

# An Novel Set of Representations for $\zeta(3)$

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2022-05-13

**ABSTRACT.** Here, we provide a novel set of expressions for zeta (3).

In our study, the following 19 expressions of zeta (3) were directly obtained by an equation or system of equations, which are called *the first kind* of expressions. As souvenirs, they are now displayed.

Note: If dig deeper, there are more than 520,000 expressions of zeta (3) called *the second kind* of.

№	<i>the First Kind</i> of expressions ( $B_{2n}$ are Bernoulli numbers)
1	$\zeta(3) = \pi^2 \cdot \frac{4}{35} \left[ \frac{3}{2} + \ln\left(\frac{2}{\pi}\right) + \sum_{n=1}^{20} \frac{ B_{2n}  \cdot (\pi)^{2n}}{2^{2n} \cdot n \cdot (2n+2)!} \right]$
2	$\zeta(3) = \pi^2 \cdot \frac{2}{7} \left[ \frac{3}{2} - \ln(\pi) + \sum_{n=1}^{20} \frac{ B_{2n}  \cdot (\pi)^{2n}}{n \cdot (2n+2)!} \right]$
3	$\zeta(3) = \pi^2 \cdot \frac{36}{35} \left[ \frac{3}{2} + \ln\left(\frac{2}{3 \cdot \pi}\right) + \sum_{n=1}^{20} \frac{3^{2n} \cdot  B_{2n}  \cdot (\pi)^{2n}}{2^{2n} \cdot n \cdot (2n+2)!} \right]$
4	$\zeta(3) = \frac{\pi^2}{8} \left[ \frac{77}{18} + \frac{7}{3} \cdot \ln\left(\frac{2}{\pi}\right) - \frac{9 \cdot \ln(3)}{4} + \sum_{n=1}^{20} \frac{(1 + 3^{2n+3}) \cdot  B_{2n}  \cdot (\pi)^{2n}}{2^{2n+2} \cdot n \cdot (2n+3)!} \right]$
5	$\zeta(3) = \frac{\pi^2}{2} \left[ \frac{11}{18} - \frac{\ln(\pi)}{3} + \sum_{n=1}^{20} \frac{ B_{2n}  \cdot \pi^{2n}}{n \cdot (2n+3)!} \right]$
6	$\zeta(3) = \pi^2 \left[ \frac{11}{9} - \frac{2}{3} \cdot \ln(2\pi) + \sum_{n=1}^{20} \frac{2^{2n+1} \cdot  B_{2n}  \cdot \pi^{2n}}{n \cdot (2n+3)!} \right]$
7	$\zeta(3) = \frac{\pi^2}{9} \left[ \frac{125}{48} - \frac{\ln(2)}{6} - \frac{5 \cdot \ln(\pi)}{4} + \sum_{n=1}^{20} \frac{(17 \cdot 2^{2n} - 2) \cdot  B_{2n}  \cdot (\pi)^{2n}}{2^{2n} \cdot n \cdot (2n+4)!} \right]$
8	$\zeta(3) = \frac{\pi^2}{55} \left[ \frac{3625}{144} + \frac{27}{2} \cdot \ln\left(\frac{2}{3}\right) - \frac{145 \cdot \ln(\pi)}{12} + \sum_{n=1}^{40} \frac{(2 \cdot 3^{2n+4} - 17 \cdot 2^{2n}) \cdot  B_{2n}  \cdot (\pi)^{2n}}{2^{2n} \cdot n \cdot (2n+4)!} \right]$

9	$\zeta(3) = \frac{\pi^2}{8} \left[ \frac{125}{36} + \frac{5}{3} \cdot \ln\left(\frac{2}{\pi}\right) - \frac{27 \cdot \ln(3)}{16} + \sum_{n=1}^{20} \frac{(3^{2n+4} - 1) \cdot  B_{2n}  \cdot (\pi)^{2n}}{2^{2n+2} \cdot n \cdot (2n+4)!} \right]$
10	$\zeta(3) = \pi^2 \left[ \frac{25}{36} - \frac{\ln(2 \cdot \pi)}{3} + \sum_{n=1}^{20} \frac{2^{2n+2} \cdot  B_{2n}  \cdot \pi^{2n}}{n \cdot (2n+4)!} \right]$
11	$\zeta(3) = \frac{\pi^2}{16} \left[ \frac{137}{20} + \frac{61 \cdot \ln(2)}{15} - \frac{81 \cdot \ln(3)}{20} - 3 \cdot \ln(\pi) + \sum_{n=1}^{20} \frac{(3^{2n+5} + 1 - 2^{2n+6}) \cdot  B_{2n}  \cdot (\pi)^{2n}}{2^{2n} \cdot n \cdot (2n+5)!} \right]$
12	$\zeta(3) = \frac{\pi^2}{16} \left[ \frac{1781}{180} - \frac{317 \cdot \ln(2)}{45} + \frac{27 \cdot \ln(3)}{20} - \frac{13 \cdot \ln(\pi)}{3} + \sum_{n=1}^{20} \frac{(2^{4n+10} + 1 - 3^{2n+5}) \cdot  B_{2n}  \cdot (\pi)^{2n}}{3 \cdot 2^{2n} \cdot n \cdot (2n+5)!} \right]$
13	$\zeta(3) = \pi^2 \left[ \frac{137}{240} - \frac{4 \cdot \ln(2)}{15} - \frac{\ln(\pi)}{4} + \sum_{n=1}^{20} \frac{(2^{2n+4} - 1) \cdot  B_{2n}  \cdot (\pi)^{2n}}{n \cdot (2n+5)!} \right]$
14	$\zeta(3) = \frac{\pi^2}{272} \left[ \frac{6517}{60} + \frac{607 \cdot \ln(2)}{9} - \frac{2673 \cdot \ln(3)}{40} - \frac{133 \cdot \ln(\pi)}{3} + \sum_{n=1}^{20} \frac{(11 \cdot 3^{2n+7} + 223 - 65 \cdot 2^{2n+7}) \cdot  B_{2n}  \cdot (\pi)^{2n}}{2^{2n} \cdot n \cdot (2n+6)!} \right]$
15	$\zeta(3) = \frac{\pi^2}{103} \left[ \frac{1421}{24} - \frac{2513 \cdot \ln(2)}{45} + \frac{81 \cdot \ln(3)}{5} - \frac{145 \cdot \ln(\pi)}{6} + \sum_{n=1}^{20} \frac{(223 \cdot 2^{4 \cdot n+6} + 65 \cdot 2^{2 \cdot n+2} - 2^3 \cdot 3^{2n+6}) \cdot  B_{2n}  \cdot (\pi)^{2n}}{2^{2n} \cdot n \cdot (2n+6)!} \right]$
16	$\zeta(3) = \frac{\pi^2}{16} \left[ \frac{539}{60} - \frac{347 \cdot \ln(2)}{45} + \frac{81 \cdot \ln(3)}{40} - \frac{11 \cdot \ln(\pi)}{3} + \sum_{n=1}^{20} \frac{(2^{4n+11} + 1 - 3^{2n+6}) \cdot  B_{2n}  \cdot (\pi)^{2n}}{2^{2n} \cdot n \cdot (2n+6)!} \right]$
17	$\zeta(3) = \frac{\pi^2}{25} \left[ \frac{1519}{120} - \frac{263 \cdot \ln(2)}{45} - \frac{31 \cdot \ln(\pi)}{6} + \sum_{n=1}^{20} \frac{(33 \cdot 2^{4 \cdot n+6} + 8 - 65 \cdot 2^{2n+2}) \cdot  B_{2n}  \cdot (\pi)^{2n}}{2^{2n} \cdot n \cdot (2n+6)!} \right]$
18	$\zeta(3) = \frac{\pi^2}{4} \left[ \frac{121}{56} - \frac{1571 \cdot \ln(2)}{630} + \frac{243 \cdot \ln(3)}{280} - \frac{5 \cdot \ln(\pi)}{6} + \sum_{n=1}^{20} \frac{(2^{4 \cdot n+12} + 3 \cdot 2^{2 \cdot n+6} - 1 - 3^{2n+7}) \cdot  B_{2n}  \cdot (\pi)^{2n}}{2^{2n} \cdot n \cdot (2n+7)!} \right]$
19	$\zeta(3) = \frac{\pi^2}{44} \left[ \frac{5327}{240} - \frac{5909 \cdot \ln(2)}{210} + \frac{23571 \cdot \ln(3)}{2240} - \frac{49 \cdot \ln(\pi)}{6} + \sum_{n=1}^{20} \frac{(65 \cdot 2^{4 \cdot n+14} + 257 \cdot 2^{2 \cdot n+8} - 415 - 97 \cdot 3^{2n+8}) \cdot  B_{2n}  \cdot (\pi)^{2n}}{3 \cdot 2^{2n} \cdot n \cdot (2n+8)!} \right]$

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