The paradox that induced electric field has energy in Maxwell's theory of classical electromagnetic field,

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***Abstract*—Those who have studied electromagnetic field theory know that the energy density of the magnetic field is proportional to the square of the magnetic field strength. The energy density of the electric field is proportional to the square of intensity of the electric field. It is assumed that the dimensions of devices such as the inductor are negligible compared with the wavelength of AC, so electromagnetic radiation can be ignore. It is no problem to calculate the energy of the magnetic field according to the above method. However, the electric field has two parts, one is the electrostatic field, and the other is the induced electric field, which is related to the time derivative of the magnetic vector. It is also clear that the electrostatic field has energy. However, it is not clear whether the induced electric field has electric energy. According to Maxwell's equation, it refers to the radiation electromagnetic field equation including displacement current, the energy of the electric field naturally includes the energy of the induced electric field. However, the induced electric field is an electromagnetic induction phenomenon, and the energy of the magnetic field has been increased in this process. It seems that the energy of the induced electric field itself should not be calculated again. On the other hand, according to the electric and magnetic quasi-static electromagnetic field equation, the induced electromagnetic field has no energy. The author believes that the electric and magnetic quasi-static electromagnetic field equation is correct, and the induced magnetic field should not have electric field energy. The author believes that this contradiction is due to the fact that Maxwell's equation (including displacement current term) is not suitable for the case of electric and magnetic quasi-static fields. As the textbook tells us, Maxwell's equations are accurate equations, and magnetic quasi-static or electric and magnetic quasi-static electromagnetic field equations are approximate equations of Maxwell's equations. The author thinks that the Maxwell equation obtained by adding the displacement current term can deduce the result of electromagnetic wave, but it is still a problem equation. The main problem is that the electric field and magnetic field obtained by Maxwell equation are not the seamless extension of the electromagnetic field under the original electric and magnetic quasi-static condition. That is to say, the electric field and magnetic field obtained according to Maxwell's equation actually do not have the properties of the original electric field and magnetic field. In particular, the electric field energy, magnetic field energy and Poynting vector formed by such electric and magnetic fields are unreliable. In the electric and magnetic quasi-static condition, the most unreliable is the energy of the induced electric field. The induced electric field should not have energy. If the induced electric field has energy, we know that the energy is a quadratic function, so the energy of the induced electric field and the electrostatic electric field will have a cross mixing part, which is even more strange. The author thinks that the Poynting theorem is still correct under the quasi-static condition of electric and magnetic field, but the Poynting theorem derived from Maxwell equation (including displacement current) is not reliable.**

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***Index Terms*—Poynting theorem, Maxwell equation, quasi-static, electric field energy, magnetic field energy. Poynting vector. Induced electric field. Electrostatic field.**

I. INTRODUCTION

A. Review of the author's electromagnetic field theory

THE author proposed the electric field mutual energy theorem [8], [19], [18] in 1987. In fact, similar formulas before and after the author are considered as reciprocity theorems [17], [13], [5], [7]. One of the main reasons why this formula is considered to be the reciprocity theorem is that it involves the advanced wave or the advanced potential. Up to now, the advanced wave has not been accepted as the objective existence of physics in the classical electromagnetic field theory and quantum physics. The advanced wave violates the causality recognized by us today. This causal relationship only allows time to move in the future direction. The time direction of the advance wave is the past. If the advanced wave is not recognized, the mutual energy theorem cannot be called as an energy theorem. Therefore, it is called reciprocity theorem at most. As the reciprocity theorem formula is similar to the Green's function, it can be regarded as a mathematical formula. The two quantities in the formula are the retarded wave and the advanced wave. The retarded wave can be regarded as a physical real quantity and the advanced wave can be regarded as a virtual quantity. However, if this formula is regarded as the energy theorem, some two quantities in the formula must be real physical quantities.

Since 2014, the author has noticed that a group of physicists support the existence of advanced waves, among which the most important ones are Wheeler and Feynman's absorber theory [1], [2]. The absorber theory is based on the theory of action and reaction [6], [14], [16]. Cramer proposed a quantum mechanical transaction interpretation based on the existence of advanced waves [3], [4]. Stephenson also proposed his advanced wave theory [15]. All these give the author great encouragement. The author first proved the theorem of mutual energy from Poynting's theorem, so the theorem of mutual energy is indeed a theorem of energy. At the same time, the author puts forward mutual energy principle, self energy principle and mutual energy flow theorem [9]. The author thinks that the mutual energy flow is photon. The mutual energy theorem is actually the conservation law of energy flow. However, according to Maxwell's equation, it cannot be proved that the mutual energy theorem is the law of energy conservation, so the author adds the time reversal wave to Maxwell's equation. The author has successively published several papers supporting advanced wave [10], [11], [12].

Recently, the author found that the electromagnetic wave and its own time reversal wave can be replaced by reactive power wave. The reactive power wave is a wave in which the electric field and the magnetic field maintain 90 degrees in phase. Maxwell's equation does not support this kind of wave, and the mutual energy theory proposed by the author supports the existence of this kind of wave. In this way, the author considers the retarded wave radiated by the dipole transmitting antenna and the advanced wave radiated by the dipole receiving antenna. These waves are reactive power waves. Therefore, although they carry electromagnetic energy, they do not transfer electromagnetic energy. The author found that electromagnetic energy is transferred only by mutual energy flow. The mutual energy flow has the characteristics of photons. In addition, the energy flow from the transformer primary coil to the transformer secondary coil is also explained by the mutual energy flow. In the past, many people tried to explain the energy flow of transformer with Poynting vector, but they did not really succeed. All these contents will be published in the near future.

In this way, the author's electromagnetic theory and the classical electromagnetic theory are quite different, not only because they contain the advanced wave. Even the calculation of the retarded wave and the advanced wave itself can no longer be calculated by Maxwell's equation. It is found that the definition of magnetic field and electric field and the field of electromagnetic quasi-static state have changed greatly due to the introduction of displacement current. The electric field and magnetic field after considering the displacement current can no longer be considered as a seamless extension of the original electric field and magnetic field. This leads to the invalidation of Poynting's theorem after considering displacement and current. This failure indicates that the calculation of Poynting vector is wrong, and the energy of electric field and magnetic field is also wrong.

If the author proposed that Poynting theorem and Maxwell equation are wrong, most people will not agree. In order to explain this problem, a simple problem is considered in this paper, that is, the energy of the induced electric field. According to Maxwell's theory including displacement current, the induced electric field should contribute to the energy of the electric field. However, according to Maxwell's own deduction of Faraday's theorem and Helmhertz's deduction of magnetic field energy, it is obvious that the induced electric field has no contribution to the electric field energy. The author also illustrates this point by using a circuit problem including a capacitor, an inductor and a resistor. The author finds that no matter whether the answer is yes or no, it will bring contradictions. If the induced electric field contributes to the electric field energy. The calculated energy must be more than the real energy. If the induced electric field does not have energy, Maxwell's electromagnetic field theory will be wrong again. This constitutes the induced electric field energy paradox.

B. Review of electromagnetic field theory

We know the energy of the electromagnetic field. First, look at the energy of the electric field

$$\mathbb{E}_E = \frac{\epsilon_0}{2}\boldsymbol{E}\cdot\boldsymbol{E}$$

The energy of the magnetic field,

$$\mathbb{E}_H = \frac{\mu_0}{2}\boldsymbol{H}\cdot\boldsymbol{H}$$

The electric and magnetic fields are defined as:

$$\boldsymbol{B} \equiv \nabla\times\boldsymbol{A}$$

$$\boldsymbol{E} \equiv -\frac{\partial}{\partial t}\boldsymbol{A} - \nabla\phi$$

"≡" means is defined as. We consider the electric and magnetic quasi-static condition,

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}dV$$

$$\phi = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho}{r}dV$$

There is no dispute about the energy of the magnetic field, but there are two opinions about the energy of the electric field. One is that the energy of the electric field is only the electrostatic field,

$$\boldsymbol{E}_s = -\nabla\phi$$

contribute to the electric field energy, and the induced electric field

$$\boldsymbol{E}_i = -\frac{\partial}{\partial t}\boldsymbol{A}$$

is no contribution to the energy of the electric field, hence,

$$\mathbb{E}_E = \frac{\epsilon_0}{2}\boldsymbol{E}_s\cdot\boldsymbol{E}_s \tag{1}$$

Here $\mathbb{E}_E$ is electric energy. Another view is that the energy of the electric field should be,

$$\mathbb{E}_E = \frac{\epsilon_0}{2}(\boldsymbol{E}_s+\boldsymbol{E}_i)\cdot(\boldsymbol{E}_s+\boldsymbol{E}_i) \tag{2}$$

The key to this problem is the induced electric field $\boldsymbol{E}_i$ should be regarded as an electric field or not, $\boldsymbol{E}_i$ and the electrostatic field $\boldsymbol{E}_s$ do they have the same energy? If $\boldsymbol{E}_i$ have also energy, another problem is the cross mixing term of electrostatic field and induced electric field

$$\epsilon_0\boldsymbol{E}_s\cdot\boldsymbol{E}_i$$

does it also have energy?

This paper answers this question and believes that the induced electromagnetic field should not have energy. The reason for this problem is that Maxwell's equation refers to the radiation electromagnetic field equation including the displacement current, which is not suitable for the electric and magnetic quasi-static condition. In other papers, the author discussed that the electric field energy, the magnetic field energy and the Poynting vector in the Poynting theorem derived from Maxwell's equation are all problematic. The author finds that Poynting's theorem is still correct under the condition of electric and magnetic quasi-static electromagnetic field.

II. Deduce Faraday's electromagnetic induction law

Let us seen how the Faraday's induction law is deduced by electromagnetic field masters.

A. Derive Faraday induction law from Neumann formula

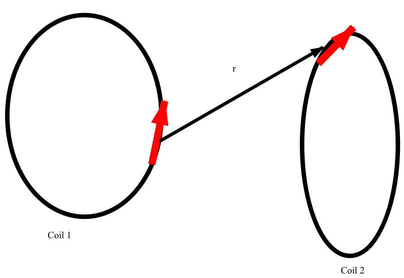

Fig. 1. Spiral tube with current.

In the field of electromagnetic field, the first person to deduce Faraday's law of electromagnetic induction was attributed to Neumann, who gave the formula of Faraday's electromagnetic induction electromotive force in 1845,

$$\mathcal{E}_{2,1} = -\frac{\partial}{\partial t}\oint_{C_2}\frac{\mu_0}{4\pi}\oint_{C_1}\frac{I_1 d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2}{r} \tag{3}$$

$\mathcal{E}_{2,1}$ is the induced electromotive force induced on coil 2 by the current coil 1. From this formula it can be defined that the magnetic vector,

$$\boldsymbol{A}_1 = \frac{\mu_0}{4\pi}\oint_{C_1}\frac{I_1 d\boldsymbol{l}_1}{r} \rightarrow \frac{\mu_0}{4\pi}\iiint_{V_1}\frac{\boldsymbol{J}_1 dV}{r}$$

"$\rightarrow$" means from something derive something, calculate

$$\nabla\times\boldsymbol{A}_1 = \frac{\mu_0}{4\pi}\iiint_{V_1}\nabla\frac{1}{r}\times\boldsymbol{J}_1 dV = \frac{\mu_0}{4\pi}\iiint_{V_1}\boldsymbol{J}_1\times\frac{\boldsymbol{r}}{r^3}dV$$

Consider the definition of the magnetic field according to Biot-Savart's Law:

$$\boldsymbol{B}_1 = \frac{\mu_0}{4\pi}\iiint_{V_1}\boldsymbol{J}_1\times\frac{\boldsymbol{r}}{r^3}dV$$

Hence, there is,

$$\boldsymbol{B}_1 = \nabla\times\boldsymbol{A}_1 \tag{4}$$

$\boldsymbol{A}_1$ is the magnetic vector potential on the coil 1. Thus,

$$\mathcal{E}_{2,1} = -\frac{\partial}{\partial t}\oint_{C_2}\boldsymbol{A}_1\cdot d\boldsymbol{l}_2$$

The induced electromotive force is defined as:

$$\mathcal{E}_{2,1} \equiv \oint_{C_2}\boldsymbol{E}_1\cdot d\boldsymbol{l}_2$$

Hence, there is,

$$\oint_{C_2}\boldsymbol{E}_1\cdot d\boldsymbol{l}_2 = -\oint_{C_2}\frac{\partial}{\partial t}\boldsymbol{A}_1\cdot d\boldsymbol{l}_2$$

or

$$\oint_{C_2}(\boldsymbol{E}_1 + \frac{\partial\boldsymbol{A}_1}{\partial t})\cdot d\boldsymbol{l}_2 = 0$$

or

$$\boldsymbol{E}_1 + \frac{\partial\boldsymbol{A}_1}{\partial t} = -\nabla\phi_1$$

or

$$\boldsymbol{E}_1 = -\frac{\partial\boldsymbol{A}_1}{\partial t} - \nabla\phi_1 \tag{5}$$

$$\nabla\times\boldsymbol{E}_1 = -\frac{\partial\nabla\times\boldsymbol{A}_1}{\partial t} - \nabla\times\nabla\phi_1$$

Considering (4),

$$\nabla\times\boldsymbol{E}_1 = -\frac{\partial\boldsymbol{B}_1}{\partial t} \tag{6}$$

The above formula is the Faraday's law of electromagnetic induction. Although the above formula (6) is not derived by Neumann, people still attribute the formula of magnetic vector potential and Faraday law to Neumann.

B. Maxwell's derivation of Faraday's law

Maxwell may be the first to obtain the following form of Faraday's law, which was included in Maxwell's 1855 paper "on Faraday force lines"

$$\boldsymbol{E}_1 = -\frac{\partial\boldsymbol{A}_1}{\partial t} - \nabla\phi_1 \tag{7}$$

Because in other people's papers, such as Kirchhoff's 1857 paper

$$\boldsymbol{J}_2 = \sigma(-\frac{\partial\boldsymbol{A}_1}{\partial t} - \nabla\phi_1)$$

$\boldsymbol{J}_2$ is the current on the secondary coil. σ is the conductivity. Maxwell was very excited when he wrote Faraday formula (7). He thought he had found an important concept of Faraday's law of electromagnetic induction. Maxwell obtained the formula of vector potential (4) from William Thomson, who was later ennobled as Lord Kelvin. Maxwell was afraid that the credit for his formula (7) would be given to Kelvin, and he specially declared that he had obtained the formula (7) after studying the Faraday experiment. The author thinks that Maxwell's derivation formula (7) is due to his contribution. But it is not unreasonable for people to attribute this credit to Neumann. It is not particularly difficult to deduce (5) from Neumann's formula (3).

Maxwell's method of obtaining the above formula (7) is quite strange. It is not as simple as what we said above today. Maxwell first learned from the Helmholtz electromagnetic energy formula that the energy of the magnetic field is defined as,

$$\mathbb{E}_B = \frac{1}{2\mu_0}\iiint_V\boldsymbol{B}\cdot\boldsymbol{B}dV \tag{8}$$

Therefore, the power of the magnetic field is

$$P = \frac{\partial}{\partial t}\mathbb{E}_B = \frac{1}{\mu_0}\iiint_V\boldsymbol{B}\cdot\frac{\partial}{\partial t}\boldsymbol{B}dV = \iiint_V\boldsymbol{H}\cdot\frac{\partial}{\partial t}\boldsymbol{B}dV$$

At that time, it was known that current can do work to the magnetic field, and the power of this work is,

$$UI = -\mathcal{E}I = -\oint_C\boldsymbol{E}\cdot d\boldsymbol{l}I$$

$$= -\iiint_V\boldsymbol{E}\cdot\boldsymbol{J}dV$$

U is the voltage on the coil and I is the current on the coil. $\mathcal{E}$ is the induced electromotive force on the coil. Where a negative sign indicates power to the magnetic field. A positive sign will indicates that power is drawn from the magnetic field. It can be seen that the magnetic field power increases to,

$$-\iiint_V \boldsymbol{E}\cdot\boldsymbol{J}dV=\frac{\partial}{\partial t}\mathbb{E}_B$$

or

$$-\iiint_V \boldsymbol{E}\cdot\boldsymbol{J}dV=\iiint_V \boldsymbol{H}\cdot\frac{\partial}{\partial t}\boldsymbol{B}dV \tag{9}$$

Consider that the magnetic vector is

$$\boldsymbol{A}=\frac{\mu_0}{4\pi}\iiint_V\frac{\boldsymbol{J}dV}{r}$$

We omit the subscript 1 and according to the mathematical formula,

$$\nabla\cdot(\frac{\partial}{\partial t}\boldsymbol{A}\times\boldsymbol{H})=\nabla\times\frac{\partial}{\partial t}\boldsymbol{A}\cdot\boldsymbol{H}-\frac{\partial}{\partial t}\boldsymbol{A}\cdot\nabla\times\boldsymbol{H}$$

Maxwell seemed to consider

$$\nabla\cdot(\frac{\partial}{\partial t}\boldsymbol{A}\times\boldsymbol{H})=0$$

Or think that the above formula is self-evident or wordless. Further,

$$\nabla\times\frac{\partial}{\partial t}\boldsymbol{A}\cdot\boldsymbol{H}-\frac{\partial}{\partial t}\boldsymbol{A}\cdot\nabla\times\boldsymbol{H}=0$$

or

$$\frac{\partial}{\partial t}\nabla\times\boldsymbol{A}\cdot\boldsymbol{H}-\frac{\partial}{\partial t}\boldsymbol{A}\cdot\nabla\times\boldsymbol{H}=0$$

Considering,

$$\boldsymbol{B}=\nabla\times\boldsymbol{A}$$
$$\nabla\times\boldsymbol{H}=\boldsymbol{J} \tag{10}$$

There is,

$$\frac{\partial}{\partial t}\boldsymbol{B}\cdot\boldsymbol{H}-\frac{\partial}{\partial t}\boldsymbol{A}\cdot\boldsymbol{J}=0$$

or

$$\iiint_V\frac{\partial}{\partial t}\boldsymbol{B}\cdot\boldsymbol{H}dV=\iiint_V\frac{\partial}{\partial t}\boldsymbol{A}\cdot\boldsymbol{J}dV \tag{11}$$

Considering the previous formula (9) obtained from the Helmholtz formula

$$-\iiint_V\boldsymbol{E}\cdot\boldsymbol{J}dV=\iiint_V\frac{\partial}{\partial t}\boldsymbol{A}\cdot\boldsymbol{J}dV \tag{12}$$

Change body current into line current

$$-\int_C\boldsymbol{E}\cdot d\boldsymbol{l}I=\int_C\frac{\partial}{\partial t}\boldsymbol{A}\cdot d\boldsymbol{l}I$$

or

$$\int_C\boldsymbol{E}\cdot d\boldsymbol{l}=-\int_C\frac{\partial}{\partial t}\boldsymbol{A}\cdot d\boldsymbol{l}$$

$$\int_C(\boldsymbol{E}+\frac{\partial}{\partial t}\boldsymbol{A})\cdot d\boldsymbol{l}=0$$

$$(\boldsymbol{E}+\frac{\partial}{\partial t}\boldsymbol{A})=-\nabla\phi$$
$$\boldsymbol{E}=-\frac{\partial\boldsymbol{A}}{\partial t}-\nabla\phi \tag{13}$$

The above is the Faraday induction law.

C. Hemhertz derived electromagnetic energy

Helmhertz derived electromagnetic energy should also use the electromagnetic induction law given by Neumann. Considering the ampere circuital law $\boldsymbol{J}=\nabla\times\boldsymbol{H}$,

$$\iiint_V\boldsymbol{E}\cdot\boldsymbol{J}dV=\iiint_V\boldsymbol{E}\cdot\nabla\times\boldsymbol{H}dV$$

Considering the mathematical formula,

$$\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})=\nabla\times\boldsymbol{E}\cdot\boldsymbol{H}-\boldsymbol{E}\cdot\nabla\times\boldsymbol{H}$$

There is,

$$\boldsymbol{E}\cdot\nabla\times\boldsymbol{H}=-\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})+\nabla\times\boldsymbol{E}\cdot\boldsymbol{H}$$

$$\iiint_V\boldsymbol{E}\cdot\boldsymbol{J}dV=\iiint_V(-\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})+\nabla\times\boldsymbol{E}\cdot\boldsymbol{H})dV$$

$$\iiint_V\boldsymbol{E}\cdot\boldsymbol{J}dV=-\oiint_\Gamma\cdot(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma+\iiint_V\nabla\times\boldsymbol{E}\cdot\boldsymbol{H}dV$$

Ignoring the radiation term,

$$\oiint_\Gamma\cdot(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma=0$$

So,

$$\iiint_V\boldsymbol{E}\cdot\boldsymbol{J}dV=\iiint_V\nabla\times\boldsymbol{E}\cdot\boldsymbol{H}dV$$

Considering (6) i.e. Faraday's law,

$$\nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t} \tag{14}$$

Obtain

$$\iiint_V\boldsymbol{E}\cdot\boldsymbol{J}dV=\iiint_V(-\frac{\partial\boldsymbol{B}}{\partial t})\cdot\boldsymbol{H}dV$$

or

$$-\iiint_V\boldsymbol{E}\cdot\boldsymbol{J}dV=\iiint_V\frac{\partial\boldsymbol{B}}{\partial t}\cdot\boldsymbol{H}dV$$
$$=\frac{\partial}{\partial t}\frac{1}{2}\iiint_V\boldsymbol{B}\cdot\boldsymbol{H}dV$$

Due to - $\iiint_5\boldsymbol{E}\cdot\boldsymbol{J}dV$ is the power of work done to the system, $\iiint_V\frac{\partial\boldsymbol{B}}{\partial t}\cdot\boldsymbol{H}dV$ can be regarded as the power of energy increase, so the magnetic field energy is

$$\mathbb{E}_B=\frac{1}{2\mu_0}\iiint_V\boldsymbol{B}\cdot\boldsymbol{B}dV$$

In fact, Helmholtz also derives the energy formula (8) according to the formula (9), and then obtains the above formula. The above derivation does not completely follow the original method of Helmhertz. Perhaps Helmhertz derives the energy of the magnetic field directly from the Neumann formula without applying the formula ([14).

However, in the final analysis, Maxwell's derivation still uses Neumann's law of electromagnetic induction. But there is a circle in the derivation. This is normal. Any discovery does not necessarily follow a straight line. It is normal to make a slight circle. However, the (10) ampere circuital law is used in this circle. If the expression "correct" according to the complete Maxwell equation should be

$$\nabla\times\boldsymbol{H}=\boldsymbol{J}+\frac{\partial}{\partial t}\boldsymbol{D} \tag{15}$$

That is, Maxwell neglected the term of displacement current $\frac{\partial}{\partial t}\boldsymbol{D}$. The approximation is used in Maxwell's derivation because

$$\nabla \cdot (\frac{\partial}{\partial t}\boldsymbol{A} \times \boldsymbol{H}) = 0$$

According to Maxwell's theory, it is actually close to Poynting's theorem,

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \nabla \cdot ((-\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\phi) \times \boldsymbol{H})$$

We know that according to Maxwell's theory of radiated electromagnetic field, the Poynting vector is not zero, and the area integral above left does not tend to zero even if the radius is infinite. So it seems that Max's derivation uses two approximations. What should be obtained is an approximate formula, but why is this approximate formula (13) so successful? Why is the derivation approximate, but the conclusion is absolutely correct? We know that the radiation electromagnetic field is based on Maxwell ampere circuital law (15) and Faraday law (16)

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{16}$$

According to the classical electromagnetic theory, these two formulas are accurate!

D. Poynting theorem

Poynting's theorem can be derived from Maxwell's equations (15-16),

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \nabla \times \boldsymbol{E} \cdot \boldsymbol{H} - \boldsymbol{E} \cdot \nabla \times \boldsymbol{H}$$

Considering Maxwell ampere loop law and Faraday law (15-16),

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = -\frac{\partial \boldsymbol{B}}{\partial t} \cdot \boldsymbol{H} - \boldsymbol{E} \cdot (\boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{D})$$

Or Poynting's theorem in the form of,

$$-\oiint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma = \iiint_{V} (\boldsymbol{E} \cdot \boldsymbol{J} + \frac{\partial \boldsymbol{B}}{\partial t} \cdot \boldsymbol{H} + \boldsymbol{E} \cdot \frac{\partial}{\partial t}\boldsymbol{D}) dV \tag{17}$$

In Poynting's theorem,

$$\frac{\partial \boldsymbol{B}}{\partial t} \cdot \boldsymbol{H}$$

Is the increase of the magnetic field density, so the energy density of the magnetic field can be defined as

$$\mathbb{E}_B = \frac{\boldsymbol{B} \cdot \boldsymbol{B}}{2\mu_0}$$

in addition

$$\boldsymbol{E} \cdot \frac{\partial}{\partial t}\boldsymbol{D}$$

The electric field energy density can be defined as,

$$\mathbb{E}_E = \frac{\boldsymbol{D} \cdot \boldsymbol{D}}{2\epsilon_0} = \frac{\epsilon_0}{2}\boldsymbol{E} \cdot \boldsymbol{E}$$

Now let's look at the energy of this electric field $\mathbb{E}_E$ How to calculate the $\boldsymbol{E}$ in? In the first method,

$$\boldsymbol{E} = \boldsymbol{E}_s = -\nabla\phi \tag{18}$$

The above electric field is an electrostatic field and the corresponding energy is the energy of the electrostatic field,

$$\mathbb{E}_E = \frac{\epsilon_0}{2}\nabla\phi \cdot \nabla\phi \tag{19}$$

The second method

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\phi \tag{20}$$

$$\mathbb{E}_E = \frac{\epsilon_0}{2}(-\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\phi) \cdot (-\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\phi)$$

In this formula, it seems that the energy of the electric field should consider the contribution of $-\frac{\partial \boldsymbol{A}}{\partial t}$ induced electric field. However, we know that the induced electric field has contributed to the energy of the magnetic field. See (12). It seems that it should no longer contribute to the energy of the electric field!

E. Spiral pipe

The following figure 2 is a spiral tube. We want to find the energy when the current I is known. Here, energy refers to energy including electric field energy and magnetic field energy.

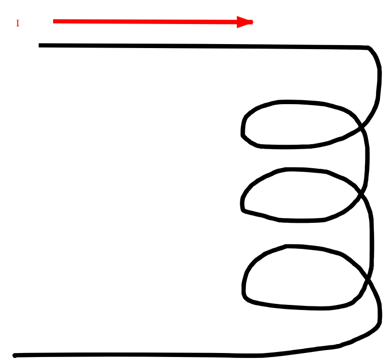

Fig. 2. Spiral tube with current.

In the spiral tube inductor, we generally think that the magnetic energy density in the spiral tube is,

$$\mathbb{E}_B = \frac{\boldsymbol{B} \cdot \boldsymbol{B}}{2\mu_0} \tag{21}$$

If the current is AC, if the energy of the electric field is zero according to (19). Only the magnetic field energy density is not zero. But if we consider (20), the energy density of the electric field is not zero. However, the author still thinks that the energy density of the electric field should be calculated according to (19). In this case, there is no electric field energy, and the energy of the electric field should only exist in the capacitor. It seems that the induced electric field in the air should not be given energy.

In fact, we use the formula (9) to get the energy density of the magnetic field, but this formula should be a simplified form Poynting's theorem,

$$-\iiint_V \boldsymbol{E}\cdot\boldsymbol{J}dV$$
$$=\iiint_V \boldsymbol{H}\cdot\frac{\partial}{\partial t}\boldsymbol{B}dV+\iiint_V \boldsymbol{E}\cdot\frac{\partial}{\partial t}\boldsymbol{D}dV+\oiint_\Gamma(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma \tag{22}$$

If we ignore the radiation energy from the above, i.e. considers the radius of the surface Γ is infinity, hence,

$$\oiint_\Gamma(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma=0$$

The increase in electric field energy must also be neglected,

$$\iiint_V \boldsymbol{E}\cdot\frac{\partial}{\partial t}\boldsymbol{D}dV \tag{23}$$

Note that this electric field energy is related to the displacement current $\frac{\partial}{\partial t}\boldsymbol{D}$. Finally get,

$$-\iiint_V \boldsymbol{E}\cdot\boldsymbol{J}dV=\iiint_V \boldsymbol{H}\cdot\frac{\partial}{\partial t}\boldsymbol{B}dV \tag{24}$$

This is (9). If Maxwell's equation is correct and the displacement current does contribute to the energy of the electric field, then the formula (9) is invalid, because the energy represented by the formula (23) cannot be ignored. (9) is at least inaccurate. It should be noted that even under the magnetic quasi-static condition ($L \ll \lambda$, the size of the device is much smaller than the wavelength), if the electric field energy (23) caused by the displacement current is absolutely not negligible in terms of value.

There are only two results, or the magnetic field energy formula (21) is not accurate. Or the displacement current contained in Maxwell's equation is not able to produce real electric field energy. Note that our discussion here is under the magnetic quasi-static condition. Or under the electric and magnetic quasi-static conditions. The following two supsection review quasi-static electromagnetic field.

F. Magnetic quasi-static electromagnetic field

The magnetic quasi-static electromagnetic field is an electromagnetic field satisfying the following equation:

$$\nabla\cdot\boldsymbol{E}=\rho/\epsilon_0$$

$$\boldsymbol{B}=\nabla\times\boldsymbol{A}\leftrightarrow\nabla\cdot\boldsymbol{B}=0$$
$$\boldsymbol{E}=-\nabla\phi-\frac{\partial}{\partial t}\boldsymbol{A}\leftrightarrow\nabla\times\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{B}$$
$$\nabla\times\boldsymbol{H}=\boldsymbol{J}$$

The magnetic quasi-static electromagnetic field equation is the Maxwell equation with displacement current removed. The Poynting theorem corresponding to this situation is,

$$-\iiint_V \boldsymbol{E}\cdot\boldsymbol{J}dV=\iiint_V \boldsymbol{H}\cdot\frac{\partial}{\partial t}\boldsymbol{B}dV+\oiint_\Gamma(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma \tag{25}$$

In this situation if the radiation term $\oiint_\Gamma(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma$ can be omit, formula (24) is obtained. The induced electric field has no contribution to electric energy.

G. Electric and magnetic quasi-static electromagnetic field equation

We now re-derive the equations of the electric and magnetic quasi-static fields, starting from the formula of the magnetic vector potential

$$\boldsymbol{A}=\frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}dV$$

Hence,

$$\begin{aligned}\nabla\cdot\boldsymbol{A}&=\nabla\cdot\frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}dV\\ &=\frac{\mu_0}{4\pi}\iiint_V \nabla\cdot(\frac{\boldsymbol{J}}{r})dV\\ &=\frac{\mu_0}{4\pi}\iiint_V \nabla(\frac{1}{r})\cdot\boldsymbol{J}dV\\ &=\frac{\mu_0}{4\pi}\iiint_V (-\nabla'(\frac{1}{r}))\cdot\boldsymbol{J}dV\\ &=\frac{\mu_0}{4\pi}\iiint_V (\frac{1}{r})\nabla'\cdot\boldsymbol{J}dV\\ &=\frac{\mu_0}{4\pi}\iiint_V (\frac{1}{r})(-\frac{\partial}{\partial t}\rho)dV\\ &=-\mu_0\epsilon_0\frac{1}{4\pi\epsilon_0}\iiint_V (\frac{1}{r})\frac{\partial}{\partial t}\rho dV\\ &=-\mu_0\epsilon_0\frac{\partial}{\partial t}\frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho}{r}dV\end{aligned}$$

or

$$\nabla\cdot\boldsymbol{A}=-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi \tag{26}$$

The above formula is Lorenz gauge condition. The above formula shows that the vector potential and the scalar potential should satisfy the Lorenz gauge condition. In the above the current continuity equation,

$$\nabla'\cdot\boldsymbol{J}=-\frac{\partial}{\partial t}\rho$$

and the formula

$$\nabla(\frac{1}{r})=-\nabla'(\frac{1}{r})$$

is considered. In addition,

$$\iiint_V \nabla'\cdot(\frac{\boldsymbol{J}}{r})dV=\oiint_\Gamma(\frac{\boldsymbol{J}}{r})\cdot\hat{n}d\Gamma=0$$

Here Γ is at outside of the $\boldsymbol{J}$. Hence,

$$\iiint_V \nabla'\cdot(\frac{\boldsymbol{J}}{r})dV=\iiint_V (\frac{\boldsymbol{1}}{r})\nabla'\cdot\boldsymbol{J}dV+\iiint_V \nabla'(\frac{1}{r})\cdot\boldsymbol{J}dV=0$$

Hence,

$$\iiint_V \nabla'(\frac{1}{r})\cdot\boldsymbol{J}dV=-\iiint_V (\frac{\boldsymbol{1}}{r})\nabla'\cdot\boldsymbol{J}dV$$

and the scale potential is defined as,

$$\phi=\frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho}{r}dV$$

Next, consider the mathematical formula,

$$\nabla\times\nabla\times\boldsymbol{A}=\nabla(\nabla\cdot\boldsymbol{A})-\nabla^2\boldsymbol{A}$$

Considering,

$$\nabla^2 \boldsymbol{A} = \nabla^2 \frac{\mu_0}{4\pi} \iiint_V \frac{\boldsymbol{J}}{r} dV$$
$$= \mu_0 \iiint_V (\frac{\nabla^2}{4\pi}\frac{1}{r}) \boldsymbol{J} dV$$
$$= \mu_0 \iiint_V (-\delta(\boldsymbol{x}-\boldsymbol{x}') \boldsymbol{J} dV$$
$$= -\mu_0 \boldsymbol{J}$$

In the above has considered,

$$\frac{\nabla^2}{4\pi}\frac{1}{r} = -\delta(\boldsymbol{x}-\boldsymbol{x}')$$

Hence,

$$\nabla \times \nabla \times \boldsymbol{A} = \nabla(\nabla \cdot \boldsymbol{A}) - (-\mu_0 \boldsymbol{J})$$

Considering,

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

There is,

$$\nabla \times \boldsymbol{B} = \nabla(\nabla \cdot \boldsymbol{A}) + \mu_0 \boldsymbol{J}$$

Consider Lorenz gauge condition (26)

$$\nabla \times \boldsymbol{B} = \mu_0 \frac{\partial}{\partial t}(-\epsilon_0 \nabla \phi) + \mu_0 \boldsymbol{J}$$

Considering $\boldsymbol{B} = \mu_0 \boldsymbol{H}$

$$\nabla \times \boldsymbol{H} = \frac{\partial}{\partial t}(-\epsilon_0 \nabla \phi) + \boldsymbol{J}$$

or

$$\nabla \times \boldsymbol{H} = \frac{\partial}{\partial t}\boldsymbol{D}_s + \boldsymbol{J} \tag{27}$$

In which,

$$\boldsymbol{D}_s = \epsilon_0 \boldsymbol{E}_s$$

$$\boldsymbol{E}_s = -\nabla \phi$$

The formula (27) is the ampere circuital law of electric and magnetic quasi-static electromagnetic field. In this formula, the displace current only includes the electric-static field, but not induced electric field.

H. Poynting's theorem under electric and magnetic quasi-static electromagnetic field

$\boldsymbol{E}_s$ is the electric field in the electric and magnetic quasi-static state. This electric field does not include an induced electric field. The Poynting theorem obtained from this formula (27) instead of the formula (15) is,

$$-\iiint_V \boldsymbol{E} \cdot \boldsymbol{J} dV$$
$$= \iiint_V \boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B} dV + \iiint_V \boldsymbol{E}_s \cdot \frac{\partial}{\partial t}\boldsymbol{D}_s dV + \oiint_\Gamma (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma$$

In this formula, the electromagnetic quasi-static radiation term $\oiint_\Gamma (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma$ can be ignored. hence,

$$-\iiint_V \boldsymbol{E} \cdot \boldsymbol{J} dV = \iiint_V \boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B} dV + \iiint_V \boldsymbol{E}_s \cdot \frac{\partial}{\partial t}\boldsymbol{D}_s dV$$

If the line has no capacitor,

$$\nabla \phi = 0$$

Considered the last item of the Poynting vector is radiation term that can be ignored. We get

$$-\iiint_V \boldsymbol{E} \cdot \boldsymbol{J} dV = \iiint_V \boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B} dV \tag{28}$$

In this way, the electromagnetic induced electric field does only generate magnetic field energy, but not electric field energy. The energy formula (28) cited by both Helmholtz and Maxwell is meaningful. The above formula shows that to make the magnetic field energy formula (21) hold, the ampere circuital law should be (27). This is a quasi-static electromagnetic field of electricity and magnetism. Therefore, the quasi-static electromagnetic fields of electricity and magnetism should be,

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{D}_s$$

Here $\boldsymbol{D}_s = \epsilon_0 \boldsymbol{E}_s$, so the problem is Maxwell's equation

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{29}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{D} \tag{30}$$

Is it a more accurate electromagnetic field equation? If it is accurate. So how should the energy of the electric field be considered? The author thinks the answer is no! In Maxwell equation,

$$\boldsymbol{D} = \epsilon_0(-\frac{\partial}{\partial t}\boldsymbol{A} - \nabla \phi)$$

Because if Maxwell's equation is an accurate electromagnetic field equation, it should also be suitable for the electric and magnetic quasi-static situation to. If so,

$$-\frac{\partial}{\partial t}\boldsymbol{A}$$

Should contribute to the energy of the electric field, i.e.,

$$\mathbb{E}_E = \epsilon_0 | -\frac{\partial}{\partial t}\boldsymbol{A} - \nabla \phi|^2$$

So, the magnetic field energy we derived earlier

$$\mathbb{E}_B = \frac{\boldsymbol{B} \cdot \boldsymbol{B}}{2\mu_0}$$

The formula (28) of is incorrect! Because then we cannot get (28). If the above formula of magnetic field energy does not hold, the edifice of electromagnetic theory will collapse. Let's look at Poynting's theorem,

$$-\iiint_V \boldsymbol{E} \cdot \boldsymbol{J} dV$$
$$= \iiint_V \boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B} dV + \iiint_V \boldsymbol{E} \cdot \frac{\partial}{\partial t}\boldsymbol{D} dV + \oiint_\Gamma (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma$$

The above Poynting theorem is obtained from Maxwell's equation with displacement current (29,30).

Ignore radiation term $\oiint_{\Gamma}(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma$, this is allowed, there is,

$$-\iiint_V \boldsymbol{E}\cdot\boldsymbol{J}dV=\iiint_V \boldsymbol{H}\cdot\frac{\partial}{\partial t}\boldsymbol{B}dV+\iiint_V \boldsymbol{E}\cdot\frac{\partial}{\partial t}\boldsymbol{D}dV$$

Because if the system has no capacitance

$$\nabla\phi=0$$

and

$$\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{A}+\nabla\phi=-\frac{\partial}{\partial t}\boldsymbol{A}=\boldsymbol{E}_i$$

$\boldsymbol{E}_i$ is a pure induced electric field. $\boldsymbol{D}_i=\epsilon_0\boldsymbol{E}_i$,

$$\boldsymbol{E}_i=-\frac{\partial}{\partial t}\boldsymbol{A}$$

there is,

$$-\iiint_V \boldsymbol{E}\cdot\boldsymbol{J}dV=\iiint_V \boldsymbol{H}\cdot\frac{\partial}{\partial t}\boldsymbol{B}dV+\iiint_V \boldsymbol{E}_i\cdot\frac{\partial}{\partial t}\boldsymbol{D}_i dV \tag{31}$$

The author believes that the above formula is wrong because it leads to the work done by the electric field $-\iiint_V \boldsymbol{E}\cdot\boldsymbol{J}dV$ not only increases the energy of the magnetic field $\frac{1}{2}\iiint_V \boldsymbol{H}\cdot\boldsymbol{B}dV$, but also increase the energy of the induced electric field $\frac{1}{2}\iiint_V \boldsymbol{E}_i\cdot\boldsymbol{D}_i dV$. This goes against our common sense. Even under electric and magnetic quasi-static conditions, where

$$\boldsymbol{E}_i=-\frac{\partial}{\partial t}\boldsymbol{A}$$

Should not contribute to electric field energy! If the reader is not clear, the examples in the next section should be better explained.

III. Oscillator

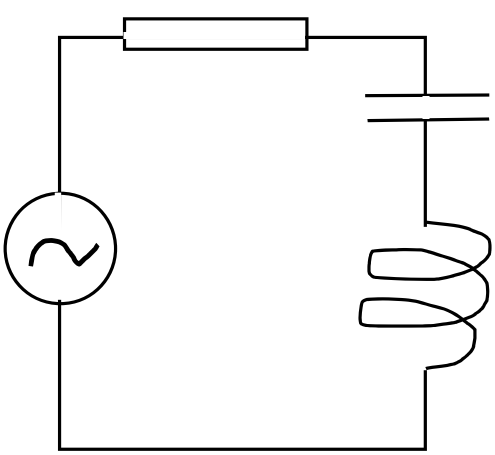

Fig. 3. Oscillator with a inductor and a capacity.

Let's study the following circuit, as shown in 3. This is an oscillator. It is assumed that the line operates at the oscillation frequency and the voltage on the inductor is,

$$U_L=L\frac{d}{dt}I=j\omega LI$$

The inductive impedance of the inductor is,

$$Z_L=j\omega L$$

The power on the inductor is

$$U_LI^*=j\omega LII^*$$

"$*$" is conjugate of complex number. For the capacitance,

$$I=C\frac{d}{dt}U_C=j\omega CU_C$$

The voltage across the capacitor is,

$$U_C=\frac{1}{j\omega C}I$$

Capacitive impedance on capacitor,

$$Z_C=\frac{1}{j\omega C}$$

The power on the capacitor,

$$U_CI^*=\frac{1}{j\omega C}II^*$$

The total impedance is,

$$Z=Z_R+Z_L+Z_C=R+j\omega L+\frac{1}{j\omega C}$$

$$=R+j(\omega L-\frac{1}{\omega C})$$

Assuming that resonance occurs, the above impedance becomes purely resistive, and the conditions are obtained

$$\omega L=\frac{1}{\omega C}$$

or

$$\omega^2=\frac{1}{LC}$$

The resonance frequency is,

$$\omega=\frac{1}{\sqrt{LC}}$$

In this case,

$$Z=R$$

The power of the inductor is,

$$U_LI^*=j\omega LII^*=j\frac{1}{\sqrt{LC}}LII^*=j\sqrt{\frac{L}{C}}II^*$$

The power of the capacitor is,

$$U_CI^*=\frac{1}{j\omega C}II^*=\frac{\sqrt{LC}}{jC}II^*=-j\sqrt{\frac{L}{C}}II^*$$

The power consumed on the resistor is,

$$U_RI^*=(RI)I^*=RII^*$$

This means that all the power of the power supply is supplied to the resistor. The inductor and the capacitor exchange energy. When they work at the resonance frequency, the power of the capacitor and the inductor is equal and the sign is opposite, indicating that the electromagnetic energy is converted between the capacitor and the inductor. Here, the energy of the capacitor is only related to the electrostatic field,

$$U_c=\phi_1-\phi_2=-\int_1^2\nabla\phi\cdot d\boldsymbol{l}$$

ϕ_1 and ϕ_2 is the points at both ends of the capacitor. That is, it is related to the electrostatic field E_s,

$$\boldsymbol{E}_s = -\nabla\phi$$

Power on the inductor in the line

$$U_L I^* = -\oint_C \boldsymbol{E}_i \cdot I^* d\boldsymbol{l} \rightarrow -\iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J} dV \qquad (32)$$

In the above formula, we replace the line circuit with the body current. Electric field above $\boldsymbol{E}_i$ is the induced electric field

$$\boldsymbol{E}_i = -\frac{\partial}{\partial t}\boldsymbol{A}$$

or considering $\boldsymbol{B} = \nabla \times \boldsymbol{A}$,

$$\nabla \times \boldsymbol{E}_i = -\frac{\partial}{\partial t}\boldsymbol{B} \qquad (33)$$

Considering the ampere circuital law in the magnetic quasi-static equation,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

The right side of formula (32) is,

$$-\iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J} dV = -\iiint_V \boldsymbol{E}_i \cdot \nabla \times \boldsymbol{H} dV \qquad (34)$$

Considering the mathematical formula,

$$\nabla \cdot (\boldsymbol{E}_i \times \boldsymbol{H}) = \nabla \times \boldsymbol{E}_i \cdot \boldsymbol{H} - \boldsymbol{E}_i \cdot \nabla \times \boldsymbol{H}$$

therefore

$$-\boldsymbol{E}_i \cdot \nabla \times \boldsymbol{H} = \nabla \cdot (\boldsymbol{E}_i \times \boldsymbol{H}) - \nabla \times \boldsymbol{E}_i \cdot \boldsymbol{H}$$

The formula (34) is

$$-\iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J} dV = \iiint_V (\nabla \cdot (\boldsymbol{E}_i \times \boldsymbol{H}) - \nabla \times \boldsymbol{E}_i \cdot \boldsymbol{H}) dV$$

or

$$-\iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J} dV = \oiint_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}) \cdot \hat{n} d\Gamma - \iiint_V \nabla \times \boldsymbol{E}_i \cdot \boldsymbol{H}) dV \qquad (35)$$

Considering that the radiation is zero,

$$\oiint_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}) \cdot \hat{n} d\Gamma = 0$$

The formula (35) is,

$$-\iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J} dV = -\iiint_V \nabla \times \boldsymbol{E}_i \cdot \boldsymbol{H} dV$$

Considering Faraday's Law (33),

$$-\iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J} dV = \iiint_V \frac{\partial}{\partial t}\boldsymbol{B} \cdot \boldsymbol{H} dV$$

or

$$-\iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J} dV = \frac{\partial}{\partial t}\iiint_V \frac{1}{2\mu}\boldsymbol{B} \cdot \boldsymbol{B} dV$$

Considering (32), we get

$$U_L I^* = \frac{\partial}{\partial t}\iiint_V \frac{1}{2\mu}\boldsymbol{B} \cdot \boldsymbol{B} dV$$

$U_L I^*$ is all converted into an increase in magnetic field energy. The system only includes the electric field energy in the capacitor and the magnetic field energy in the inductor. Here we see that there is no reason and no room to calculate the energy of the induced electric field,

$$\frac{\epsilon}{2}||\boldsymbol{E}_i||^2$$

This part of energy does not exist! There is no reason to calculate the energy of the electric field according to the following formula:

$$\frac{\epsilon}{2}||\boldsymbol{E}||^2 = \frac{\epsilon}{2}||-\frac{\partial}{\partial t}\boldsymbol{A} - \nabla\phi||^2$$

The energy of the electric field is absolutely only the energy of the electrostatic field,

$$\frac{1}{2}||\boldsymbol{E}_s||^2 = \frac{1}{2}||-\nabla\phi||^2$$

However, for the radiated electromagnetic field, the electromagnetic field satisfies Maxwell's equation and of course includes the displacement current. At this case, Poynting's theorem is,

$$-\iiint_V \boldsymbol{E} \cdot \boldsymbol{J} dV$$

$$= \iiint_V \boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B} dV + \iiint_V \boldsymbol{E} \cdot \frac{\partial}{\partial t}\boldsymbol{D} dV + \oiint_\Gamma (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma$$

among them,

$$\iiint_V \boldsymbol{E} \cdot \frac{\partial}{\partial t}\boldsymbol{D} dV$$

And should be considered as the energy of the electric field,

$$\boldsymbol{E} = \boldsymbol{E}_s + \boldsymbol{E}_i = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A}$$

This makes Maxwell's equation and Poynting's theorem really confusing. The author thinks that the magnetic quasi-static electromagnetic field equation or the electric and magnetic quasi-static electromagnetic field equation is an accurate, but the Maxwell equation including displacement current is not accurate or confusing.

IV. Lorentz Retarded Potential Method

The author began to doubt the Maxwell equation including displacement current, but the author thinks that the wave equation of scalar potential and vector potential should still be correct. Because the wave equation is so beautiful, it seems that there should never be any problem. The wave equation of vector potential and scalar potential was first introduced by Lorenz in 1867.

A. Derivation of Maxwell equation using Lorenz retarded potential

The author acknowledges that Lorenz's retarded potential method is correct, that is come from,

$$\boldsymbol{A}(x,t) = \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}(x',t)}{r} dV' \qquad (36)$$

$$\phi(x,t) = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(x',t)}{r} dV' \qquad (37)$$

Lorenz thus directly generalized and guessed the retarded potential,

$$\boldsymbol{A}^{(r)}(x,t)=\frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}(\boldsymbol{x}',t-r/c)}{r}dV' \tag{38}$$

$$\phi^{(r)}(x,t)=\frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\boldsymbol{x}',t-r/c)}{r}dV' \tag{39}$$

Superscript (r) means retarded, and the above can be converted to frequency domain,

$$\boldsymbol{A}^{(r)}(x,t)=\frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}\exp(-jkr)dV' \tag{40}$$

$$\phi^{(r)}(x,t)=\frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho}{r}\exp(-jkr)dV' \tag{41}$$

In the above $\boldsymbol{J}=\boldsymbol{J}_0\exp(j\omega t)$ and $\rho=\rho_0\exp(j\omega t)$. Both potentials satisfy the Lorenz gauge condition,

$$\nabla\cdot\boldsymbol{A}(x,t)=-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi$$

and

$$\nabla\cdot\boldsymbol{A}^{(r)}(x,t)=-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi^{(r)} \tag{42}$$

It is also assumed the electromagnetic fields for the retarded potential,

$$\boldsymbol{E}^{(r)}=-\frac{\partial}{\partial t}\boldsymbol{A}^{(r)}-\nabla\phi^{(r)} \tag{43}$$

$$\boldsymbol{B}^{(r)}=\nabla\times\boldsymbol{A}^{(r)} \tag{44}$$

First of all, we should note that Lorenz follows Kirchhoff, and they do not establish the concept of electric field and magnetic field. Perhaps they do not think that there are electric and magnetic fields in space. They are concerned with the current of the conductor, so for Lorenz only wrote,

$$\boldsymbol{J}=\sigma(-\frac{\partial}{\partial t}\boldsymbol{A}-\nabla\phi)$$

$$\boldsymbol{J}=\sigma(-\frac{\partial}{\partial t}\boldsymbol{A}^{(r)}-\nabla\phi^{(r)})$$

Where σ is the conductivity. Lorenz and Kirchhoff do not use the formula (43,44), which are only used by Maxwell, so we put a question mark on it. The author thinks that the formula of magnetic field under magnetic quasi-static or electromagnetic quasi-static conditions,

$$\boldsymbol{B}=\nabla\times\boldsymbol{A}$$

Still correct. However, when the displacement current is increased or the retarded potential is adopted, the magnetic field is not so reliable. Considering that the retarded potential satisfies the wave equation,

$$\nabla^2\boldsymbol{A}^{(r)}-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\boldsymbol{A}^{(r)}=-\mu_0\boldsymbol{J} \tag{45}$$

$$\nabla^2\phi^{(r)}-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi^{(r)}=-\rho/\epsilon_0 \tag{46}$$

Considering the mathematical formula,

$$\nabla\times\nabla\times\boldsymbol{A}^{(r)}=\nabla(\nabla\cdot\boldsymbol{A}^{(r)})-\nabla^2\boldsymbol{A}^{(r)}$$

Consider definitions (44) and Lorenz gauge condition (42)

$$\nabla\times\boldsymbol{B}^{(r)}=\nabla(-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi^{(r)})-\nabla^2\boldsymbol{A}^{(r)}$$

or

$$\nabla^2\boldsymbol{A}^{(r)}=\nabla(-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi^{(r)})-\nabla\times\boldsymbol{B}^{(r)}$$

Substitute the above formula (45)

$$(\nabla(-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi^{(r)})-\nabla\times\boldsymbol{B}^{(r)})-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\boldsymbol{A}^{(r)}=-\mu_0\boldsymbol{J}$$

or

$$(\mu_0\epsilon_0\frac{\partial}{\partial t}(-\nabla\phi^{(r)})-\nabla\times\boldsymbol{B}^{(r)})-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\boldsymbol{A}^{(r)}=-\mu_0\boldsymbol{J}$$

or

$$\mu_0\epsilon_0\frac{\partial}{\partial t}(-\nabla\phi^{(r)}-\frac{\partial}{\partial t}\boldsymbol{A}^{(r)})-\nabla\times\boldsymbol{B}^{(r)}=-\mu_0\boldsymbol{J}$$

or

$$\mu_0\epsilon_0\frac{\partial}{\partial t}\boldsymbol{E}^{(r)}-\nabla\times\boldsymbol{B}^{(r)}=-\mu_0\boldsymbol{J}$$

or

$$\mu_0\epsilon_0\frac{\partial}{\partial t}\boldsymbol{E}^{(r)}+\mu_0\boldsymbol{J}=\nabla\times\boldsymbol{B}^{(r)}$$

or

$$\epsilon_0\frac{\partial}{\partial t}\boldsymbol{E}^{(r)}+\boldsymbol{J}=\nabla\times\boldsymbol{H}^{(r)}$$

or

$$\nabla\times\boldsymbol{H}^{(r)}=\boldsymbol{J}+\epsilon_0\frac{\partial}{\partial t}\boldsymbol{E}^{(r)} \tag{47}$$

The above formula is Maxwell ampere circuital law. Considering the wave equation of scalar potential,

$$\nabla^2\phi^{(r)}-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi^{(r)}=-\rho/\epsilon_0$$

or

$$\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi^{(r)}-\nabla^2\phi^{(r)}=\rho/\epsilon_0$$

Consider Lorenz gauge condition $\mu_0\epsilon_0\frac{\partial}{\partial t}\phi^{(r)}=-\nabla\cdot\boldsymbol{A}^{(r)}$

$$(-\frac{\partial}{\partial t}\nabla\cdot\boldsymbol{A}^{(r)}-\nabla\cdot\nabla\phi^{(r)})=\rho/\epsilon_0$$

or

$$\nabla\cdot(-\frac{\partial}{\partial t}\boldsymbol{A}^{(r)}-\nabla\phi^{(r)})=\rho/\epsilon_0$$

Considering,

$$\boldsymbol{E}^{(r)}\equiv-\frac{\partial}{\partial t}\boldsymbol{A}^{(r)}-\nabla\phi^{(r)} \tag{48}$$

By substituting the above formula,

$$\nabla\cdot\boldsymbol{E}^{(r)}=\rho/\epsilon_0 \tag{49}$$

Although we can prove Maxwell's equation by Lorenz retarded potential method. So Maxwell equation is equivalent to retarded potential method. Even if the method of retarded potential is correct. So in the final analysis, Maxwell's method is a retarded potential method.

The question is: after (36,36) to (38,39), do the definitions of electric field and magnetic field (43-44) remain correct? If the electric field and magnetic field change during the conversion from non retarded potential to retarded potential, so that (44 and 48) change, we still cannot obtain Maxwell equations (49) and (47). Therefore, even if Lorenz's retarded potential is reasonable, Maxwell's equations (49, 47, 44, 48) may not be obtained. In fact, what the author suspects most is that the formula of magnetic field is:

$$\boldsymbol{B}^{(r)} = \nabla \times \boldsymbol{A}^{(r)}$$

This is because,

$$\nabla \times \boldsymbol{A}^{(r)} = \frac{\mu_0}{4\pi} \iiint_V \nabla \frac{\exp(-jkr)}{r} \times \boldsymbol{J} dV$$

$$= \frac{\mu_0}{4\pi} \iiint_V (\nabla \frac{1}{r}) \exp(-jkr) + \frac{(-j\boldsymbol{k})\exp(-jkr)}{r}) \times \boldsymbol{J} dV$$

$$= \frac{\mu_0}{4\pi} \iiint_V ((-\frac{\boldsymbol{r}}{r^3}) \exp(-jkr) + \frac{(-j\boldsymbol{k})\exp(-jkr)}{r}) \times \boldsymbol{J} dV$$

$$= \frac{\mu_0}{4\pi} \iiint_V ((-\frac{\boldsymbol{r}}{r^3}) \exp(-jkr) \times \boldsymbol{J} + \frac{(-j\boldsymbol{k})\exp(-jkr)}{r} \times \boldsymbol{J}) dV$$

$$= \frac{\mu_0}{4\pi} \iiint_V \frac{\exp(-jkr)}{r} ((-\frac{\hat{r}}{r}) \times \boldsymbol{J} + (-j\boldsymbol{k}) \times \boldsymbol{J}) dV$$

$$\lim_{k \to 0} \nabla \times \boldsymbol{A}^{(r)} = \frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} ((-\frac{\hat{r}}{r}) \times \boldsymbol{J} + (-j\boldsymbol{k}) \times \boldsymbol{J}) dV$$

$$= \boldsymbol{B}_0 + \boldsymbol{B}_1$$

Where

$$\boldsymbol{B}_0 = \frac{\mu_0}{4\pi} \iiint_V \boldsymbol{J} \times \frac{\hat{r}}{r^2} = \boldsymbol{B}_s$$

and

$$\boldsymbol{B}_1 = \frac{\mu_0}{4\pi} \iiint_V \boldsymbol{J} \times \frac{j\boldsymbol{k}}{r} dV$$

Hence,

$$\lim_{k \to 0} \nabla \times \boldsymbol{A}^{(r)} = \boldsymbol{B}_s + \boldsymbol{B}_1$$

So we have,

$$\lim_{k \to 0} \boldsymbol{B}^{(r)} \neq \boldsymbol{B}_s \tag{50}$$

$k \to 0$, because $k = \frac{2\pi}{\lambda}$, λ is the wavelength. This is equivalent to $\lambda \to \infty$. This condition is satisfied when the scale of the current area is $l \ll \lambda$. For example, if the alternating current is 100 Hz, the speed of light is $c = 3 * 10^8$ meters. Wavelength,

$$\lambda = c/f = \frac{3 * 10^8}{100} = 3 * 10^6$$

So the wavelength is 3000 km. If the scale of our inductance device is less than 1 meter, then the magnetic quasi-static condition is well satisfied. on the other hand,

$$\boldsymbol{E}^{(r)} = -\frac{\partial}{\partial t} \boldsymbol{A}^{(r)} - \nabla \phi^{(r)}$$

have two terms,

$$\boldsymbol{E}_i^{(r)} \equiv -\frac{\partial}{\partial t} \boldsymbol{A}^{(r)}$$

$$\boldsymbol{E}_s^{(r)} \equiv -\nabla \phi^{(r)}$$

The subscript "i" means inductive, and the subscript "s" means static. Similarly, we also define:

$$\boldsymbol{E}_i \equiv -\frac{\partial}{\partial t} \boldsymbol{A} = -j\omega \boldsymbol{A}$$

$$\boldsymbol{E}_s \equiv -\nabla \phi$$

$$\boldsymbol{E}_i^{(r)} = -\frac{\partial}{\partial t} \boldsymbol{A}^{(r)} = -j\omega \boldsymbol{A}^{(r)} = -j\omega \frac{\mu_0}{4\pi} \iiint_V \frac{\boldsymbol{J}}{r} \exp(-jkr) dV'$$

$$\lim_{k \to 0} \boldsymbol{E}_i^{(r)} = -j\omega \frac{\mu_0}{4\pi} \iiint_V \frac{\boldsymbol{J}}{r} dV' = -j\omega \boldsymbol{A} = \boldsymbol{E}_i \tag{51}$$

Induced electric field obtained by retarded potential $\boldsymbol{E}_i^{(r)}$ when $k \to 0$ and $\boldsymbol{E}_i$ are the same. $k \to 0$ means that the scale of the region V where the current is located is much smaller than the wavelength. We usually think that this is the condition for the establishment of the magnetic quasi-static field. In this case, the radiated electromagnetic field should be the same as the magnetic quasi electromagnetic field. For the term of induced electric field, $\boldsymbol{E}_i{}^{(r)}$. The condition is satisfied.

$$\boldsymbol{E}_s^{(r)} = -\nabla \phi^{(r)} = -\nabla \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho}{r} \exp(-jkr) dV'$$

$$= -\frac{1}{4\pi\epsilon_0} \iiint_V \nabla \frac{\exp(-jkr)}{r} \rho dV'$$

$$= -\frac{1}{4\pi\epsilon_0} \iiint_V (\exp(-jkr) \nabla \frac{1}{r} + \nabla \exp(-jkr) \frac{1}{r}) \rho dV'$$

$$= -\frac{1}{4\pi\epsilon_0} \iiint_V \nabla \frac{1}{r} (\exp(-jkr) + (-j\boldsymbol{k}) \exp(-jkr) \frac{1}{r}) \rho dV'$$

$$\lim_{k \to 0} \boldsymbol{E}_s^{(r)} = -\frac{1}{4\pi\epsilon_0} \iiint_V \nabla \frac{1}{r} + \frac{(-j\boldsymbol{k})}{r}) \rho dV'$$

$$= -\nabla \frac{1}{4\pi\epsilon_0} \iiint_V \frac{1}{r} \rho dV' - \frac{1}{4\pi\epsilon_0} \iiint_V \frac{(-j\boldsymbol{k})}{r}) \rho dV'$$

$$= -\nabla \phi - \frac{1}{4\pi\epsilon_0} \iiint_V \frac{(-j\boldsymbol{k})}{r}) \rho dV'$$

$$= \boldsymbol{E}_s - \frac{1}{4\pi\epsilon_0} \iiint_V \frac{(-j\boldsymbol{k})}{r}) \rho dV'$$

$$\lim_{k \to 0} \boldsymbol{E}_s^{(r)} \neq \boldsymbol{E}_s \tag{52}$$

We can see that there is (50) for the magnetic field, which indicates that the radiation retarded magnetic field of Maxwell is not consistent with the magnetic quasi-static magnetic field even when the magnetic quasi-static conditions are satisfied. For the induced electric field, when the magnetic quasi-static condition is satisfied, the induced electric field of Maxwell's radiation retarded field $\boldsymbol{E}_i^{(r)}$ and magnetic quasi-static electric field $\boldsymbol{E}_i$ is consistent, but Maxwell's radiation retarded field static electric field $\boldsymbol{E}_s^{(r)}$ and magnetic quasi-static electric field $\boldsymbol{E}_s$ is inconsistent. In many special cases, $\boldsymbol{E}_s^{(r)} = 0$, so we have $\boldsymbol{E}^{(r)}$ consistent with $\boldsymbol{E}$ in case $k \to 0$. An example of these special cases is the radiated electromagnetic field of an infinite plate current. It is assumed that the current on the plate is constant everywhere. Due to symmetry, $\boldsymbol{E}_s^{(r)} = 0$, $\boldsymbol{E}_s = 0$. In such an example, it can be ensured that the radiation retarded electric field $\boldsymbol{E}^{(r)}$ and the electric field $\boldsymbol{E}$ are consistent when the magnetic quasi-static condition $k \to 0$ is established. But the magnetic field $\boldsymbol{B}$ does not have this property.

B. Derivation of Lorenz retarded potential equation by using Maxwell equation

It is assumed that Maxwell's equation holds

$$\nabla \cdot \boldsymbol{E} = \rho/\epsilon_0$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{D}$$

We deduce the wave equation of the retarded potential to see if there is anything wrong in the derivation process.

$$\nabla \cdot \boldsymbol{B} = 0 \to \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

Then, according to the electrostatic field condition, that is $\frac{\partial}{\partial t}\boldsymbol{B} = 0$,

$$\nabla \cdot \boldsymbol{E} = \rho/\epsilon_0$$

$$\nabla \times \boldsymbol{E} = 0$$

The above formula indicates that when there is no induced electric field

$$\boldsymbol{E} = -\nabla\phi$$

$$\nabla \cdot (-\nabla\phi) = \rho/\epsilon_0$$

$$\nabla^2\phi = -\rho/\epsilon_0 \tag{53}$$

$$\phi = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho}{r}dV$$

Considering now the presence of an induced electric field,

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \to \nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\nabla \times \boldsymbol{A}$$

$$\nabla \times \boldsymbol{E} = -\nabla \times \frac{\partial}{\partial t}\boldsymbol{A}$$

$$\nabla \times (\boldsymbol{E} + \frac{\partial}{\partial t}\boldsymbol{A}) = 0$$

$$\boldsymbol{E} + \frac{\partial}{\partial t}\boldsymbol{A} = -\nabla\psi$$

$$\boldsymbol{E} = -\nabla\psi - \frac{\partial}{\partial t}\boldsymbol{A}$$

suppose

$$\psi = \phi$$

$$\boldsymbol{E} = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A}$$

$$\nabla \times \boldsymbol{B} = \mu_0\boldsymbol{J} + \mu_0\epsilon_0\frac{\partial}{\partial t}\boldsymbol{E}$$

$$\nabla \times (\nabla \times \boldsymbol{A}) = \mu_0\boldsymbol{J} + \mu_0\epsilon_0\frac{\partial}{\partial t}(-\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A})$$

Consider

$$\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2\boldsymbol{A}$$

$$\nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2\boldsymbol{A} = \mu_0\boldsymbol{J} + \mu_0\epsilon_0\frac{\partial}{\partial t}(-\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A})$$

$$\nabla(\nabla \cdot \boldsymbol{A} + \mu_0\epsilon_0\frac{\partial}{\partial t}\phi) - \mu_0\boldsymbol{J} = \nabla^2\boldsymbol{A} - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\boldsymbol{A}$$

$$\nabla^2\boldsymbol{A} - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\boldsymbol{A} = \nabla(\nabla \cdot \boldsymbol{A} + \mu_0\epsilon_0\frac{\partial}{\partial t}\phi) - \mu_0\boldsymbol{J}$$

For Maxwell, adhere to the Coulomb criterion

$$\nabla \cdot \boldsymbol{A} = 0$$

$$\nabla^2\phi = -\rho/\epsilon_0$$

$$\nabla^2\boldsymbol{A} - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\boldsymbol{A} = \nabla(\mu_0\epsilon_0\frac{\partial}{\partial t}\phi) - \mu_0\boldsymbol{J}$$

It makes sense for him to do so because if the Lorenz gauge condition is adopted,

$$\nabla \cdot \boldsymbol{A} = -\mu_0\epsilon_0\frac{\partial}{\partial t}\phi$$

So for

$$\boldsymbol{E} = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A}$$

There is,

$$\rho/\epsilon_0 = \nabla \cdot \boldsymbol{E} = -\nabla \cdot \nabla\phi - \frac{\partial}{\partial t}\nabla \cdot \boldsymbol{A} = -\nabla^2\phi + \frac{\partial}{\partial t}(\mu_0\epsilon_0\frac{\partial}{\partial t}\phi)$$

or

$$\nabla^2\phi - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi = -\rho/\epsilon_0 \tag{54}$$

It means that the Poisson equation of scalar potential (53) is replaced by the wave equation of scalar potential (54) from the beginning. In that case, it means that the same retarded potential method as Lorenz is adopted from the beginning. Generally speaking, the wave equation of the retarded potential derived from Maxwell's equation is stable, and there are not many loopholes.

However, Maxwell's equation should be regarded as the definition of electromagnetic field. When the displacement current $\frac{\partial}{\partial t}\boldsymbol{D}$ added, it is certain that the electromagnetic field $\boldsymbol{E}, \boldsymbol{B}$ changes. The question is: after this change, can the new electromagnetic field (including displacement current) and the original electromagnetic field under quasi-static condition still be regarded as electromagnetic fields with the same properties? Or is it a seamless extension of quasi-static electromagnetic fields?

V. Conclusions

Under the electric and magnetic quasi-static or electric and magnetic quasi-static conditions, there is no doubt about the energy of the magnetic field, but the energy of the electric field is different. According to the traditional understanding, this is handled according to the magnetic quasi-static or electric and magnetic quasi-static electromagnetic field conditions, and the energy of the electric field only includes the energy of the electric potential. This energy is the energy stored in the capacitor. But according to Maxwell's equation, including the displacement current, the energy of the electric field includes the energy of the induced electric field. The author thinks that the energy of this induced electric field is fictitious and does not exist. This indicates that there is a problem in the calculation of energy from Maxwell's equation including displacement current. This paper only shows that the energy of the induced electric field is fictitious, and the author also explains in other papers that Maxwell's electromagnetic theory calculates the phase difference between the electric field and the magnetic field incorrectly in the calculation of the radiated electromagnetic field. For plane waves, the phase of the electric and magnetic fields should be 90 degrees, but Maxwell's equation calculates it as 0 degrees or in phase. The author believes that the calculation error of the energy of the induced electric field is also one of the reasons for the calculation error of the phase difference between the radiated electric field and the radiated magnetic field in Maxwell's theory. The conclusion of this paper further supports the author's mutual energy theory that the phase difference between the radiated electric field and the magnetic field should be 90 degrees, not in phase.

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