Collatz Directed Graph

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Abstract

We present the construction of a directed graph based on the iterating $T(n) = (2n - b)/3^b$, where b=0 and b=1 if n ≡ 2mod(3) . Let G be a directed graph with nodes V(G) represented by integers generated from T(n) starting with n =1. This graph is called the Collatz directed graph. Let A be a set consists of all nodes in G and $N = \{1, 2, 3, \ldots\}$ the set of all natural numbers . It is proved that (N- A) is an empty set.

1. **Introduction**

Define the iterating function introduced by R. Terras[1] :

$$T(n) = (2n - b)/3^b, \tag{1}$$

where b=0 and b=1 if n ≡ 2mod(3)

2. **The Collatz directed graph**

Let G be a directed graph with nodes V(G) represented by integers generated from (1) starting with n =1. This graph is called the Collatz directed graph as shown in Figure 1.

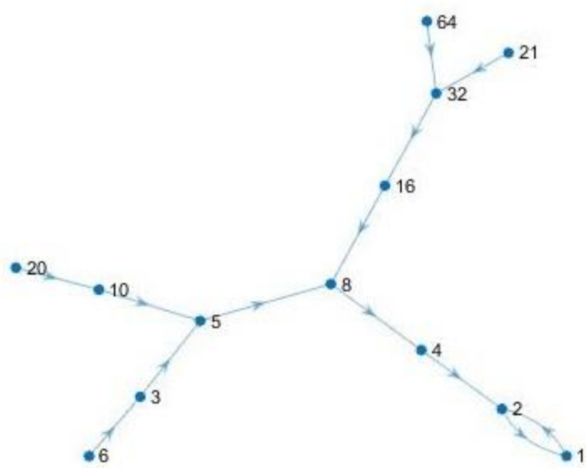

Figure 1. Some part of G starting at 6, 20, 21, 64

Define a recursive equation on N by setting b = 1 when a_n is odd and b = 0 when a_n is even,

$$a_{n+1} = (3^b a_n + b)/2 \qquad (2)$$

Denote each natural number as node in a graph $\hat{G}$, by eq.(1) each node in $\hat{G}$ can have one or two incoming node, and by (2) each node can have only one outgoing node as shown in Figure 2.

2← 1←2

1← 2←4

5← 3←6

2← 4←8

8← 5←3, 10

3← 6←12

11← 7←14

4← 8←5, 16

: : :

Figure 2 Structure of $\hat{G}$

The Collatz conjecture asserts that by using (2) repeatedly always lead to 1 for any $a_0 \in N$. Let B be a set consists of all nodes of $\hat{G}$ then B = N. The validity of the Collatz conjecture is established by proving that (N-A) is an empty set ,where A is a set of all nodes in G. This assertion will be proved by contradiction as follows:

Assume (N-A) = { $n_1, n_2, n_3, \ldots\ldots.. n_m$ } is not an empty set. Since each n_i, i=1,2,…m , m ∈N always has $2n_i$ as its incoming node, but $2n_i$ is not in (N-A).

Thus, (N-A) is an empty set.

References

[1] R . Terras, (1976). " A stopping time problem on the positive integers".

Acta Arithmetica, 30(3), 241-252.