

## Direct Proof of Fermat's Last Theorem

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### Abstract

In 1994 the mathematician Andrews Wiles proved, using concepts of modern mathematics, namely the Modular Elliptic Curves, the Fermat's Last Theorem. That demonstration is long and it is understandable to mathematic specialists, moreover, these mathematical concepts were not known at the time when Fermat lived, so he could not prove it through this road. I thought that there might be another simpler proof which would use the properties of algebraic equations and inequalities. These mathematical concepts were known at the time in which Fermat lived and which, therefore, could apply for the proof of the theorem of which he wrote, in the margin of a page of the book of Arithmetica of Diophantus he was reading, to have found a demonstration "wonderful," but that he had not could to write to the narrowness of the margin itself. I propose, therefore, the following demonstration that uses directly the mathematical properties of algebraic equations and inequalities that are understood by all those who study algebra.

### Theorem

It is possible to divide a power of degree  $n$  in the sum of two powers of the same degree, only if  $n$  is equal to 2.

### Demonstration

$$1) - X^n + Y^n = Z^n$$

$X, Y, Z, n$  are integeres numbers such that  $X < Y < Z$ ;  
we also have that

$$2) - \frac{X^n + Y^n}{Z^n} = 1$$

we have also that

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3) -  $\frac{X^n}{Z^n} > 0 < \frac{1}{2}$  and also

4) -  $\frac{Y^n}{Z^n} > 0 > \frac{1}{2}$  we extract root **n** at **3)** and **4)** and we have

5) -  $\frac{X}{Z} > 0 < \left(\frac{1}{2}\right)^{1/n}$  and yet

6) -  $\frac{Y}{Z} > 0 > \left(\frac{1}{2}\right)^{1/n}$  and so adding the **5)** and the **6)** being  $X+Y > Z$  we have

7) -  $\frac{X+Y}{Z} > 1 < 2 \left(\frac{1}{2}\right)^{1/n}$  extracting the nth roots to the second terms of the inequality, we get that each of the two terms:  $\left(\frac{1}{2}\right)^{1/n}$  **tend to 1** and by adding they **tend to 2**.

In order for the inequality **7)** to be verified, it is necessary that the second member equals the term  $2^{1/n}$  with the same common value of the exponent **n**, therefore we can write

$$8) - \frac{X+Y}{Z} > 1 < 2^{1/n}$$

the inequalities 7) e 8) are verified only for the same values of the exponent  $n$ . If in equation 1) we set the exponent  $n$  equal to 2 after performing the operations from 1) to 6) we obtain that the inequality 7) at second member we have, replacing  $n$  equal to 2 :

$$< 2 \frac{(1)^{1/2}}{2} \text{ that developed it's the same } < \frac{2^{1/2}}{2}$$

Replacing at second member of the inequation 8)  $n$  equal to 2 :  $< \frac{2^{1/2}}{2}$  we see that the results **coincide**, therefore they satisfy the two inequalities 7) and 8). If, for exemple, on the other hand, we set the exponent  $n$  equal to 3 by carrying out the same procedure, we obtain that the inequality 7)

at second member we have, replacing  $n$  equal to 3:  $< 2 \frac{(1)^{1/3}}{2}$  that developed it's the same

$< \frac{4^{1/3}}{2}$ . Replacing at second member of the inequality 8)  $n$  equal to 3 :  $< \frac{2^{1/3}}{2}$  we see that the results **does not coincide**, therefore they **not** satisfy the two inequalities 7) and 8).

With any other value of the exponent  $n$  the values can never coincide and therefore satisfy the inequalities. In fact the values **are divergent**: the value of the 7) tends to 2, while the value of the 8) tends to 1. Raising first and second member of the inequality to 2 we obtain to the numerator also Newton's, binomial which contains the 1)

$$9) - \left( \frac{X+Y}{Z} \right)^2 > 1 < (2^{1/2})^2 \quad \text{and also}$$

$$10) - \frac{X^2 + Y^2}{Z^2} + \frac{2XY}{Z^2} > 1 < 2 \quad \text{from which it is clear that } \frac{X^2 + Y^2}{Z^2} = 1$$

and  $\frac{2XY}{Z^2} < 1$  Therefore we conclude that equation 1), with integeres number, is possible to divide a power of degree  $n$  in the sum of two powers of the same degree, only if  $n$  is equal to 2, that is to say:  $X^2 + Y^2 = Z^2$ .

**Q. E. D.**

## References

- [1] A. Wiles - Modular elliptic curves and Fermat's Last Theorem  
<http://math.stanford.edu/~lekheng/flt/wiles.pdf>
- [2] Simon Singh, L'ultimo teorema di Fermat, Milano, Rizzoli, 1999
- [3] Wikipedia: Fermat's Last Theorem