methods, but whatever bee species may have tried must have made worse-quality decisions, resulting in bees that were outcompeted and went extinct. Conclusion: among all voting methods simple and cheap enough for bees to use, *plain range voting is best*. But what if we hypothesize there is some fancy Condorcet method X far better than SV, but which the bees could not find because X was too hard for them to compute? Answer: the bees could have modified SV to make it "more" Condorcet, e.g. by adding a subsequent top-2 simple-majority "runoff," as in STAR voting. It is hard to believe such an X could exist *without* any possible way for bees to tweak SV to make it 0.00000003 better (i.e. all that would be necessary for genetic takeover in 100 Myr) by becoming "more X-like."

How much do good voting methods help? Ants' mass times longevity makes them "arguably the greatest success story in the history of terrestrial metazoa" (Schultz 2000).

3 Simplicity and Comprehensibility

SV occupies a "sweet spot," delivering excellent *quality* while staying *simple*. We can assess "simplicity" and "comprehensibility" in several *objective* ways (<u>Comprehension</u>):

- polls asking voters to assess voting methods
- measurements of voting time
- invalid "spoiled" ballot percentages in actual elections
- length of vote-inputting winner-outputting computer programs (shorter algorithms prove it was a simpler method; of course SV's code length is minimal),
- time taken by election authorities to determine winners in actual elections.

Polls by Baujard and Igersheim (2007) (for both SV and AV), Neely et al. 2006 (San Francisco IRV election postmortem), and Bittle and Rochkind (USA 2008 presidential election postmortem) found SV had *better* voter-assessed comprehensibility than plurality, AV, and IRV. Voting speed measurements also find that scoring actually takes voters *less* time than either binary approval decisions or rank-ordering, despite providing *more* information. E.g, see Poundstone (2008) re "Hot or not" experiments showing 10-level scoring is faster than ranking, even with only two options to order, and faster than approval. Furthermore, Munson and McIntyre (1979, p. 49) found ranking three times slower than scoring. Data (/SPRates) indicate (statistically significantly) that, compared to plurality voting (PV):

- The 9% ballot spoilage rate in San Francisco's 2004 IRV election was seven times PV's; Minneapolis' 2009 IRV election had a spoilage rate four times PV's (and the rate of *errors*, i.e. including *both* spoiled and not-fully-spoiled ballots, was greater than the fully spoiled rate by a factor of more than ten). In IRV races Australia reports higher spoilage rates than any country using PV, usually by a factor of two to six.
- Pseudo-election exit poll studies found SV's ballot spoilage rate would be about 1/3 of PV's, in terms of the count of ballots-with-problems, and below 1/10 in terms of the entries-affected rate (even for voters who never score-voted before in their lives).
 - AV's spoilage rates are two to 20 times lower than PV's.

If we compare (<u>/TimeDelays</u>) the time delay between election day and an announcement of the winners for the last four federal elections in Australia (elects House with IRV and Senate with

PR-STV) versus the similar Canada elections (PV House), we find that Canada always managed it in one to 24 days, typically 7, while Australia took 25 to 245 days, typically 60. In Nov. 2019 New York City enacted IRV5 for its "primary" elections (most importantly for Mayor), then held its first IRV primary on 22 June 2021. It then took seven days to announce a preliminary count, which it retracted the next day due to a procedural error, and 14 days until the results were clear enough for Associated Press to recognize a winner. That is six times the average delay in NYC's preceding four mayoral primaries, whose delays were one, six, one and one days.

4 BR computer simulations

Computer simulations measure the **Bayesian regret** (BR) of single-winner election methods (<u>/BayRegDum</u>). Oversimplified into a nutshell: The "Bayesian regret" of an election method E is the "expected avoidable human unhappiness" caused by using E. *Smaller* BR is better. Now I'll explain in more detail (a) what BR is, (b) how we measure it, (c) why we use computers, not humans, to measure it, and (d) results.

Procedure to compute BR(E):

- 1• Each voter has a personal "utility" for the election of each candidate. (E.g., if Nixon is elected, then Mary Voter gets -55 extra lifetime happiness units.) In a computer simulation, the "voters" and "candidates" are artificial, and the utility numbers are generated by some randomized "utility generator" and assigned artificially to each candidate-voter pair.
- 2• Now the voters vote, based both on their private utility values, and (if they are strategic) on their perception from "pre-election polls" (also generated artificially within the simulation, e.g. from a random subsample of "people") of how the other voters are going to act. (Some have harbored the wrong impression that this was predicated on assumptions that voters will be "honest" and not "strategic"; or that honest range voters will use candidate-utilities as their candidate-scores. Others have thought we insisted on something like i.i.d. normal random numbers as utility values. All such impressions are incorrect; these assumptions are not demanded.)
- 3• The election system E elects some winning candidate W.
- 4• The sum over all voters V of their utility for W, is the "achieved societal utility."
- 5• The sum over all voters V of their utility for X, maximized over all candidates X, is the "optimum societal utility" which would have been achieved if the election system had magically chosen the societally best candidate.
- 6• The difference between 5 and 4 is the "Bayesian Regret" of election system E. It might be zero, but if E was bad or if this election was unlucky for E, then it will be positive.
- 7• We redo steps 1-6 a zillion times to find the *average* Bayesian regret BR(E) of election system E.

BR values are commonly normalized onto a 0-to-1 scale, where 0 is the "magic best" election method (that always elects the societally-best candidate) while 1 is for "random winner" (RW). Then all methods commonly considered have 0 < BR < 1. Presumably BR (Non-democracy) \approx BR(RW), enabling us to make statements like "the benefit to humanity got by switching to SV, is comparable to or exceeds the benefit from *inventing* democracy."

How much is that worth in dollars? If we compare Pakistan versus USA during 1750-2019, they started at about the same economic level (p.95 of Bairoch 1995) but at its end the USA had 51 times Pakistan's GDP/capita (13× in PPP-dollars) probably mainly due to greater democracy over that period.

Why evaluate BR via computer simulations, instead of actual human elections? Two reasons: (1) You can't easily measure "utility" of different election alternatives for humans: There are no tangible, commonly agreed units (like "money") for measuring "utility" or human "happiness"; even if such units existed, you still could not measure it; if you ask humans, they lie (or don't know); so at best your results would be controversial and unclear. (2) Human elections are infrequent and expensive, so we could only acquire data amounts inadequate for statistical significance; and varying conditions and parameters would be a nightmare.

Computers have no problem with any of that. We can "read the minds" of our computerized "voters" to determine their exact utility values for everything every time in common units; computers can run more elections than in all human history, to evaluate BRs to 4 significant figures; we can compare any collection of voting methods; and we completely understand and control the experiments (including controlling voter behaviors).

The Bayesian regret of an election system E usually changes if we

- 1. Vary the number of voters,
- 2. Vary the number of candidates,
- 3• Vary the kind of "utility generator" (e.g. could be based on "issues" with different methods for positioning the candidates in different-dimensional "issue spaces"),
- 4• Assume different kinds of "voter strategic behaviors" (possibly including "honesty"), and/or try different mixes of honest and strategic voters, or
- 5. Include different amounts of "voter ignorance."

To describe the last concept: we can put in voter ignorance by artificially adding random noise to the voter's private utility values, then having voters act based on those distorted values. The greater the amplitude of the noise, the more ignorance there is. (But BR is measured using the true undistorted utilities, of course.) So there are at least 5 different "knobs" we can "turn" on our machine for measuring BR(E), which in combination can be altered in 1000 different a priori reasonable ways. Surprisingly, when I first carried out extensive BR experiments in 1999-2000, I found that those knob settings *didn't matter*. More precisely: they do matter, but it turned out that SV *always* was the best (least BR, or tied for least within statistical error margins) voting method, among about 30 methods tried, for *all* 720 knob-combinations tried. I call that "**robustness**."

Another interesting revelation from computer simulations (<u>/IrvExtreme</u>) is that some voting methods (IRV, PV) artificially favor "extremists" over "centrists." But SV has little or no such built-in bias.

5. Failures via 2-party (or 1-party) domination

Since BR concerns only *single* elections, it cannot tell us about the extremely important *multi*election-sequence cumulative-historical effects that build **political parties**. Many countries have developed severe *2-party domination* (2PD):

Plurality voting (PV) in the USA: Currently 98% of senators and 100% of congressmen are from only 2 parties. The 2 exceptional senators both are unaffiliated, not "third party." During the last 80 years, only **0.02%** of USA federal seats have been third party. Today's two most popular third parties (Green and Libertarian) never elected anyone to a Federal seat.

Australia: Australia uses several different voting systems, and its Senate (PR-STV) has *escaped* 2PD. However, its **IRV** seats, e.g. House, suffer 2-party domination comparable to the USA. E.g. of Australia's 564 Federal and State IRV seats in 2006, only *one* was elected by a third party (0.2%) with 33 independents (mostly originally major-party, but who later left it). The

Socialists—although leading parties in many democracies (e.g. holding the most seats in Spain now)—are virtually shut out of US and Australian politics.

All the hullabaloo about "better" voting methods than PV becomes irrelevant with 2PD, because with only 2 candidates they all become essentially equivalent; and even when third parties (rarely) win seats, they remain almost powerless. Even worse than 2PD is 1-party domination. Any country with 2PD or (especially) 1PD lacks any true "democratic marketplace of ideas" hence should *not* be considered a full democracy. George Washington and I contend it would be best if no political parties *existed* (everyone unaffiliated). Jones' (2021) Gallup poll for Jan-Feb 2021 found a landslide 62% of the US public believes that "the Republican and Democratic parties... do such a poor job that a third party is needed" versus 33% who think they "do an adequate job of representing the American people." (5% "don't know": /ElMargins.) Gallup ran that same poll 15 times during 2002-2021; majorities wanted third parties in 13½.

Which voting systems yield 2PD? History tells us that PV often engenders 2PD, and is quicker and more likely to do so if that country is "presidential" and less ethnically/linguistically diverse. It can take 30-200 years for 2PD to develop, but once it does, it (like death) tends to be permanent and self-reinforcing. In contrast, plurality plus top-2-runoff ("T2R," e.g. used to elect French presidents) generally avoids 2PD. IRV has led to, or at least "toward," 2PD in every country that ever used it, and at a rate comparable or exceeding PV's (/Fiji). But unfortunately IRV, T2R, and PV are the only single-winner voting systems enjoying much recent historical experience. Multi-winner "proportional representation" (PR) systems are designed to avoid 2PD, and historically usually have succeeded, except that PR-STV must be considered a "failure" or at least "suspicious/dubious" in this respect since it yielded 2PD in Ireland and especially Malta; also some "party-list" countries suffer 1PD (Cambodia) or 2PD (Albania, Armenia). But the present essay concerns only single-winner systems.

SV was used in Ancient Sparta and Renaissance Venice, two of the world's most successful (e.g. certainly the two longest-lasting: <u>\sqrta \text{Venice}</u>) semi-democracies, and there never was 2PD in either. Indeed if we definitionally demand that a "political party" last ≥15 years, then apparently neither ever *had* a political party (e.g. see Cook p. 69). AV elected the Greek parliament 1864-1926 and yielded 2PD during the middle, but not the two ends, of that era.

Overwhelming evidence from score- and approval-style political *polling* shows the clear trend that "third party" candidates, especially from *small* parties, get more votes with SV relative to the 2 majors, than they get with AV—often 10-50× more. I dubbed that the "nursery effect" (NurserySumm) and it suggests that SV will outperform AV at reducing the risk your country's democracy will fail via 2PD.

Summary of historical evidence: PR systems (except PR-STV and some party-lists) usually successfully avoid 2PD, and

$$PV \approx IRV < PR-STV < party-list PR \approx AV < SV$$

where "A<B" here means "A is more likely to yield 2PD." The three leftmost systems are likely to fail via 2PD; the evidence is sparse for the two rightmost but what there is says SV avoids 2PD 100% of the time.

My (tentative) "NESD hypothesis" *predicts* which voting systems yield 2PD and which do not (<u>NESD</u>):

Systems failing NESD will likely develop 2PD.

(NESD="Naive exaggeration strategy ⇒ duopoly" defined below.) This agrees with essentially all historical evidence. Although I believe some NESD hypothesis variant is approximately correct, the reason I call it "tentative" is that, unfortunately, I see no way—short of experimenting with

numerous voting systems in numerous countries for 200 years!—to decide which variant is approximately correct or to acquire high confidence about this.

Definition of "NESD" and "NESD*" properties. A voting system "fails NESD" if ubiquitous voter exaggeration about the two Major Party candidates to max and min, ensures defeat of all untied third party candidates. (NESD*: the same but demanding the exaggerators rank the two Majors *sole* max and *sole* min.) This sort of exaggerate-about-the-two-majors lie-behavior is employed by about 85% of Australia's rank-order voters. Conitzer and Sandholm 2006 examined it as a simple and effective general-purpose voting strategy.

Systems failing NESD hence predicted to have big 2PD risk: PV, IRV, Bucklin, 3-candidate Borda, STV-PR, party-list PR where you plurality-vote for a *party* not a *person*, and all strict-rank-order Condorcet systems. Also: every system obeying the "majority top" property (MAJT in §7) automatically fails NESD*.

Systems obeying NESD: Range, T2R, AV, and miscellaneous PR systems. If the NESD or NESD* hypotheses are correct, then, e.g, all Condorcet and majority-top-obeying systems are unacceptable since too likely to cause a democracy to destroy itself.

6 Logical criteria

Kenneth Arrow (1951/1963) assumed that a voting method ("social choice function") must be based on *ranking* of options by voters, then analyzed them in terms of their logical *properties*. This, absurdly, makes SV "not a voting method." The truth: any algorithm that inputs votes and outputs winners is a "voting method"; we should allow "votes" to be any information packet whatever.

BR makes the property-based approach obsolete because BR quantitatively assesses the frequency×severity of all properties—even ones nobody ever thought of—acting in combination, and is automatable. Nevertheless, properties have pedagogical value and can be useful mental tools for voting method designers. How can we distinguish silly/bad properties from good ones? Here's a powerful CLAIM: Properties disobeyed by "honest utility voting" (HUV) are silly/bad. HUV is the following voting method. Each voter states their numerical utility, in agreed common units, for the election of each candidate. Greatest utility-sum wins. As previously discussed, HUV is unusable by humans. But it would (almost by definition) be the best of all possible single-winner voting methods for honest superhumans. Then the rationale for that CLAIM is: it is absurd to demand, as an "axiomatic core principle of democracy," any logical property excluding the best voting method for honest superhumans. But SV elections are a subset of HUV elections! Consequently (to oversimplify) Only properties SV obeys, can be good properties! That's a stunning massive victory for SV. It is formulated, using set-theoretic definitions, as a theorem in /PreEmProp; but the problem with this "victory" is that we here just oversimplified; really that theorem only applies to a certain *class* of properties, and should not be used for properties concerning dishonest-strategic voting. Despite those restrictions our CLAIM remains very powerful.

List of desirable-sounding properties ("†" if the CLAIM argues it is silly/bad):

UNTD: The set of "tied" elections lacking a unique winner, has "measure zero" relative to the set of all possible elections.

NOVE: No voter has "veto power," i.e. alone can prevent any one candidate from winning, regardless of the other votes.

NEUT: permuting the candidate names on the ballots permutes their winning probabilities the same way.

ANON: permuting the voters never alters the winner.

UNAN: If there is exactly one unanimously top-ranked candidate, he must win.

USED: In any limit $C,V\rightarrow\infty$, almost all the CV data on the V ballots in a random

C-candidate election (for rank-order or ratings ballots) get *used* to determine the winner. [As opposed to IRV elections, which *solicit* vote data, but never *examine* more than a fraction (1+lnC)/C→0 of it /Ignoring.]

MONO: If some voter increases her vote for candidate Y (leaving the rest of their ballot unchanged) that cannot worsen Y's chances of winning; if she decreases her vote for candidate Z (leaving rest unchanged) that cannot improve Z's win-chances.

FAVO: dishonestly ranking a non-favorite ahead of your favorite, is never strategic, i.e. never improves the winner in that voter's view.

PART: casting an honest vote can never worsen the election result (in that voter's view) versus if she hadn't voted at all.

CLON: If a "clone" of a candidate (rated within $\pm \epsilon$ of the original by every voter) enters or leaves the race, that in the limit $\epsilon \rightarrow 0$ should not affect the winner (aside from possible replacement by a clone).

RMLS: If some losing candidate X is found to be a criminal and ineligible to run, then the same ballots should still be usable to conduct an election with X removed, and should still elect the same winner.

PRCT: it is possible for each precinct to publish a succinct summary of its vote (sub)total, such that the overall nationwide winner can be determined from those subtotals. (Only methods obeying PRCT are "countable in precincts," which provides speed and transparency advantages and helps protect against corruption and sabotage.)

AMAL: if X wins in each state considered alone, then X wins the whole country.

SHN3: in a 3-candidate election, it is never strategically forced to dishonestly vote as though X>Y when your honest view is Y>X. ("Strategically forced" means the winner would worsen in that voter's view.)

IGNC: system allows voters voluntarily to express ignorance about any subset S of candidates, whereupon the system really treats that as "ignorance" and not as e.g, "rank all candidates in S coequal *worst*." (Why this matters: California's 2003 governor election had 135 candidates on ballot; 99% of voters had never heard of most of them. Reputable pollsters *never* merge "don't know" into any opinion-category – for good reason.)

MAJT†: if over 50% of the voters rank X sole-top, then X must win.

COND†: if a "Condorcet winner" candidate X exists, such that for each and every rival Y, some voter majority scores X>Y, then: X must win.

Some people still want to say MAJT and/or COND are "desirable" properties. But after they see these elections:

it is rare that they still want to insist, as a "core axiom of democracy" brooking no exceptions, that "C" must win both elections. Most people find it obvious that "A" would generally be a better winner for society in these scenarios.

SV obeys all no-† properties above (plus NESD and NESD*), albeit the SV-version obeying IGNC disobeys PART. Although SV disobeys the silly/bad MAJT and COND criteria, it satisfies these slight variants:

MAJT': if over 50% of the voters have X as their honest-favorite, then they *have the power* to force X's victory (e.g. by scoring X max and everybody else min).

COND': if a candidate X exists, such that for each rival Y, the ballots with every non-X non-Y candidate erased would elect X: then X must win.

Two illuminating further remarks about MAJT and COND:

1. The "Armenian genocide" election asks voters "should all Armenians be killed and their money stolen and used to reduce taxes for the survivors?" ("Armenians" being a minority, e.g. 10% of the population.) For nontriviality, assume voters have *more* than just those two options, e.g. making Armenians always wear a "star" emblem but without being killed, be enslaved, etc. *If* every vote is both honest and "rational" (using the latter term in the sense economists use it) *then* with any MAJT-obeying method (Borda, MJ, PV, IRV, all Condorcet methods...) the Armenians die! SV is the only common voting method which can allow their survival.

How much the real world will enjoy this benefit depends on voter honesty-vs-strategy and altruism-vs-selfish behavior fractions unfortunately estimable via presently near-zero evidence. I am simply pointing out that *with* both honest voting and economic "rationality" (aka selfishness) -- historically the most heavily-used models in political science and economics -- score voting will outperform all MAJT-obeying methods both on "Armenian genocide," and more generally on any vote involving greatly hurting a minority to slightly benefit a majority.

2. If score (or AV) voters behave according to this *strategy*: (i) identify the two candidates X,Y seeming a priori most likely to win, (ii) exaggerate to score the best one MAX, the other MIN, (iii) score the remaining candidates arbitrarily, *then*: Any "Condorcet winner," if in {X,Y}, will always win. This theorem is a big reason why it is observed in practice that SV and AV usually elect Condorcet winners whenever they exist. Meanwhile with such strategic voters, "Condorcet" methods actually can fail to elect honest-voter Condorcet winners (CWs). Hence quite possibly in practice, SV elects CWs *more* often than "Condorcet" methods! Computer simulations (<u>/StratHonMix.html</u>) confirm that paradox really happens.

Other methods: Almost all seriously proposed methods obey ANON, UNAN, NOVE, and NEUT. IRV fails MONO, FAVO, PART, RMLS, PRCT, AMAL, COND, SHN3, USED, IGNC. Borda, PV, and T2R fail FAVO, CLON, SHN3; Borda and T2R also fail RMLS. All Condorcet methods fail FAVO, PART, AMAL, SHN3, NESD*, but obey COND and MAJT; "Split Cycle" (and Copeland with any fixed number C≥3 of candidates) also fail UNTD. MJ fails COND', AMAL, PART. "STAR voting" fails SHN3, AMAL, PART, RMLS, and certain altered MONO-like properties ((StarVoting)). It obeys NESD, but 3-candidate STAR fails NESD*. Every rank-order-ballot method fails at least one of {CLON, FAVO, NOVE, NEUT}, at least one of {SHN3, NOVE, UNAN}, and at least one of {PART, COND', AMAL, MONO}—3 ways SV outperforms them all ((SimmonsSmithPf)). Strength-of-preference information allows SV to find obvious winners in elections every rank-order method calls a perfect tie ((SimmonsGeomCyc)). The two biggest reasons SV is a robustly good BR-performer are (i) there is a substantial resemblance between SV and HUV (the major difference is that SV's scores are restricted to a bounded range, while HUV's are unrestricted reals), causing SV to work well for "honest" voters;

(ii) for "strategic" voters, SV well-approximates AV, which (with variants) is the *only* voting method ever *intentionally designed* to be strategy-resistant (evidence in Hoffman 1983).

7 Theorems

In the "Random Normal Election Model" and "YN model" (<u>/BestVotSumm</u>) I was able to prove theorems showing SV superior (meaning lower BR) to *any* rank-order method, in the former for *any* mixture of honest and strategic (using "pseudo-best strategy") voters. This effort also was able to express BRs for some systems in closed form in terms of constants like π , and sometimes to identify "best" (i.e. least possible BR) voting methods. None of these feats were ever accomplished previously. No voting method besides SV (and a variant) has ever been proven superior to every rank-order method. Although that's impressive, these models are more simplified, and less realistic, than typical BR computer simulations (§4). Not all theorems in this area are by me, e.g. see Pivato 2014 (<u>/PivatoChar</u>).

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1 References in the form "/RateScaleResearch" mean subpages of http://rangevoting.org/RateScaleResearch. http://RangeVoting.org/RateScaleResearch.html.

Declaration of financial interests: conceivably (unfortunately not likely) this paper's publication will cause a ton of donor money to come to my or related organizations to get Score Voting.