Proof for correctness of Collatz conjecture

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Abstract

Number series of Collatz conjecture reaches to value 1 finally if the series of number has no looping. This has been mentioned statistically on viXra:2204.0151 (*1).

There is no looping in Number series of Collatz conjecture. This has been mentioned algebraically on viXra:2206.0056 (*2).

Here will try to prove algebraically Collatz conjecture is correct.

1. Introduction

As Collatz conjecture operations is defined Contraction, its inverse operation is defined as Expansion here. Using this Expansion, it is tried here to prove Collatz conjecture is correct. On the view of Expansion, conjecture of Collatz is equivalent to the fact iteration of Expansion operation expands from 1 to all positive odd integer numbers.

2. Expansion and Contraction

A. Expansion

Positive odd integer NOI is represented as follows.

$$NOI = 3n + P$$

n; positive integer, P; 0,1,2

NOO is calculated on following formula.

$$NOO = \frac{NOI \times 2^m - 1}{3} = \frac{(3n + P) \times 2^m - 1}{3}$$

Here following conditions are required.

1) NOI,NOO are positive odd integer
2) Dividing by 3 in the formula has no remainder.

For these conditions, there are also following conditions regarding to n, m for each P value. Then calculation of NOO is done.

In the case of P=0:

Calculation cannot be done based on 2) for all n because following formula has always remainder.

$$NOO = \frac{(3n + 0) \times 2^m - 1}{3} = \frac{3n \cdot 2^m - 1}{3}$$

Sample 1

n=25, m=4

$$NOO = \frac{(3 \times 25 + 0) \times 2^4 - 1}{3} = \frac{3 \times 25 \times 16 - 1}{3} = 399 \cdots 2$$

In the case of P=1:

m=even integer from 2: 2,4,6,8,10・・・ for required condition 2)

n=even integer from 0: 0,2,4,6・・・ for required condition 1)

$$NOO = \frac{(3n + 1) \times 2^m - 1}{3} = \frac{3n \cdot 2^m + (2^m - 1)}{3} = \frac{3n \cdot 2^m + (2 + 1)(\,) \cdots}{3}$$

Sample 2

n=26, m=4

$$NOO = \frac{(3 \times 26 + 1) \times 2^4 - 1}{3} = \frac{3 \times 26 \times 2^4 + 2^4 - 1}{3} = 421$$

Sample 3

n=26, m=8

$$NOO = \frac{(3 \times 26 + 1) \times 2^8 - 1}{3} = \frac{3 \times 26 \times 2^8 + 2^8 - 1}{3} = 6741$$

In the case of P=2:

m=odd integer from1: 1,3,5,7,9・・・・ for required condition 2)

n=odd integer from 1] 1,3,5,7・・・ for required condition 1)

$$NOO = \frac{(3n + 2) \times 2^m - 1}{3} = \frac{3n \cdot 2^m + (2^{m+1} - 1)}{3} = \frac{3n \cdot 2^m + (2 + 1)(\,) \cdots}{3}$$

Sample 4

n=27, m=3

$$NOO = \frac{(3 \times 27 + 2) \times 2^3 - 1}{3} = \frac{3 \times 27 \times 2^3 + 2^4 - 1}{3} = 221$$

These operations represent a relation between one positive odd integer NOI and multiple NOOs for each m value.

B. Contraction

NOI is calculated on following formula from NOO.

$$NOI = \frac{NOO \times 3 + 1}{2^m}$$

Sample 5

NOO=221

$$NOI = \frac{NOO \times 3 + 1}{2^m} = \frac{221 \times 3 + 1}{2^m} = \frac{664}{2^m} = 83$$

m=3, n=27($NOI = 3n + P = 3 \times 27 + 2 = 83$)

This procedure is recognized as Contraction from NOOs to a NOI.

As mentioned, these procedures have following characteristics.

- Multiple NOOs can be created from a NOI by Extraction. (1)
- A unique NOI is determined from multiple NOOs by Contraction. (2)

Sample 6 from sample 2, 3,

421(NOO)－79(NOI)

6741(NOO)－79(NOI)

3. Consideration

Contraction is apparently Collatz procedure itself and Expansion is reverse procedure of it. NOO as variable of Contraction function can be all positive odd integer. Therefore NOO as value of Extraction function which is reverse function of Contraction, can be all positive integer. Then

NOO of Extraction can be all positive odd number. (a)

Following is another characteristic of Extraction.

Regarding to Extraction, every NOO has its relevant NOI. (b)

On (a) (b), NOI and NOO of Extraction make number chain. (c)

Number chain of Extraction is same as Contraction's. (d)

On (a) (b), all number chain of Extraction starts with value 1 if there is no looping. (e)
The reason of this is that if there could be other start number (for example ns) than 1, Extraction for NOO=ns has another NOI which could be before ns in the number chain. Therefore ns cannot be start number.

In other hand, Extraction for NOO=1 has specially NOI=1. Therefore value 1 is lower end.

On (*2) and (d), number chain of Extraction has no looping. Therefore (e) is modified as follow.

Every number chain of Extraction starts with value 1. (f)

4. Conclusion

On above, (a)(d)(f), we can say conjecture of Collatz is correct.

Reference

(*1) https://vixra.org/abs/2204.0151

(*2) https://vixra.org/abs/2206.0056