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Proof of N^2+1 Conjecture

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Abstract:

The n^2+1 conjecture states that there are infinitely many natural numbers n such that n^2+1 is a prime number.

This paper defines the distribution type of prime numbers as deterministic random distribution. The distribution is characterized by limited degrees of freedom and a certain degree of predictability. This paper proves that the number of prime numbers in an interval is equivalent to the cumulative probability value. According to this, the number of prime numbers in a certain region can be determined by calculating the cumulative probability value. Therefore, the problems related to prime numbers can be studied, analyzed and proved by using probability and statistics methods.

The conjecture is proved by judging the convergence of the series using probability statistics method.

Key words:

n^2+1 conjecture, probability and statistics, prime number theorem, series, infimum, deterministic random distribution

The n^2+1 conjecture states that there are infinitely many natural numbers n such that n^2+1 is a prime number.

This paper uses the probability and statistics method to prove the conjecture, and now the method is introduced as follows:

1. Proof method of the n^2+1 conjecture

Since prime numbers do not have any mathematical model to represent them with complete accuracy, prime numbers are randomly distributed on the number line. The existing prime number theorem and many related studies show that the number of prime numbers smaller than the integer x is always greater than x/ln(x). In fact, the value of the function x/ln(x) is the infimum of the actual number of prime numbers in the interval. When x tends to infinity, the ratio of the number of prime numbers predicted by the function x/ln(x) to the actual number of prime numbers is close to 1[1][2]. Anyone who counts at any time, the prime numbers appear in a fixed number and in a fixed position, rather than appearing in the way of dice. A large amount of data indicates that the probability of the existence of prime numbers in adjacent regions has a certain degree of similarity, so the distribution type of prime numbers can be defined as deterministic random distribution. The distribution is characterized by limited degrees of freedom and a certain degree of predictability, and the number of interval prime numbers is equivalent to the cumulative probability value. For example, the probability that an integer less than an integer x is a prime number is 1/ln(x), because there are x integers in the interval, so the number of prime numbers and the cumulative probability value are both x/ln(x). According to this, the number of prime numbers that appear in a certain area can be determined by calculating the cumulative probability value. Therefore, the problems related to prime numbers can be studied, analyzed and proved by means of probability and statistics [3][4].

Let n be any positive integer. According to the prime number theorem, the probability of n^2+1 being a prime number is about $1/\ln(n^2+1)$, so when n is any positive integer, the number of prime numbers (cumulative probability) not greater than n is about for

$$\sum_{x=1}^{n}\frac{1}{ln(x^2+1)}$$

Because as x increases, ln(x2+1) approaches the value of ln(x2), the above formula can be simplified to

$$\frac{1}{2}\sum_{x=2}^{n}\frac{1}{ln(x)}$$

And because when n approaches infinity, each term of the function is greater than the corresponding term of the harmonic series, and therefore the function value, that is, the number of prime numbers, tends to infinity. This proves the n2+1 prime number conjecture, there are infinitely many prime numbers in that form.

2. Experimental verification

See Table 1 for the predicted number and actual number of prime numbers of n^2+1 types in different integer regions.

Table 1 Number of prime numbers of n^2+1 types in different integer regions

Integer value n (≤)	Number of prime numbers less than or equal to n^2+1 (cumulative probability)	Actual number of prime numbers less than or equal to n^2+1
20	4.9	8
40	7.9	12
60	10.4	14
80	12.8	16
100	15.0	18
120	17.1	21
140	19.2	24
160	22.2	27
180	23.1	30
200	25.0	31
220	26.9	34

3. Conclusion

Among the positive integers n^2+1, there are infinitely many prime numbers.
Conjecture: Among the positive integers less than or equal to n^2+1, the more accurate estimate of the n^2+1 type prime number is about

$$\frac{5}{8}\sum_{x=2}^{n}\frac{1}{ln(x)}$$

4. References

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