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N-theory: Do Quarks Have Memory?

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Abstract

N-theory deals with the space-time nonlocality in nature which in time domain results in memory effect. Recent results show that memory can be considered as a fundamental characteristic of all fundamental interactions. In this short note we deal with the strong interaction and address this question that: do quarks as fundamental building blocks of our universe exhibit some kind of memory effects? We investigate this idea through the framework of fractional dynamics and also, we discus about a possible fractional quark theory. The idea is based on some behavior and characteristics of quarks and quark matter that naturally can be described using fractional operators which intrinsically incorporate memory kernels and nonlocality. Finally, we examine the fractional quark theory's application in the exotic baryon spectroscopy and its power to prediction of possible future LHC results. We show that this powerful framework can reproduce the experimental mass for the exotic baryons.

Keywords: Nonlocality, memory effect, fractional dynamics, fractional quark theory, exotic baryons.

1. Introduction

The quark model was proposed in 1964 [1, 2] to describe the physics of hadrons produced at high-energy accelerators. Recently in [3,4] authors proposed a description of the Higgs boson as top-antitop quark bound state within a nonlocal relativistic quark model of Nambu-Jona-Lasinio (NJL) type which in contrast to models with local four-fermion interaction and in accordance with phenomenology, in the nonlocal generalization the mass of the scalar bound state can be lighter than the sum of its constituents. In their nonlocal effective theory, they considered the ansatz of a local current-current vertex, but with nonlocal particle currents with effective action for non-local NJL model in the form of:

$$S = \int d^4x \left(\bar{t}(x)(\partial_\mu \gamma^\mu + m)t(x) - \frac{N}{2} J(x)J(x) \right) \tag{1}$$

with the nonlocal scalar current:

$$J(x) = \int d^4y \; F(y)\bar{t}(x+\frac{y}{2})t(x-\frac{y}{2}) \tag{2}$$

where $F(y)$ is the form-factor responsible for the nonlocality and N is coupling strength of their model [3,4].

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In addition to above mentioned nonlocality, dissipative phenomena in quark-gluon plasma (QGP) [5] and non-equilibrium hydrodynamics of QGP [6] have been investigated and reported.

On the other hand, recently in [7] the author showed that fractional Schrödinger equation can successfully describe in some way the nature of strong interaction and quarks' behavior e.g. the important phenomenon of quark confinement based on this fact that the solutions of a free fractional Schrödinger equation, are localized in a very small region of space. He also showed that fractional calculus [8, 9] can provide a reliable framework for a successful explanation of hadron spectra while many of recently discovered hadrons have not been covered by other approaches in hadrons' spectroscopy [7]. In addition, more recently by use of fractional calculus, authors of [10] tried to provide some insights for physical interpretations of the dependence of the total cross-section, which can be obtained from the imaginary part of the forward elastic scattering amplitude A(s, t) through the optical theorem and at high energies can be defined as:

$$\sigma_{tot}(s) = \frac{\mathrm{Im}\ \mathrm{A(s,t=0)}}{s} \tag{3}$$

where s and t are the energy and momentum transfer squared in the center of mass system (Mandestam variables) and t = 0 means the forward direction, with the energy and its asymptotic rise in high-energy hadron-hadron collisions which is still an open problem in quantum chromodynamics (QCD) [10].

A final theory of nature has to consider space-time nonlocality which in time domain means memory effect described by memory kernel. This idea along with the above-mentioned characteristics and evidences altogether can all be considered as strong reasons for applying fractional calculus to all area related to quark matter and building a fractional theory of quark. This natural emergence of fractional calculus is just due to the fact that it previously was successful in describing various phenomena which were nonlinear, nonlocal, dissipative and thermodynamically out of equilibrium [11-16]. And so, we can immediately ask this question that is there a special kind of memory effect in phenomena related to quark matter because memory and memory effect is another main and intrinsic aspect of fractional operators? For answering to such this question, at first, we need to build a fractional theory of quark. So, in the next section we try to propose some ideas.

2. Fractional quark theory

One approach to construction of a fractional quark theory (FQT) is that we at first build a fractional group theory [7, 14]. In this framework we can introduce $SO^{\alpha}(3N)$ as a fractional generalization of the standard rotation group $SO(3N)$ and then define the set of Casimir operators C_k^2, where the index *k* indicates the Casimir operator associated with $SO^{\alpha}(k)$, as:

$$C_k^2 = \frac{1}{2}\sum_{i,j}^{k}\left(L_{ij}^{\alpha}\right)^2 \qquad k = 2,\ldots,3N \tag{4}$$

Where L_{ij}^{α} are the generators of infinitesimal fractional rotations in the i, j plane in $\boldsymbol{R}^N$ with ($i, j = 1, ..., 3N$) defined as [7]:

$$L_{ij}^{\alpha} = \hat{X}_i^{\alpha}\hat{P}_j - \hat{X}_j^{\alpha}\hat{P}_i^{\alpha} = -i\hbar\left(x_i^{\alpha}\partial_j^{\alpha} - x_j^{\alpha}\partial_i^{\alpha}\right) \tag{5}$$

Where ∂^{α} denotes the well-known Riemann–Liouville (RL) partial fractional derivatives. Left (forward) and right (backward) Riemann–Liouville (RL) partial fractional derivatives (which respectively are nonlocal causal operators and nonlocal non-causal operators) of order α_{μ} , β_{μ} (which are positive real or even complex numbers) of a real valued function f of $d+1$ real variables $x^0, x^1, ..., x^d$ with respect to x_{μ}, i.e.:

$$_{a_\mu}\partial_{\mu}^{\alpha_\mu} f(x^0,...,x^d) = \frac{1}{\Gamma(n_\mu - \alpha_\mu)}\partial_{x^\mu}^{n_\mu}\int_{a_\mu}^{x}\frac{f(x^0,...,x^{\mu-1},u,x^{\mu+1},...,x^d)}{(x^\mu - u)^{1+\alpha_\mu - n_\mu}}du \quad \text{(left RL)} \tag{6}$$

$$_{\mu}\partial_{b_\mu}^{\beta_\mu} f(x^0,...,x^d) = \frac{(-1)^{n_\mu}}{\Gamma(n_\mu - \beta_\mu)}\partial_{x^\mu}^{n_\mu}\int_{x}^{b_\mu}\frac{f(x^0,...,x^{\mu-1},u,x^{\mu+1},...,x^d)}{(u - x^\mu)^{1+\beta_\mu - n_\mu}}du \quad \text{(right RL)} \tag{7}$$

where $\partial_{x^\mu}^{n_\mu}$ is the ordinary partial derivative of integer order n with respect to the variable x and a_{μ} , b_{μ} are real number which define the domain [11,15]. And finally with the definitions $\hat{L}_z(\alpha) = C_2$ and $\hat{L}^2(\alpha) = C_3^2$, we will have eigenvalues of Casimir operator in terms of the quantum numbers L and M as [7, 14]:

$$\hat{L}_z(\alpha)\left|LM\right\rangle = \pm\hbar\frac{\Gamma(1+|M|\alpha)}{\Gamma(1+(|M|-1)\alpha)}\left|LM\right\rangle \qquad M = 0, \pm1, \pm2, ..., \pm L \tag{8}$$

$$\hat{L}^2(\alpha)\left|LM\right\rangle = \pm\hbar^2\frac{\Gamma(1+(L+1)\alpha)}{\Gamma(1+(L-1)\alpha)}\left|LM\right\rangle \qquad \mathrm{L} = 0, +1, +2, ... \tag{9}$$

Now introducing the model Hamilton operator H^{α} as:

$$H^{\alpha} = m_0 + a_0 L^2(\alpha) + b_0 L_z(\alpha) \tag{10}$$

where coefficients m_0, a_0 and b_0 are free parameters, one can derive in lowest order an analytic expression for the splitting of the energy levels and mass spectrum of a non-relativistic charged fractional spinless particle in a constant fractional magnetic field as [7,14]:

$$E_R^{\alpha} = m_0 + a_0\frac{\Gamma(1+(L+1)\alpha)}{\Gamma(1+(L-1)\alpha)} \pm b_0\frac{\Gamma(1+|M|\alpha)}{\Gamma(1+(|M|-1)\alpha)} \tag{11}$$

that such particle can be associated with a quark and the fractional magnetic field with a color magnetic field and finally this model should allow a description of the hadron spectrum. It is showed that the above formula reproduces the full baryon spectrum with an error less than 1% [7, 14].

Based on the above successful agreement of baryon spectrum in the framework of fractional quark theory with experimental data one can propose fractional generalized lagrangian formalism for models which describe quarks' physics. For instance, one can consider a fractional generalization of NJL model which is an effective model for quarks and describes the mass of nucleons through spontaneous chiral symmetry breaking and drive some new results. The advantages of this approach to quark theory is that such these models will be intrinsically nonlocal and also will consider memory effects which seem to be an important unconsidered characteristic of quarks and quark matter. So in the framework of fractional dynamics especially when the space-time dependent order of fractional derivative α is close to unity [17], QCD lagrangian for n_f flavors of quark fields and the gluon fields is in the form of:

$$L_{FQCD} = \sum_f \bar{q}_f (i\gamma_\alpha{}^\mu D_\mu^\alpha + I_\alpha m_f{}^\alpha) q_f - \frac{1}{4} G_{\mu\nu,\alpha}^a G^{a\mu\nu,\alpha} - \frac{\eta}{2} G_{\mu\nu,\alpha}^a G^{a\mu\nu,\alpha} \tag{12}$$

where in which we have:

$$D_\mu^\alpha = \partial_\mu^\alpha - ig_s G_\mu \tag{13}$$

$$G_{\mu\nu,\alpha} = \partial_\mu^\alpha G_\nu - \partial_\nu^\alpha G_\mu + g_s f_{abc} G_\mu G_\nu \tag{14}$$

$\gamma_\alpha{}^\mu$ - matrices are built from triads of traceless, unitary $n \times n$ matrices, which span a subspace of $SU(\mathrm{n})$ and obey an extended Clifford algebra [14,18]:

$$\sum_{\{\pi\}} \prod_{i=1}^{n} \gamma_\alpha{}^{\mu_i} = n!\,\delta^{\mu_1\mu_2\cdots\mu_n} \tag{15}$$

where $\{\pi\}$ denotes all permutations of $\gamma_\alpha{}^{\mu_i}$ and finally η is a real parameter and I_α is the corresponding unit matrix. So based on the above formalism we can drive fractional generalization of NJL model in the form of:

$$S_\alpha = \int d^4x \left(\bar{t}(x)(i\gamma_\alpha{}^\mu \partial_\mu^\alpha + I_\alpha m_t{}^\alpha) t(x) - \frac{N}{2} J(x) J(x) \right) \tag{16}$$

with local scalar current.

3. Exotic baryons

In this section we consider the special case of exotic baryons in the above mentioned framework i.e. fractional quark theory and we examine whether it can play a role in future exotic baryon spectroscopy. Recently in an analysis of Run 1 data, the LHCb collaboration reported the observation of significant $J/\psi p$ pentaquark structures in $\Lambda_b^0 \to J/\psi p K^-$ decays. A model-dependent six-dimensional amplitude analysis of invariant masses and decay angles describing the Λ_b^0 decay revealed a $P_c(4450)^+$ structure peaking at $4449.8 \pm 1.7 \pm 2.5\ MeV$ with a width of $39 \pm 5 \pm 19\ MeV$ [19 and Refs. therein]. The $P_c(4450)^+$ is confirmed and observed to consist of two narrow overlapping peaks, $P_c(4440)^+$ and $P_c(4457)^+$, where the statistical significance of this two-peak interpretation is 5.4σ [19 and Refs. therein]. The optimum fit parameter set α, m_0, a_0, b_0 of Eq. (11) for baryon spectroscopy were obtained respectively as: 0.112, $-17171.6\,[MeV]$, $10971.8\,[MeV]$, $8064.6\,[MeV]$ [7, 14]. Surprisingly the fractional baryon mass spectra for the values of $L=14$ and $M=13$ will result in $4444.5\,[MeV]$ which is completely close to the observed and experimentally calculated mass of P_c pentaquark [20] and again with an error less than 1%.

4. Conclusion

A final theory of nature has to consider space-time nonlocality which in time domain means memory effect described by memory kernel. This theory can be called N-theory. Based on this theory it seems that memory is a fundamental quantity of our nature. In our previous works we addressed electromagnetic memory and gravitational memory [11, 12]. This work deals with strong interaction using the tool of fractional calculus as the best tool for this purpose. So based on some characteristics which are exhibited by quarks and quark matter i.e. nonlinearity, nonlocality, non-equilibrium dynamics [21], viscosity [22], we immediately concluded that all of them can be perfectly and effectively described by use of the powerful tool of fractional calculus that intrinsically incorporate memory in system. We have proposed this idea that there will be a fractional theory of quarks and quark matter which will be able to describe and predict many phenomena in this field in future. As an example we showed that this powerful framework can reproduce the experimental mass for the exotic baryons. And finally this framework tells us that other exotic baryons can be observed and approved experimentally in near future.

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