Connections between the prime factorization exponents of a natural number and the possible sums of 2+ consecutive naturals

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This paper identifies a correlation between a multiplicative property of number and the additive property of all the sequences of consecutive number with equal sum

Premise

All this research has been done using Mathematica©, a symbolic computation environment that uses a programming language called Wolfram Language.

Consider initially the sum of k>1 consecutive positive naturals starting from n:

Theorem 1

This sum cannot be expressed as a power of 2.

Demonstration

The sum of k>1 consecutive integers starting with n>0 is m=k*(2n+k−1)/2, hence 2m=k*(2n+k−1). If m is a power of 2 so is 2m, then both k and 2n+k−1 must be powers of two, but they have different parities, so one of them must be 1. Since k<(2n+k−1), so k must be 1 and this contradicts the fact that k>1.

Now I start to investigate how a number not power of 2 can be represented as some given sum of consecutive k numbers starting from n. Here is a function to do this:

```
In[ ]:= sumconsecint[m_Integer]:=Reduce[(2n+k-1)*k/2==m&&n>0&&k>1,{n,k},Integers]/.{And→List}
```

We start with the 3,5,7,11 multiplied with the powers of 2 followed:

In[ ]:=
```
sumconsecint[3]
sumconsecint[2*3]
sumconsecint[2^2*3]
sumconsecint[2^3*3]
sumconsecint[2^4*3]
sumconsecint[5]
sumconsecint[2*5]
sumconsecint[2^2*5]
sumconsecint[2^3*5]
sumconsecint[2^4*5]
sumconsecint[7]
sumconsecint[2*7]
sumconsecint[2^2*7]
sumconsecint[2^3*7]
sumconsecint[2^4*7]
sumconsecint[11]
sumconsecint[2*11]
sumconsecint[2^2*11]
sumconsecint[2^3*11]
sumconsecint[2^4*11]
```

Out[ ]= {n == 1, k == 2}

Out[ ]= {n == 1, k == 3}

Out[ ]= {n == 3, k == 3}

Out[ ]= {n == 7, k == 3}

Out[ ]= {n == 15, k == 3}

Out[ ]= {n == 2, k == 2}

Out[ ]= {n == 1, k == 4}

Out[ ]= {n == 2, k == 5}

Out[ ]= {n == 6, k == 5}

Out[ ]= {n == 14, k == 5}

Out[ ]= {n == 3, k == 2}

Out[ ]= {n == 2, k == 4}

Out[ ]= {n == 1, k == 7}

Out[ ]= {n == 5, k == 7}

Out[ ]= {n == 13, k == 7}

Out[ ]= {n == 5, k == 2}

Out[ ]= {n == 4, k == 4}

Out[ ]= {n == 2, k == 8}

Out[ ]= {n == 3, k == 11}

Out[ ]= {n == 11, k == 11}

As you can see all the numbers with pattern 2^m*p have always A SINGLE solution.

Now consider a slightly different pattern 2^m*p1*p2

In[ ]:=
```
sumconsecint[3*5]
sumconsecint[2*3*5]
sumconsecint[2^2*3*5]
sumconsecint[2^3*3*5]
sumconsecint[2^4*3*5]
sumconsecint[3*7]
sumconsecint[2*3*7]
sumconsecint[2^2*3*7]
sumconsecint[2^3*3*7]
sumconsecint[2^4*3*7]
sumconsecint[3*11]
sumconsecint[2*3*11]
sumconsecint[2^2*3*11]
sumconsecint[2^3*3*11]
sumconsecint[2^4*3*11]
sumconsecint[5*7]
sumconsecint[2*5*7]
sumconsecint[2^2*5*7]
sumconsecint[2^3*5*7]
sumconsecint[2^4*5*7]
sumconsecint[5*11]
sumconsecint[2*5*11]
sumconsecint[2^2*5*11]
sumconsecint[2^3*5*11]
sumconsecint[2^4*5*11]
sumconsecint[7*11]
sumconsecint[2*7*11]
sumconsecint[2^2*7*11]
sumconsecint[2^3*7*11]
sumconsecint[2^4*7*11]
```

```
Out[ ]= {n == 1, k == 5} || {n == 4, k == 3} || {n == 7, k == 2}
Out[ ]= {n == 4, k == 5} || {n == 6, k == 4} || {n == 9, k == 3}
Out[ ]= {n == 4, k == 8} || {n == 10, k == 5} || {n == 19, k == 3}
Out[ ]= {n == 1, k == 15} || {n == 22, k == 5} || {n == 39, k == 3}
Out[ ]= {n == 9, k == 15} || {n == 46, k == 5} || {n == 79, k == 3}
Out[ ]= {n == 1, k == 6} || {n == 6, k == 3} || {n == 10, k == 2}
Out[ ]= {n == 3, k == 7} || {n == 9, k == 4} || {n == 13, k == 3}
Out[ ]= {n == 7, k == 8} || {n == 9, k == 7} || {n == 27, k == 3}
Out[ ]= {n == 3, k == 16} || {n == 21, k == 7} || {n == 55, k == 3}
Out[ ]= {n == 6, k == 21} || {n == 45, k == 7} || {n == 111, k == 3}
Out[ ]= {n == 3, k == 6} || {n == 10, k == 3} || {n == 16, k == 2}
Out[ ]= {n == 1, k == 11} || {n == 15, k == 4} || {n == 21, k == 3}
Out[ ]= {n == 7, k == 11} || {n == 13, k == 8} || {n == 43, k == 3}
Out[ ]= {n == 9, k == 16} || {n == 19, k == 11} || {n == 87, k == 3}
Out[ ]= {n == 1, k == 32} || {n == 43, k == 11} || {n == 175, k == 3}
Out[ ]= {n == 2, k == 7} || {n == 5, k == 5} || {n == 17, k == 2}
Out[ ]= {n == 7, k == 7} || {n == 12, k == 5} || {n == 16, k == 4}
```

Out[ ]= {n == 14, k == 8} || {n == 17, k == 7} || {n == 26, k == 5}

Out[ ]= {n == 10, k == 16} || {n == 37, k == 7} || {n == 54, k == 5}

Out[ ]= {n == 2, k == 32} || {n == 77, k == 7} || {n == 110, k == 5}

Out[ ]= {n == 1, k == 10} || {n == 9, k == 5} || {n == 27, k == 2}

Out[ ]= {n == 5, k == 11} || {n == 20, k == 5} || {n == 26, k == 4}

Out[ ]= {n == 15, k == 11} || {n == 24, k == 8} || {n == 42, k == 5}

Out[ ]= {n == 20, k == 16} || {n == 35, k == 11} || {n == 86, k == 5}

Out[ ]= {n == 12, k == 32} || {n == 75, k == 11} || {n == 174, k == 5}

Out[ ]= {n == 2, k == 11} || {n == 8, k == 7} || {n == 38, k == 2}

Out[ ]= {n == 9, k == 11} || {n == 19, k == 7} || {n == 37, k == 4}

Out[ ]= {n == 23, k == 11} || {n == 35, k == 8} || {n == 41, k == 7}

Out[ ]= {n == 31, k == 16} || {n == 51, k == 11} || {n == 85, k == 7}

Out[ ]= {n == 23, k == 32} || {n == 107, k == 11} || {n == 173, k == 7}

As before the presence of a power of 2 is irrelevant and for every example of the pattern the number of solutions is 3. We continue the exploration with the pattern 2^m*p1*p2^2:

In[ ]:=
```
sumconsecint[3*5^2]
sumconsecint[2*3*5^2]
sumconsecint[2^2*3*5^2]
sumconsecint[2^3*3*5^2]
sumconsecint[2^4*3*5^2]
sumconsecint[3*7^2]
sumconsecint[2*3*7^2]
sumconsecint[2^2*3*7^2]
sumconsecint[2^3*3*7^2]
sumconsecint[2^4*3*7^2]
sumconsecint[3*11^2]
sumconsecint[2*3*11^2]
sumconsecint[2^2*3*11^2]
sumconsecint[2^3*3*11^2]
sumconsecint[2^4*3*11^2]
sumconsecint[5*7^2]
sumconsecint[2*5*7^2]
sumconsecint[2^2*5*7^2]
sumconsecint[2^3*5*7^2]
sumconsecint[2^4*5*7^2]
sumconsecint[5*11^2]
sumconsecint[2*5*11^2]
sumconsecint[2^2*5*11^2]
sumconsecint[2^3*5*11^2]
sumconsecint[2^4*5*11^2]
sumconsecint[7*11^2]
sumconsecint[2*7*11^2]
sumconsecint[2^2*7*11^2]
sumconsecint[2^3*7*11^2]
sumconsecint[2^4*7*11^2]
```

Out[ ]= {n == 3, k == 10} || {n == 10, k == 6} || {n == 13, k == 5} || {n == 24, k == 3} || {n == 37, k == 2}

```
Out[]= {n == 3, k == 15} || {n == 7, k == 12} || {n == 28, k == 5} || {n == 36, k == 4} || {n == 49, k == 3}
Out[]= {n == 1, k == 24} || {n == 13, k == 15} || {n == 34, k == 8} || {n == 58, k == 5} || {n == 99, k == 3}
Out[]= {n == 12, k == 25} || {n == 30, k == 16} || {n == 33, k == 15} || {n == 118, k == 5} || {n == 199, k == 3}
Out[]= {n == 22, k == 32} || {n == 36, k == 25} || {n == 73, k == 15} || {n == 238, k == 5} || {n == 399, k == 3}
Out[]= {n == 4, k == 14} || {n == 18, k == 7} || {n == 22, k == 6} || {n == 48, k == 3} || {n == 73, k == 2}
Out[]= {n == 4, k == 21} || {n == 19, k == 12} || {n == 39, k == 7} || {n == 72, k == 4} || {n == 97, k == 3}
Out[]= {n == 13, k == 24} || {n == 18, k == 21} || {n == 70, k == 8} || {n == 81, k == 7} || {n == 195, k == 3}
Out[]= {n == 1, k == 48} || {n == 46, k == 21} || {n == 66, k == 16} || {n == 165, k == 7} || {n == 391, k == 3}
Out[]= {n == 24, k == 49} || {n == 58, k == 32} || {n == 102, k == 21} || {n == 333, k == 7} || {n == 783, k == 3}
Out[]= {n == 6, k == 22} || {n == 28, k == 11} || {n == 58, k == 6} || {n == 120, k == 3} || {n == 181, k == 2}
Out[]= {n == 6, k == 33} || {n == 55, k == 12} || {n == 61, k == 11} || {n == 180, k == 4} || {n == 241, k == 3}
Out[]= {n == 28, k == 33} || {n == 49, k == 24} || {n == 127, k == 11} || {n == 178, k == 8} || {n == 483, k == 3}
Out[]= {n == 37, k == 48} || {n == 72, k == 33} || {n == 174, k == 16} || {n == 259, k == 11} || {n == 967, k == 3}
Out[]= {n == 13, k == 96} || {n == 160, k == 33} || {n == 166, k == 32} || {n == 523, k == 11} || {n == 1935, k == 3}
Out[]= {n == 11, k == 14} || {n == 20, k == 10} || {n == 32, k == 7} || {n == 47, k == 5} || {n == 122, k == 2}
Out[]= {n == 4, k == 28} || {n == 15, k == 20} || {n == 67, k == 7} || {n == 96, k == 5} || {n == 121, k == 4}
Out[]= {n == 5, k == 40} || {n == 11, k == 35} || {n == 119, k == 8} || {n == 137, k == 7} || {n == 194, k == 5}
Out[]= {n == 16, k == 49} || {n == 39, k == 35} || {n == 115, k == 16} || {n == 277, k == 7} || {n == 390, k == 5}
Out[]= {n == 56, k == 49} || {n == 95, k == 35} || {n == 107, k == 32} || {n == 557, k == 7} || {n == 782, k == 5}
Out[]= {n == 17, k == 22} || {n == 50, k == 11} || {n == 56, k == 10} || {n == 119, k == 5} || {n == 302, k == 2}
Out[]= {n == 6, k == 44} || {n == 51, k == 20} || {n == 105, k == 11} || {n == 240, k == 5} || {n == 301, k == 4}
Out[]= {n == 17, k == 55} || {n == 41, k == 40} || {n == 215, k == 11} || {n == 299, k == 8} || {n == 482, k == 5}
Out[]= {n == 21, k == 80} || {n == 61, k == 55} || {n == 295, k == 16} || {n == 435, k == 11} || {n == 966, k == 5}
Out[]= {n == 20, k == 121} || {n == 149, k == 55} || {n == 287, k == 32} || {n == 875, k == 11} || {n == 1934, k == 5}
Out[]= {n == 28, k == 22} || {n == 54, k == 14} || {n == 72, k == 11} || {n == 118, k == 7} || {n == 423, k == 2}
Out[]= {n == 17, k == 44} || {n == 47, k == 28} || {n == 149, k == 11} || {n == 239, k == 7} || {n == 422, k == 4}
Out[]= {n == 6, k == 77} || {n == 33, k == 56} || {n == 303, k == 11} || {n == 420, k == 8} || {n == 481, k == 7}
Out[]= {n == 5, k == 112} || {n == 50, k == 77} || {n == 416, k == 16} || {n == 611, k == 11} || {n == 965, k == 7}
Out[]= {n == 52, k == 121} || {n == 138, k == 77} || {n == 408, k == 32} || {n == 1227, k == 11} || {n == 1933, k == 7}
```

If we check the pattern 2^m*p1^2*p2^2 we have:

In[ ]:=
```
sumconsecint[3^2*5^2]
sumconsecint[2*3^2*5^2]
sumconsecint[2^2*3^2*5^2]
sumconsecint[2^3*3^2*5^2]
sumconsecint[2^4*3^2*5^2]
sumconsecint[3^2*7^2]
sumconsecint[2*3^2*7^2]
sumconsecint[2^2*3^2*7^2]
sumconsecint[2^3*3^2*7^2]
sumconsecint[2^4*3^2*7^2]
sumconsecint[3^2*11^2]
sumconsecint[2*3^2*11^2]
sumconsecint[2^2*3^2*11^2]
sumconsecint[2^3*3^2*11^2]
sumconsecint[2^4*3^2*11^2]
sumconsecint[5^2*7^2]
sumconsecint[2*5^2*7^2]
sumconsecint[2^2*5^2*7^2]
sumconsecint[2^3*5^2*7^2]
sumconsecint[2^4*5^2*7^2]
sumconsecint[5^2*11^2]
sumconsecint[2*5^2*11^2]
sumconsecint[2^2*5^2*11^2]
sumconsecint[2^3*5^2*11^2]
sumconsecint[2^4*5^2*11^2]
sumconsecint[7^2*11^2]
sumconsecint[2*7^2*11^2]
sumconsecint[2^2*7^2*11^2]
sumconsecint[2^3*7^2*11^2]
sumconsecint[2^4*7^2*11^2]
```

Out[ ]=
```
{n == 4, k == 18} || {n == 8, k == 15} || {n == 18, k == 10} || {n == 21, k == 9} ||
 {n == 35, k == 6} || {n == 43, k == 5} || {n == 74, k == 3} || {n == 112, k == 2}
```

Out[ ]=
```
{n == 6, k == 25} || {n == 13, k == 20} || {n == 23, k == 15} || {n == 32, k == 12} ||
 {n == 46, k == 9} || {n == 88, k == 5} || {n == 111, k == 4} || {n == 149, k == 3}
```

Out[ ]=
```
{n == 3, k == 40} || {n == 24, k == 25} || {n == 26, k == 24} || {n == 53, k == 15} ||
 {n == 96, k == 9} || {n == 109, k == 8} || {n == 178, k == 5} || {n == 299, k == 3}
```

Out[ ]=
```
{n == 14, k == 48} || {n == 18, k == 45} || {n == 60, k == 25} || {n == 105, k == 16} ||
 {n == 113, k == 15} || {n == 196, k == 9} || {n == 358, k == 5} || {n == 599, k == 3}
```

Out[ ]=
```
{n == 11, k == 75} || {n == 58, k == 45} || {n == 97, k == 32} || {n == 132, k == 25} ||
 {n == 233, k == 15} || {n == 396, k == 9} || {n == 718, k == 5} || {n == 1199, k == 3}
```

Out[ ]=
```
{n == 11, k == 21} || {n == 16, k == 18} || {n == 25, k == 14} || {n == 45, k == 9} ||
 {n == 60, k == 7} || {n == 71, k == 6} || {n == 146, k == 3} || {n == 220, k == 2}
```

Out[ ]=
```
{n == 7, k == 36} || {n == 18, k == 28} || {n == 32, k == 21} || {n == 68, k == 12} ||
 {n == 94, k == 9} || {n == 123, k == 7} || {n == 219, k == 4} || {n == 293, k == 3}
```

Out[ ]=
```
{n == 4, k == 56} || {n == 12, k == 49} || {n == 62, k == 24} || {n == 74, k == 21} ||
 {n == 192, k == 9} || {n == 217, k == 8} || {n == 249, k == 7} || {n == 587, k == 3}
```

Out[ ]=
```
{n == 25, k == 63} || {n == 48, k == 49} || {n == 50, k == 48} || {n == 158, k == 21} ||
 {n == 213, k == 16} || {n == 388, k == 9} || {n == 501, k == 7} || {n == 1175, k == 3}
```

Out[ ]=
```
{n == 26, k == 96} || {n == 81, k == 63} || {n == 120, k == 49} || {n == 205, k == 32} ||
 {n == 326, k == 21} || {n == 780, k == 9} || {n == 1005, k == 7} || {n == 2351, k == 3}
```

```
Out[]= {n == 17, k == 33} || {n == 39, k == 22} || {n == 52, k == 18} || {n == 94, k == 11} ||
  {n == 117, k == 9} || {n == 179, k == 6} || {n == 362, k == 3} || {n == 544, k == 2}
Out[]= {n == 28, k == 44} || {n == 43, k == 36} || {n == 50, k == 33} || {n == 176, k == 12} ||
  {n == 193, k == 11} || {n == 238, k == 9} || {n == 543, k == 4} || {n == 725, k == 3}
Out[]= {n == 6, k == 88} || {n == 25, k == 72} || {n == 116, k == 33} || {n == 170, k == 24} ||
  {n == 391, k == 11} || {n == 480, k == 9} || {n == 541, k == 8} || {n == 1451, k == 3}
Out[]= {n == 12, k == 121} || {n == 39, k == 99} || {n == 158, k == 48} || {n == 248, k == 33} ||
  {n == 537, k == 16} || {n == 787, k == 11} || {n == 964, k == 9} || {n == 2903, k == 3}
Out[]= {n == 84, k == 121} || {n == 127, k == 99} || {n == 134, k == 96} || {n == 512, k == 33} ||
  {n == 529, k == 32} || {n == 1579, k == 11} || {n == 1932, k == 9} || {n == 5807, k == 3}
Out[]= {n == 1, k == 49} || {n == 18, k == 35} || {n == 37, k == 25} || {n == 81, k == 14} ||
  {n == 118, k == 10} || {n == 172, k == 7} || {n == 243, k == 5} || {n == 612, k == 2}
Out[]= {n == 26, k == 49} || {n == 53, k == 35} || {n == 74, k == 28} || {n == 86, k == 25} ||
  {n == 113, k == 20} || {n == 347, k == 7} || {n == 488, k == 5} || {n == 611, k == 4}
Out[]= {n == 60, k == 56} || {n == 76, k == 49} || {n == 103, k == 40} || {n == 123, k == 35} ||
  {n == 184, k == 25} || {n == 609, k == 8} || {n == 697, k == 7} || {n == 978, k == 5}
Out[]= {n == 32, k == 112} || {n == 83, k == 80} || {n == 176, k == 49} || {n == 263, k == 35} ||
  {n == 380, k == 25} || {n == 605, k == 16} || {n == 1397, k == 7} || {n == 1958, k == 5}
Out[]= {n == 25, k == 175} || {n == 43, k == 160} || {n == 376, k == 49} || {n == 543, k == 35} ||
  {n == 597, k == 32} || {n == 772, k == 25} || {n == 2797, k == 7} || {n == 3918, k == 5}
Out[]= {n == 28, k == 55} || {n == 36, k == 50} || {n == 109, k == 25} || {n == 127, k == 22} ||
  {n == 270, k == 11} || {n == 298, k == 10} || {n == 603, k == 5} || {n == 1512, k == 2}
Out[]= {n == 11, k == 100} || {n == 83, k == 55} || {n == 116, k == 44} || {n == 230, k == 25} ||
  {n == 293, k == 20} || {n == 545, k == 11} || {n == 1208, k == 5} || {n == 1511, k == 4}
Out[]= {n == 40, k == 121} || {n == 94, k == 88} || {n == 193, k == 55} || {n == 283, k == 40} ||
  {n == 472, k == 25} || {n == 1095, k == 11} || {n == 1509, k == 8} || {n == 2418, k == 5}
Out[]= {n == 50, k == 176} || {n == 140, k == 121} || {n == 263, k == 80} || {n == 413, k == 55} ||
  {n == 956, k == 25} || {n == 1505, k == 16} || {n == 2195, k == 11} || {n == 4838, k == 5}
Out[]= {n == 39, k == 275} || {n == 223, k == 160} || {n == 340, k == 121} || {n == 853, k == 55} ||
  {n == 1497, k == 32} || {n == 1924, k == 25} || {n == 4395, k == 11} || {n == 9678, k == 5}
Out[]= {n == 12, k == 98} || {n == 39, k == 77} || {n == 97, k == 49} || {n == 259, k == 22} ||
  {n == 417, k == 14} || {n == 534, k == 11} || {n == 844, k == 7} || {n == 2964, k == 2}
Out[]= {n == 38, k == 121} || {n == 116, k == 77} || {n == 218, k == 49} || {n == 248, k == 44} ||
  {n == 410, k == 28} || {n == 1073, k == 11} || {n == 1691, k == 7} || {n == 2963, k == 4}
Out[]= {n == 136, k == 121} || {n == 226, k == 88} || {n == 270, k == 77} || {n == 396, k == 56} ||
  {n == 460, k == 49} || {n == 2151, k == 11} || {n == 2961, k == 8} || {n == 3385, k == 7}
Out[]= {n == 182, k == 176} || {n == 332, k == 121} || {n == 368, k == 112} || {n == 578, k == 77} ||
  {n == 944, k == 49} || {n == 2957, k == 16} || {n == 4307, k == 11} || {n == 6773, k == 7}
Out[]= {n == 94, k == 352} || {n == 312, k == 224} || {n == 724, k == 121} || {n == 1194, k == 77} ||
  {n == 1912, k == 49} || {n == 2949, k == 32} || {n == 8619, k == 11} || {n == 13549, k == 7}
```

Again we have the same number of solutions for all the members of the pattern equal 8. Now we focus on the pattern that consider only the odd primes, like p1*p2*p3 or p1*p2*p3*p4:

In[ ]:=
```
For[i=2,i<20,i++,
Print[sumconsecint[Prime[i]*Prime[i+1]*Prime[i+2]]]
]
For[i=2,i<20,i++,
Print[sumconsecint[Prime[i]*Prime[i+1]*Prime[i+2]*Prime[i+3]]]
]
```

```
{n == 1, k == 14} || {n == 6, k == 10} || {n == 12, k == 7} ||
 {n == 15, k == 6} || {n == 19, k == 5} || {n == 34, k == 3} || {n == 52, k == 2}
{n == 7, k == 22} || {n == 21, k == 14} || {n == 30, k == 11} ||
 {n == 34, k == 10} || {n == 52, k == 7} || {n == 75, k == 5} || {n == 192, k == 2}
{n == 26, k == 26} || {n == 35, k == 22} || {n == 65, k == 14} ||
 {n == 71, k == 13} || {n == 86, k == 11} || {n == 140, k == 7} || {n == 500, k == 2}
{n == 55, k == 34} || {n == 81, k == 26} || {n == 100, k == 22} ||
 {n == 135, k == 17} || {n == 181, k == 13} || {n == 216, k == 11} || {n == 1215, k == 2}
{n == 92, k == 38} || {n == 107, k == 34} || {n == 149, k == 26} ||
 {n == 212, k == 19} || {n == 239, k == 17} || {n == 317, k == 13} || {n == 2099, k == 2}
{n == 139, k == 46} || {n == 177, k == 38} || {n == 202, k == 34} ||
 {n == 312, k == 23} || {n == 382, k == 19} || {n == 429, k == 17} || {n == 3714, k == 2}
{n == 190, k == 58} || {n == 253, k == 46} || {n == 315, k == 38} ||
 {n == 423, k == 29} || {n == 540, k == 23} || {n == 658, k == 19} || {n == 6336, k == 2}
{n == 303, k == 62} || {n == 328, k == 58} || {n == 427, k == 46} ||
 {n == 652, k == 31} || {n == 699, k == 29} || {n == 888, k == 23} || {n == 10 338, k == 2}
{n == 413, k == 74} || {n == 506, k == 62} || {n == 545, k == 58} ||
 {n == 881, k == 37} || {n == 1058, k == 31} || {n == 1133, k == 29} || {n == 16 631, k == 2}
{n == 533, k == 82} || {n == 599, k == 74} || {n == 728, k == 62} ||
 {n == 1127, k == 41} || {n == 1253, k == 37} || {n == 1502, k == 31} || {n == 23 513, k == 2}
{n == 716, k == 86} || {n == 755, k == 82} || {n == 845, k == 74} ||
 {n == 1496, k == 43} || {n == 1571, k == 41} || {n == 1745, k == 37} || {n == 32 615, k == 2}
{n == 835, k == 94} || {n == 921, k == 86} || {n == 970, k == 82} ||
 {n == 1740, k == 47} || {n == 1906, k == 43} || {n == 2001, k == 41} || {n == 41 430, k == 2}
{n == 958, k == 106} || {n == 1093, k == 94} || {n == 1203, k == 86} ||
 {n == 1995, k == 53} || {n == 2256, k == 47} || {n == 2470, k == 43} || {n == 53 556, k == 2}
{n == 1187, k == 118} || {n == 1334, k == 106} || {n == 1517, k == 94} ||
 {n == 2462, k == 59} || {n == 2747, k == 53} || {n == 3104, k == 47} || {n == 73 484, k == 2}
{n == 1503, k == 122} || {n == 1558, k == 118} || {n == 1747, k == 106} ||
 {n == 3097, k == 61} || {n == 3204, k == 59} || {n == 3573, k == 53} || {n == 95 373, k == 2}
{n == 1733, k == 134} || {n == 1916, k == 122} || {n == 1985, k == 118} ||
 {n == 3566, k == 67} || {n == 3923, k == 61} || {n == 4058, k == 59} || {n == 120 566, k == 2}
{n == 1973, k == 142} || {n == 2099, k == 134} || {n == 2318, k == 122} ||
 {n == 4052, k == 71} || {n == 4298, k == 67} || {n == 4727, k == 61} || {n == 145 088, k == 2}
{n == 2306, k == 146} || {n == 2375, k == 142} || {n == 2525, k == 134} ||
 {n == 4721, k == 73} || {n == 4856, k == 71} || {n == 5150, k == 67} || {n == 173 630, k == 2}
{n == 7, k == 42} || {n == 16, k == 35} || {n == 19, k == 33} || {n == 24, k == 30} || {n == 42, k == 22} ||
 {n == 45, k == 21} || {n == 70, k == 15} || {n == 76, k == 14} || {n == 100, k == 11} || {n == 111, k == 10} ||
 {n == 162, k == 7} || {n == 190, k == 6} || {n == 229, k == 5} || {n == 384, k == 3} || {n == 577, k == 2}
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{n == 10, k == 91} || {n == 27, k == 77} || {n == 37, k == 70} || {n == 45, k == 65} || {n == 64, k == 55} ||
 {n == 126, k == 35} || {n == 180, k == 26} || {n == 217, k == 22} || {n == 351, k == 14} || {n == 379, k == 13} ||
 {n == 450, k == 11} || {n == 496, k == 10} || {n == 712, k == 7} || {n == 999, k == 5} || {n == 2502, k == 2}

{n == 3, k == 182} || {n == 34, k == 154} || {n == 48, k == 143} || {n == 84, k == 119} || {n == 142, k == 91} ||
 {n == 183, k == 77} || {n == 484, k == 34} || {n == 642, k == 26} || {n == 763, k == 22} || {n == 993, k == 17} ||
 {n == 1209, k == 14} || {n == 1303, k == 13} || {n == 1542, k == 11} || {n == 2428, k == 7} || {n == 8508, k == 2}

{n == 19, k == 286} || {n == 64, k == 247} || {n == 99, k == 221} ||
 {n == 117, k == 209} || {n == 154, k == 187} || {n == 252, k == 143} || {n == 1197, k == 38} ||
 {n == 1342, k == 34} || {n == 1764, k == 26} || {n == 2089, k == 22} || {n == 2422, k == 19} ||
 {n == 2709, k == 17} || {n == 3547, k == 13} || {n == 4194, k == 11} || {n == 23 094, k == 2}

{n == 3, k == 437} || {n == 52, k == 391} || {n == 138, k == 323} ||
 {n == 174, k == 299} || {n == 268, k == 247} || {n == 327, k == 221} || {n == 2077, k == 46} ||
 {n == 2523, k == 38} || {n == 2824, k == 34} || {n == 3702, k == 26} || {n == 4188, k == 23} ||
 {n == 5074, k == 19} || {n == 5673, k == 17} || {n == 7423, k == 13} || {n == 48 288, k == 2}

{n == 11, k == 646} || {n == 116, k == 551} || {n == 191, k == 493} ||
 {n == 275, k == 437} || {n == 356, k == 391} || {n == 506, k == 323} || {n == 3686, k == 58} ||
 {n == 4661, k == 46} || {n == 5651, k == 38} || {n == 6320, k == 34} || {n == 7415, k == 29} ||
 {n == 9356, k == 23} || {n == 11 330, k == 19} || {n == 12 665, k == 17} || {n == 107 720, k == 2}

{n == 13, k == 874} || {n == 195, k == 713} || {n == 256, k == 667} ||
 {n == 373, k == 589} || {n == 438, k == 551} || {n == 681, k == 437} || {n == 6306, k == 62} ||
 {n == 6745, k == 58} || {n == 8518, k == 46} || {n == 10 320, k == 38} || {n == 12 658, k == 31} ||
 {n == 13 533, k == 29} || {n == 17 070, k == 23} || {n == 20 668, k == 19} || {n == 196 431, k == 2}

{n == 94, k == 1147} || {n == 177, k == 1073} || {n == 402, k == 899} ||
 {n == 474, k == 851} || {n == 717, k == 713} || {n == 814, k == 667} || {n == 10 302, k == 74} ||
 {n == 12 309, k == 62} || {n == 13 162, k == 58} || {n == 16 609, k == 46} || {n == 20 659, k == 37} ||
 {n == 24 664, k == 31} || {n == 26 367, k == 29} || {n == 33 252, k == 23} || {n == 382 524, k == 2}

{n == 141, k == 1517} || {n == 438, k == 1271} || {n == 553, k == 1189} ||
 {n == 616, k == 1147} || {n == 735, k == 1073} || {n == 1068, k == 899} || {n == 16 591, k == 82} ||
 {n == 18 393, k == 74} || {n == 21 966, k == 62} || {n == 23 485, k == 58} || {n == 33 243, k == 41} ||
 {n == 36 841, k == 37} || {n == 43 978, k == 31} || {n == 47 013, k == 29} || {n == 681 891, k == 2}

{n == 266, k == 1763} || {n == 476, k == 1591} || {n == 575, k == 1517} ||
 {n == 851, k == 1333} || {n == 956, k == 1271} || {n == 1190, k == 1147} || {n == 23 471, k == 86} ||
 {n == 24 620, k == 82} || {n == 27 290, k == 74} || {n == 32 585, k == 62} || {n == 47 006, k == 43} ||
 {n == 49 301, k == 41} || {n == 54 635, k == 37} || {n == 65 216, k == 31} || {n == 1 011 080, k == 2}

{n == 507, k == 2021} || {n == 628, k == 1927} || {n == 858, k == 1763} ||
 {n == 894, k == 1739} || {n == 1132, k == 1591} || {n == 1263, k == 1517} || {n == 32 569, k == 94} ||
 {n == 35 607, k == 86} || {n == 37 348, k == 82} || {n == 41 394, k == 74} || {n == 65 208, k == 47} ||
 {n == 71 278, k == 43} || {n == 74 757, k == 41} || {n == 82 843, k == 37} || {n == 1 532 928, k == 2}

{n == 518, k == 2491} || {n == 788, k == 2279} || {n == 935, k == 2173} || {n == 1163, k == 2021} ||
 {n == 1316, k == 1927} || {n == 1610, k == 1763} || {n == 41 378, k == 106} ||
 {n == 46 673, k == 94} || {n == 51 023, k == 86} || {n == 53 516, k == 82} || {n == 82 835, k == 53} ||
 {n == 93 416, k == 47} || {n == 102 110, k == 43} || {n == 107 093, k == 41} || {n == 2 195 816, k == 2}

{n == 458, k == 3127} || {n == 893, k == 2773} || {n == 1223, k == 2537} ||
 {n == 1292, k == 2491} || {n == 1634, k == 2279} || {n == 2117, k == 2021} || {n == 53 498, k == 118} ||
 {n == 59 567, k == 106} || {n == 67 184, k == 94} || {n == 73 442, k == 86} || {n == 107 084, k == 59} ||
 {n == 119 213, k == 53} || {n == 134 438, k == 47} || {n == 146 948, k == 43} || {n == 3 159 833, k == 2}

{n == 692, k == 3599} || {n == 1157, k == 3233} || {n == 1304, k == 3127} ||
 {n == 1694, k == 2867} || {n == 1847, k == 2773} || {n == 2354, k == 2491} || {n == 73 424, k == 122} ||
 {n == 75 917, k == 118} || {n == 84 524, k == 106} || {n == 95 327, k == 94} || {n == 146 939, k == 61} ||
 {n == 151 922, k == 59} || {n == 169 127, k == 53} || {n == 190 724, k == 47} || {n == 4 482 554, k == 2}
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{n == 1084, k == 4087} || {n == 1257, k == 3953} || {n == 1752, k == 3599} || {n == 1824, k == 3551} ||
 {n == 2337, k == 3233} || {n == 2524, k == 3127} || {n == 95 307, k == 134} || {n == 104 694, k == 122} ||
 {n == 108 247, k == 118} || {n == 120 514, k == 106} || {n == 190 714, k == 67} ||
 {n == 209 479, k == 61} || {n == 216 582, k == 59} || {n == 241 107, k == 53} || {n == 6 390 024, k == 2}

{n == 1221, k == 4757} || {n == 1788, k == 4331} || {n == 1993, k == 4189} || {n == 2146, k == 4087} ||
 {n == 2355, k == 3953} || {n == 2958, k == 3599} || {n == 120 496, k == 142} || {n == 127 698, k == 134} ||
 {n == 140 271, k == 122} || {n == 145 030, k == 118} || {n == 241 098, k == 71} ||
 {n == 255 496, k == 67} || {n == 280 633, k == 61} || {n == 290 148, k == 59} || {n == 8 560 221, k == 2}

{n == 1496, k == 5183} || {n == 1886, k == 4891} || {n == 2075, k == 4757} || {n == 2531, k == 4453} ||
 {n == 2726, k == 4331} || {n == 3140, k == 4087} || {n == 145 016, k == 146} || {n == 149 105, k == 142} ||
 {n == 158 015, k == 134} || {n == 173 570, k == 122} || {n == 290 141, k == 73} ||
 {n == 298 316, k == 71} || {n == 316 130, k == 67} || {n == 347 231, k == 61} || {n == 10 591 460, k == 2}

{n == 1874, k == 5767} || {n == 2087, k == 5609} || {n == 2537, k == 5293} || {n == 2702, k == 5183} ||
 {n == 3164, k == 4891} || {n == 3389, k == 4757} || {n == 173 552, k == 158} || {n == 187 829, k == 146} ||
 {n == 193 124, k == 142} || {n == 204 662, k == 134} || {n == 347 222, k == 79} ||
 {n == 375 767, k == 73} || {n == 386 354, k == 71} || {n == 409 424, k == 67} || {n == 13 716 809, k == 2}
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Even in this case we have the same number of solutions for all the members of the pattern equal 7. Now we can guess the general rule that connect a number with the number of possible sequences that sum to that number.

Conclusion

Let a number have the prime decomposition 2^e_0*p_1^e_1*p_2^e_2*...*p_m^e_m then the total number of solution is independent from the power of 2 and equal to: (e_1+1)*(e_2+1)*...*(e_m+1)-1. So we have a correlation between a multiplicative property of numbers, its factorization, and the addictive property of consecutive sequences with equal sum.

In[∘]:=
```
seqnum[n_Integer]:=If[FactorInteger[n][[1,1]]==2,Times@@(Last/@Rest@FactorInteger@n+1)-1,Times@
seqnum[3*5*7]
seqnum[5*7*11*13]
seqnum[3^2*5^2]
```

Out[∘]= 7

Out[∘]= 15

Out[∘]= 8