Is Special Relativity compatible with General Relativity?

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Abstract

Is special relativity theory applicable to the universe, as this theory was developed for places with no gravity? Is Special Relativity compatible with General Relativity?

Yes, both theories are compatible although there is a difference between Special Relativity (SR) and General Relativity (GR).

SR relates to speed and time, it was developed by Einstein in 1905 and was based on works done by Newton, Lorentz, Maxwell, Michelson-Morley, and others. GR was suggested by Einstein in 1916 to generalize SR, by including acceleration. According to the equivalence principle, acceleration and gravity are equivalent. As the universe is full of celestial bodies gravity exists everywhere.

SR is based on two main postulates:

1. The laws of physics are invariant in all inertial frames of reference (that is, frames of reference with no acceleration or gravity).
2. The speed of light in the vacuum is the same for all observers, regardless of the motion of the light source or observer.

From postulate one of SR, it can be concluded that SR is applicable only in deep space, far away from any celestial body, where there is no gravity. Therefore, using SR to solve problems in the universe where gravity exists is not applicable.

It should be noted that Einstein had the same claim on the validity of SR in presence of gravity. A quote from the collected papers of Albert Einstein Volume 7: The Berlin Years: Writings, 1918-1921

https://einsteinpapers.press.princeton.edu/vol7-trans/156?highlightText=%22spatially%20variable%22

"Second, this consequence shows that the law of the constancy of the speed of light no longer holds, according to the general theory of relativity, in spaces that have gravitational fields. As a simple geometric consideration shows, the curvature of light rays occurs only in spaces where the speed of light is spatially variable. **From this, it follows that the entire conceptual system of the theory of special relativity can claim rigorous validity only for those space-time domains where gravitational fields (under appropriately chosen coordinate systems) are absent. The theory of special relativity, therefore, applies only to a limiting case that is nowhere precisely realized in the real world**. Nevertheless, this limiting case ⟨also⟩ is of fundamental significance for the theory of general relativity; because of the fact from which we started, namely that no gravitational field exists near a free-falling observer, this very fact shows that in the vicinity of every world point the results of the theory of special relativity are valid (in the infinitesimal) for a suitably chosen local coordinate system."

Nevertheless, SR has been validated, despite Einstein's claim, for over a century by many experiments and is used in places that are influenced by gravity, e.g., in the vicinity of Earth.

I will bring here the example of the global positioning system (GPS), which is used many million times a day by users all over the world.

Fig 1- shows schematically the structure of the GPS. The GPS is comprised of three different segments: the space segment, the control segment, and the user segment. The space segment includes 30+ navigation satellites circling Earth at an altitude of 20,000 km and a speed of 14,000km/h. The control segment is a ground station and the user segment e.g., cellular phone.

It is shown that the reference frame direction of the satellite orbiting Earth is constantly changing relative to the reference frame of the ground station

At the start of measurement, the reference frames of ground station G_0 and satellite frame S_0 are parallel. If the measurement time is big then the frames G_f and S_f are no longer parallel and are rotated by angle Φ. For example, if the measurement duration is one hour then Φ1=60deg and Φ2=30deg. Thus, the relative rotation Φ =Φ1- Φ2=30deg. The smaller the measurement time, the smaller Φ, and the reference frames stay nearly parallel.

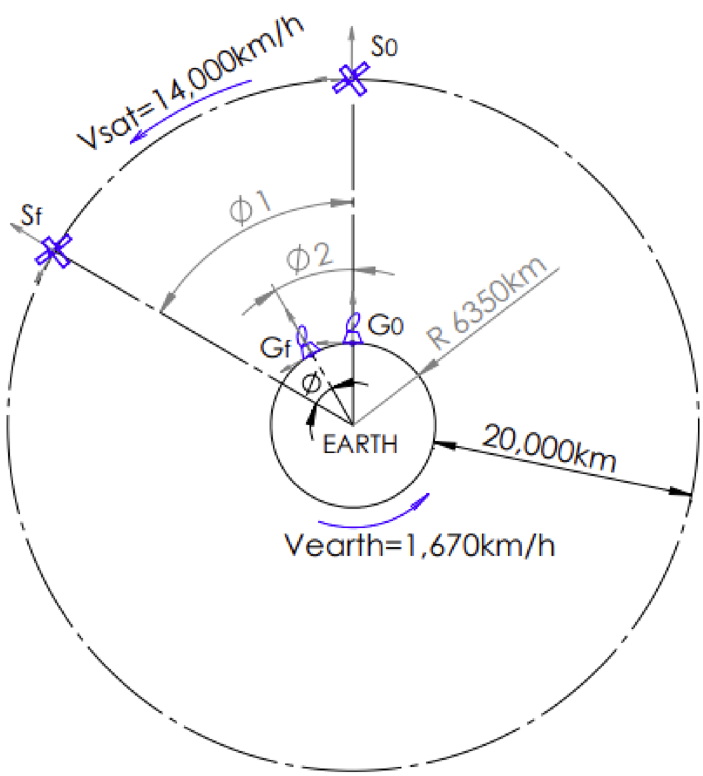

Fig 1. – Global Positioning System

According to Einstein, the rate of advance of two identical clocks placed one on the satellite and the other on the ground station, will differ due to the difference in the gravitational potential (GR) and the relative speed between them (SR). From GR, it can be calculated that the orbiting clocks in the satellites tick slightly faster than the clock at the ground station, by about 45 microseconds per day. On the other hand, from SR calculations the orbiting clock ticks slower, about 7 microseconds per day. The net result is that time on a GPS satellite clock advances faster than a clock on the ground by about 38 microseconds per day. If relativity compensation is not taken into account, it would cause navigational errors that accumulate to 10 km per day! **Note:** a detailed calculation is given in **Appendix A**.

Thus, my answer to the question of how SR is applicable in gravity fields, is that SR is applicable in limited cases where the measurement duration is "small enough" so that the change of references frame is negligible. The definition of "small enough" means that SR is an engineering approximation. It will be accurate if the time of measurement is infinitesimal.

An additional example of the usage of SR in the gravity field of Earth is the measurement of the atmospheric muon. This is also a case when the reference frames are inertial. On the other hand, I claim that sometimes SR is used in such cases that result in absurdity. For example, the tween paradox, or the grandfather paradox. I claim that these paradoxes are not valid for SR because they are done during a long time where the frame references are not inertial.

SR has been used to solve cases such as the Sagnac effect, Bradley stellar aberration, and the Michelson-Morley experiment. I think that using SR for these cases is not valid. I claim that all these observations can be explained by another phenomenon. This is the frame-dragging of space by a spinning celestial body. Frame dragging was predicted by GR and verified by the Gravity Probe A experiment.

The dragging of space by a spinning celestial body was suggested by Stokes (and others), to explain the Bradley stellar aberration. However, observations showed that Stokes was wrong. Nevertheless, I accept Stokes' proposal that space is dragged by any spinning celestial body. But it is not only space that is dragged by Earth but rather the entire universe being dragged by a universal spinning central neutron star. For a detailed description see:

https://www.academia.edu/45575390/The_structure_of_the_Pivot_Universe

Note: There is one experiment that was done by Hafele and Keating that I cannot explain by my previous assumptions. Time dilation is not accumulated thus the time dilation between the two clocks, calibrated on the ground, one that flew around Earth and finally brought near the second clock that stayed on Earth must be zero. The final time dilation of the two clocks, as explained in the paragraph on GPS, has the same gravitational time delay and the same SR delay.

Appendix A

This appendix is a citation from:

https://www.scienceofgadgets.com/post/how-relativistic-time-dilation-and-gps-are-related

Time dilation due to velocity - SR

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{86\,400}{\sqrt{1-\frac{3889^2}{(3\cdot 10^8)^2}}} = 86\,400{,}000007\,s$$

Δt - time measured on satellite

Δt_0 - time measured on Earth

v - satellite speed (3889 m/s)

c - speed of light ($3\cdot 10^8$ m/s)

Time dilation due to gravity - GR

$$\Delta t_1 = \frac{\Delta t_0}{\sqrt{1 - \frac{2GM}{rc^2}}} = \frac{86400}{\sqrt{1 - \frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 5.972 \cdot 10^{24}}{6.37 \cdot 10^{3} \cdot (3 \cdot 10^{8})^2}}} = 86400{,}00006\,s$$

Δt_1 - time with gravity
Δt_0 - time without gravity
G - gravitational constant
M - mass of the Earth
r - radius of the Earth
c - speed of light

The GPS satellite orbits about 20 000 kilometers above the Earth's surface. At this distance (**r** + **h**) from the center of mass of the Earth the clock onboard experiences a time dilation of 0.000015 seconds.

$$\Delta t_2 = \frac{\Delta t_0}{\sqrt{1 - \frac{2GM}{(r+h)c^2}}} = \frac{86400}{\sqrt{1 - \frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 5.972 \cdot 10^{24}}{2.638 \cdot 10^{7} \cdot (3 \cdot 10^{8})^2}}} = 86400{,}000015\,s$$

Δt_2 - time with gravity
Δt_0 - time without gravity
G - gravitational constant
M - mass of the Earth
r - radius of the Earth
h - height above the ground
c - speed of light

So, we get a time difference of 45 microseconds (0.00006 - 0.000015 = 0.000045) per day between the time measured on Earth and the time measured on the satellite. From the first equation, we found that the clock on the fast-moving satellite lags behind the receiver's clock by 7 microseconds per day. Subtracting time dilation of 7 microseconds from 45 microseconds, we find that the receiver clock runs 38 microseconds slower per day than the clock onboard of GPS satellite (0.000045 - 0.000007 = 0.000038 sec = 38 ms).