A mathematical analysis of zero-dimensionality in deriving the natural numbers, offering a solution to Goldbach's conjecture and the Riemann hypothesis

Stephen H. Jarvis

ORCiD: http://orcid.org/0000-0003-3869-7694

web:www.xemdir.com

email: stephen.jarvis@xemdir.com

Abstract: Examined here is zero-dimensional space, or more simply the mathematics and physics of a hypothetical point. The infinitesimal-infinite (scaling) paradox that exists with a point as zero-dimensional space is acknowledged and rectified. The solution presented is central to proposing a singular dimensional moment of time for zero-dimensional space from an infinitesimal scale to an infinite scale. The next step is to then locate a time-point relative to another time-point, creating the non-local dimensions of time-after and time-before. From such are derived 1d, 2d, and 3d spatial dimensions for zero-dimensional references as timespace. A mathematical formalism is developed to establish the natural numbers and associated primes. By this process Goldbach's conjecture can be shown to be upheld in using 1d timespace. Similarly, the Riemann hypothesis is shown to be upheld in using a 2d timespace complex plane. From there 3d timespace is shown to exhibit prime number features relevant to physical phenomena.

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1. Introduction

Do numbers require the ideas of time and space? Do time and space require the idea of numbers? Do they require each other? Could time and space be connected with a mathematics of zero dimensionality, with the mathematics of a point?

Although physics tries to explain time and space using numbers, here time and space shall be used as fundamental concepts if not objects to *explain* numbers, to explain mathematics. Here it will be shown that numbers have a primary function and co-dependency with the fundamental ideas of time and space via a proposed mathematics of zero-dimensionality, or more simply the mathematics of a point. To achieve such, new objectivity for the number 0 is required in comparison with 1 and the idea of ϖ in accompanying the objects if not characters of time and space.

Of note in this paper is how this new time and space objectivity of numbers, fundamentally for 0, 1, and the idea of ϖ, can thence derive the set of prime numbers from 0 to ϖ, and how those primes would then relate to each other. Of particular note is how the prime number grouping can reveal a golden ratio code derived from the mathematical analysis of zero-dimensional space as *timespace*. That derivation will be demonstrated to form the basis for a time-equation for space and thence how that golden ratio time-equation in providing an absolute account of numbers from 0 to ϖ can then relate specifically to both Euler's number e and the complex number plane. In this way both Goldbach's conjecture and Riemann's hypothesis are visualized and confirmed.

In achieving such, the paper will be sectioned as follows:

1. Introduction
2. Numbers: 0, 1, and ϖ
3. Zero-dimensionality and the $\odot_0^{\varpi}$ realm
4. Zero-dimensional *positions* as *timespace*
5. Zero-dimensional *processes* as 1d *timespace*: resolving Goldbach's conjecture and twin-primes
6. Zero-dimensional *processes* as 2d *timespace*: resolving the Riemann hypothesis
7. Zero-dimensional *processes* as 3d *timespace:* Temporal Mechanics
8. Conclusion

Although Riemann presented his 2d complex number and log-scale bridge between the two sides of Euler's Zeta function equation in bringing out the prime-numbers, the absoluteness of the complex number plane and log scale was in question, hence the uncertainty around Riemann's hypothesis thence requiring constant calculation approaching infinite prime values.

Here, the proof for the absoluteness of Euler's number and Riemann's associated equations comes with understanding the absoluteness of 0 to ϖ as the mathematics of zero-dimensionality and its relationship to Euler's number e and the complex 2d plane. That proof is offered in sections 5-6 with the mathematics of 0-dimensional space deriving the underlying golden ratio (φ and $\frac{-1}{\varphi}$) code, namely how the golden ratio features of the mathematics of zero-dimensional space are highlighted in Goldbach's conjecture (section 5) and the Riemann hypothesis (section 6), thus establishing a zero-dimensional reference for a grid of primes in 1d (section 5), 2d (section 6), and thence 3d (section 7) *timespace*.

First though, the basis for numbers and associated scale, namely 0, 1, and ϖ, needs to be described for this new zero-dimensional mathematical context.

2. Numbers: 0, 1, and ∞

A number quite simply is a *mathematical object* used to count, measure, identify, and label. The natural numbers (positive integers) 1, 2, 3…, are the whole numbers used for counting. From there are the relationships between numbers most simply as fractions which are termed rational numbers. Those fractions that are not expressions of real numbers are the irrational numbers. There are positive numbers and negative numbers, generally dependent on their reference to the value 0 or even more simply as per their utility for addition and subtraction in calculations. There are also other numbers such as i ($\sqrt{-1}$) forming the basis of what are known as complex numbers.

In short, numbers quantify things, either as a basic process of labelling, or used in mechanisms of calculations in their association with each other (addition, subtraction, multiplication, division, and exponentiation).

Calculations with numbers are performed with arithmetical operations, commonly addition, subtraction, multiplication, division, and exponentiation. Here, equations are used with numbers as the vehicle to carry calculations, a vehicle that represents a formula as the equality of two expression.

With all of such, in numbers being primarily mathematical objects, they do not specify anything exclusive or particular unless requested to, namely unless applied to something exclusive or particular.

There is nonetheless a possibility that numbers (such as primes) may by their particular association with each other represent a foundation for objective phenomena, and such will be examined here by first addressing the number 0 as an ideal mathematical reference object.

As a mathematical object, the number 0 has an interesting past, only being formally employed as a mathematical object in the early Renaissance period. Traditionally 0 was considered as a type of void, an uncountable. Today 0 is considered as a reference point, namely the reference between the positive and negative numbers. 0 is also considered as a "0" result for calculations resulting in 0.

The question though this paper asks is how the mathematics of such can work for 3d space, namely how can a zero-dimensional point as zero-dimensional space relate with another zero-dimensional space *in the context of 3d space*?

The proposal in this paper is that the process of analysis and description for one zero-dimensional reference of space to relate with another zero-dimensional reference of space requires the idea of a universal reference not as space yet as that which is not space, proposed here as *time*, as a universal moment, as a unit value, 1.

Consider this as *time-now=1*, as $t_N 1$.

Here, $t_N 1$ is proposed to exist for a zero-dimensional spatial realm of any size, *infinitesimal* or *infinite*, for of course the scale of space in this context has yet to be defined. Such is not an intuitive thing to imagine, as our imagination is adjusted to thinking in terms of 1d time and 3d space regarding small and large scales. Yet the basis here is proposing that any infinite number of zero-dimensional spatial points are related to one another temporally in being a part of the one moment, the *one* realm, $t_N 1$.

The next question is how one zero-dimensional spatial point can be related to another zero-dimensional spatial point in the context of $t_N 1$. To address this the idea of $t_N 1$ as a mathematical object needs to be addressed.

The number 1 is merely a standard unit which can be applied to any type of measurement scaling system, whether with length-distance, mass-weight, or time.

Here, the number 1 is to be *primarily* used with the idea of "time", as the concept of a moment. Such is not to be confused with a length of time, namely *not* to be confused with the idea of *1-second*, yet a moment as a *primary* consideration for time and not a *secondary* consideration for time (as with addition and subtraction, as shall become apparent).

Here the value $t_N 1$ will form a key link with zero-dimensional space as an intrinsic link with the idea of zero-dimensional space, proposed as *timespace*.

Here also $t_N 1$ presents with the feature that if space as a fundamental consideration is a zero-dimensional construct, at its core, and time as a moment is proposed to represent the concept of 1, then $t_N 1$ can be applied to any concept, any number, as with multiplication and division, and still have no effect on that any number's value.

1 of course added to or subtracted from any value changes that value by an increment of 1, which is considered as a *secondary* application of the number 1. Yet here on a fundamental level of consideration, as an *axiom*, the idea of 1 is to be fundamentally considered for a process of the datum reference of *time-now* in zero-dimensional space as $t_N 1$.

Indeed, regardless of location in space, time in that space is always still existing in the moment *regardless of one's relative motion*. Such is a self-evident thing, and thus the idea of $t_N 1$ for zero-dimensional space can be considered as an *axiom* in being self-evident:

> *The temporal moment is given symbolic mathematical value as time-now=1* ($t_N 1$) *for any zero-dimensional point reference of space.*

The next question is how far can this $t_N 1$ realm extend in 0-space? Simply, how can *infinity* be defined?

Infinity (∞) as a non-numerical value has as many ideas as perhaps can be organized with numbers attempting to calculate it. Simply, infinity is a hypothetical large-scale non-numerical value which continues indefinitely.

Here, the idea of infinity (∞) is similar to that of 0 in that 0 can be infinite and still be 0, and that ∞ can be any multiple of itself and still be ∞. The question here therefore is the *scales* at play, and that is where the idea of "1" can assist in determining a relationship between 0 and ∞.

In all, the proposal here is to consider a new basic numerical *objectivity* for 0, 1, and ∞ as with the concepts of time and space as per the mathematics of zero-dimensionality.

3. Zero-dimensionality and the $\odot_0^\infty$ realm

Zero-dimensional (0d) space (or nildimensional space) is space with no dimension, simply imagined as a point. Zero-dimensional space is the idea of something without scale as a point, usually and commonly considered to be the concept of an infinitesimally sized point, despite such having no scale of size in being 0d. Simply, a 0d point could be any size as a point, as it has nothing to bear reference to as an *a priori*, as a standalone entity.

Let this problem be considered as the 0-∞ paradox, namely whether zero-dimensional space as a point is *infinitesimal* (0) or *infinite* (∞).

To resolve this issue, let us consider the infinitesimal and infinite zero-dimensional realms *as one*, here proposed as $\odot_0^\infty$, a symbol of a point surrounded by a circle in between and including the mathematical scales of 0 to ∞, *as a single overall infinite set of infinitesimal zero-dimensional points*; the proposal here is to consider a *continuum between* the *infinitesimal* zero-dimensional reference and the *infinite* zero-dimensional reference, and nothing more just yet.

Consider this proposed *model* as the $\odot_0^\infty$ model.

The obvious issue here though is the idea of a point within a point, namely a lack of precise reference and scale.

If it were therefore required to find an infinitesimal zero-dimensional reference in that infinite zero-dimensional $\odot_0^\infty$ realm, a core infinitesimal point in that infinity, how would it be done?

The proposed process involves nominating that the entire $\odot_0^\infty$ realm represents a moment in time as the datum-reference of *time-now*. In other words, the concept of "time" is being employed to explain zero-dimensional space, nominated here as a moment (not a period) of time as the value of "1" noting that time by this description cannot be as "0" yet must represent a value, nominated as the value of 1, as $t_N = 1$ $(t_N 1)$.

A hypothetical infinitesimal *time-point*, namely $t_N 1$ associated to a zero-dimensional reference, is thence proposed to exist anywhere and everywhere in the $\odot_0^\infty$ realm such that there would exist an infinite-endless number of infinitesimal $t_N 1$ time-points in this $\odot_0^\infty$ realm.

Simply, time as a $t_N 1$ time-point bearing reference to another $t_N 1$ time-point is still a moment in time in the context of an overall $t_N 1$ moment of time for the $\odot_0^\infty$ realm.

Yet, what separates these time-points if they each represent the idea of $t_N 1$?

Mathematically, "nothing", 0 exists between each time point; the "thing" between one $t_N 1$ time point to another $t_N 1$ time-point is 0-space, as much as the difference between $t_N 1$ to $t_N 1$ is 0, which is fine as a mathematical value, namely $1 - 1 = 0$.

In short, here, $t_N 1$ implies that for every infinitesimal $t_N 1$ zero-dimensional reference in the proposed $\odot_0^\infty$ realm then time is as a moment as though there is a universal moment entanglement of $t_N 1$ infinitely everywhere in the $\odot_0^\infty$ realm, and that separating the $t_N 1$ time-points is the idea of 0-space.

What is to be now proposed is that the $\odot_0^\infty$ realm is in fact *indivisible*, as much as zero-dimensional space is indivisible, the key implication therefore being that ***infinity*** as a value ***must be a prime*** in being divisible only by itself or $t_N 1$. The importance of this infinity-prime (ϖ-prime) precedent becomes apparent for sets of $t_N 1$ time-points as primes forming all the integers, as shall be demonstrated ahead.

First though, the question arises as to how time and space can develop as dimensions and not infinitesimal zero-dimensional point-analogues merely associated as integer sets of $t_N 1$ time-points. Namely, how can an infinitesimal point in time and space be *located/positioned* in reference to another infinitesimal point in time and space in the context of this entire infinite datum-reference of $t_N 1$ to create sets or dimensional extensions of $t_N 1$ time-points represented by the natural numbers?

4. Zero-dimensional *positions* as *timespace*

Here the idea of *position* enters the $\odot_0^{\varpi}$ realm, which requires bearing reference from one zero-dimensional reference to another zero-dimensional reference as an altogether new event, as a spatial *dimensional* event, namely the spatial position of a nominated zero-dimensional reference in the $\odot_0^{\varpi}$ *time-now* ($t_N 1$) realm compared to another zero-dimensional reference point.

Time here though as an infinitesimal time-point ($t_N 1$) bearing reference to another infinitesimal time-point ($t_N 1$) is still a moment in time.

Therefore, in order to generate dimensionality for space as distance, time must develop as a dimensional entity from its $t_N 1$ status in order for space to also develop as a dimensional entity. The question is how.

The proposal here is to create two new *temporal positions* as *time-before* and *time-after* in regard to *time-now* ($t_N 1$).

Why? *Time-now* must be *time-now* by definition of the general *infinitesimal* and *infinite* zero-dimensional reference realm ($\odot_0^{\varpi}$), as a universal moment, and so to create another infinitesimal *time-now* is to herald back to the reference, which can't be done as a new step, and so a new concept of a *position* of time relative to *time-now* must be created, and here is the concept of *time-after* as a new reference of time, say t_A, *time-after* being that step *beyond time-now*.

What is the position of *time-after*? The position of *time-after* is proposed to be *unknown*, as much as space is still 0-space and the reference grid scale is still indeterminant other than space being a 0-space non-dimensional point reference in the context of time-points all representing a moment.

Therefore, as a proposal thus far, $t_N = 1$ ($t_N 1$), and t_A=?.

To say though there is a *time-after* event is to imply a *time-before* event relative to $t_N 1$, and thus there must be a *time-before* event also, somehow, say as t_B.

Thus, there would be three features for time, *time-now* (t_N), *time-after* (t_A), and *time-before* (t_B).

The proposal is that *time-now* (t_N) in alliance with this potential *time-before* (t_B) results in *time-after* (t_A).

The solution proposed here is that t_B in regard to t_N requires a negative sign for t_B (equation 1) given t_B would be a "backward/negative" step in reference to t_N if indeed *time-after* is a forward step ahead of *time-now*, namely t_B as a "before" concept in regard to $t_N 1$. Thus:

$$(-t_B)\ +1(t_N1) =\ fundamental\ property\ A \tag{1}$$

Yet, if time as t_N1 is the *time-now* basis, as a $\odot_0^\infty\ t_N1$ realm basis, t_N can also be per "$-t_B$" as another equation, as technically t_B would already be positioned *within* the t_N1 reference, as it would have already happened. Thus:

$$\frac{1(t_N1)}{(-t_B)} =\ fundamental\ property\ B \tag{2}$$

Thus, if these two equations represent fundamental properties of time, and time itself is being defined as a $\odot_0^\infty\ t_N1$ realm, then <u>fundamental property A</u> must equate to <u>fundamental property B</u>:

$$(-t_B)\ +1(t_N1) = \frac{1(t_N1)}{(-t_B)} \tag{3}$$

From equation 3:

$$t_B^2 - t_B\ = 1(t_N1) \tag{4}$$

$$t_B + 1(t_N1) =\ t_B^2 \tag{5}$$

Given there are only 3 proposed concepts for time, namely t_B, t_N, and t_A, then t_B^2 must be equivalent to t_A:

$$t_B + 1(t_N1) =\ t_A \tag{6}$$

Equation 6 is the proposed time-equation, noting that the solution to equation 5 as t_B is φ and $\frac{-1}{\varphi}$, the *<u>golden ratio</u>*.

These two values (φ and $\frac{-1}{\varphi}$) as the golden ratio are now proposed to function as two distinct references for time which can thence be used to formulate *spatial dimensionality* and thus *positioning*.

To now work with these features, let us take two Pythagorean algebraic vectors for t_B, one as φ the other as $\frac{-1}{\varphi}$, giving the hypotenuse as the value of $\sqrt{3}$, arriving at equation 7:

$$\left(\frac{-1}{\varphi}\right)^2 (t_A) + \varphi^2 (t_A) \cong 3(t_N 1)$$

(7)

How this 3 value manifests as *spatial dimensionality* is proposed to be how space is incorporated with *time-now* ($t_N 1$) as a dimensional entity, namely 3d space associated to a universal time-now $t_N 1$ event, simply as $3(t_N 1)$.

Here the proposal is that $3(t_N 1)$ from equation 7 represents a 3d vector grid as the 3 dimensions of 0-space with an accompanying time component serving as the dimensional definition of a 3d spatial position in regard to $t_N 1$. This is proposed as a 3d *timespace* grid.

To note is that the $\sqrt{3}$ value can also be expressed with $t_N 1$ as Pythagorean algebraic vectors resulting in a value of 2 as the hypotenuse; here it is proposed that the 2 value represents a double $t_N 1$ as $2(t_N 1)$, meaning there are proposed to be two $t_N 1$ applications for each of the 3 dimensions of space. Of course, there are two golden ratio values, yet these two values have already been factored, thus a new concept must be considered when applying this $2(t_N 1)$ factor to 3d space from a zero-dimensional (0d) reference point.

Here regarding $2(t_N 1)$, 0-space is proposed to have 3 time-related dimensions (3d) incorporating 2 temporal outcomes for each of the 3 time-related axes; in creating a 0-space reference for each 3d time-spatial (*timespace*) vector grid, the $2(t_N 1)$ value would represent the *dual* directions on each $t_N 1$ vector axis from the 0-space point reference for 3d space.

To note is that in this process both t_B and t_A as non-localities (non-$t_N 1$) are used together according to Pythagorean algebra to set a zero-dimensional reference for 3d space, as $t_N 1$ points (zero-dimensional) in *timespace*. Although the values of the golden ratio are irrational, they are defined as being *non-local* in not being as $t_N 1$, yet together via Pythagorean algebra they form the locality for time-now ($t_N 1$) as 3d space for a zero-dimensional point reference 0.

Thus, the idea of locality for zero-dimensionality comes by the golden ratio Pythagorean relationship in the context of the $\odot_0^{\infty}$ set.

The product of golden ratio values can be considered as a “plane” (2d) value, and when added to $t_N 1$ results in 0, and thus by default a 0-dimensional reference of focus:

$$\varphi \cdot \frac{-1}{\varphi}(t_B) + 1(t_N 1) = 0(t_A)$$

(8)

This $(\varphi \cdot \frac{-1}{\varphi})$ 2d plane value is negative and thus would represent a natural complex number plane.

As a complex number plane, the work of Leonard Euler has shown that $e^{i\pi} = -1$, namely $e^{ix} = \cos x + i \sin x$ where $x = \pi$, and thus $e^{i\pi}$ also representing a complex plane of the same value of $\varphi \cdot \frac{-1}{\varphi}$.

Thus:

$$\varphi \cdot \frac{-1}{\varphi} = e^{i\pi} \tag{9}$$

Thus, equation 8 becomes:

$$e^{i\pi}(t_B) + 1(t_N 1) = 0(t_A) \tag{10}$$

The suggestion here is that the golden ratio time-equation and its two golden ratio results of t φ and $\frac{-1}{\varphi}$ represent the basis for a natural complex number 2d plane *instructed* by $e^{i\pi}$.

Thus, on the one hand the time-equation presents a natural 3d spatial grid, and on the other hand there also exists a 2d complex number plane awaiting fulfilment and description with the varying complex plane features of $e^{i\pi}$.

The next question therefore is, "how do the 1d, 2d, and 3d *timespace* grids *work*?".

5. Zero-dimensional *processes* as 1d *timespace*: resolving Goldbach's conjecture

It can be assumed that the overall $\odot_0^{\varpi}$ realm in being a prime, namely ϖ-prime, would contain all the set of prime numbers, and thence the natural number system as mathematical object values from 0 to ϖ.

For any two primes added together along a hypothetical 1d time-spatial (*timespace*) grid, the condition of those two $t_N 1$ sets (each as primes) by that addition of primes must be upheld in the general context of an ϖ-prime set (as presented in section 3), ϖ being a prime.

It would naturally follow that in the context of ϖ-prime, for the addition of two primes there would be a resultant value that must be divisible by 2 (two $t_N 1$ sets, each as primes). Hence, any prime number added to another prime number must result in an even number.

The key point to consider here is ϖ as a prime, and thus ultimately if any two scales of ϖ are added together as primes (which incorporates the set of primes), and thus any two primes added together, then that value of those two sets of $t_N 1$ each as primes must still be divisible by 2 and thus together as a value must represent an even number:

$$\infty_{prime} + \infty_{prime} = 2\infty\ (even) \tag{11}$$

Even though 2∞ is beyond ∞, the only allowable condition for such thence must be *within* the $\odot_0^{\infty}$ *set of natural numbers*, thence equation 11 where x can equal any value, including ∞:

$$x\infty = \infty \tag{12}$$

This issue is underwritten by the $\odot_0^{\infty}$ realm, as presented in section 3.

In this way it is now possible to visualize a pattern of twin-primes, namely primes separated by the value of 2. For indeed, does $\infty + 2 = prime$? According to the logic of equation 12 then $\infty + 2 = \infty$ must also be upheld for any value of x.

In short, the limit for infinity is set at ∞ by definition, and thus technically $\infty + x = \infty$ must be upheld where x is any number including ∞, noting $2\infty = \infty$ (eq.12).

As much as zero (0) is proposed to be a unique number concept with attributes ***unique for*** addition and multiplication, infinity (∞) is also proposed to be a unique number concept with attributes ***common to both*** addition and multiplication.

As such, mathematical operators are proposed to only work from 0 to ∞ where at any value for ∞ mathematical operators are proposed to become irrelevant in reaching the ∞-prime level (∞ as a prime). Simply, as much as ∞ is unbound, the mathematical operators there are proposed to serve no purpose other than ∞ being (by the proposed *a priori*) a prime number holding an associated infinite set of primes including all other numbers making up the natural numbers.

It can thus be deduced that there would be an infinite number of twin-primes ***in*** an infinite set of primes if ∞ is a prime.

To be noted here therefore is that:

- any two primes added together in that context of equation 11 must result in an even number, thus resolving Goldbach's conjecture [1] which states that any two primes added together result in an even number.
- Goldbach's conjecture would be limited by ∞-prime given equation 12.

Essentially, here a 1d mathematical grid is formed from the mathematics of zero-dimensionality for all primes and even numbers thence relating all the natural numbers by default.

The question now is whether the location of prime numbers as sets of $t_N 1$ can be calculated.

The proposal is that the location of primes on a hypothetical number line can be calculated in using Euler's equations [2] and Gauss' conjecture [3][4], particularly in using the 2d complex number plane and associated log-scale for primes, as proposed by Bernhard Riemann, as per the Riemann hypothesis [5].

6. Zero-dimensional *processes* as 2d *timespace*: resolving the Riemann hypothesis

Euler's number e is a tool of number relationships in describing how numbers relate in functions:

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \cdots . \cong 2.718282 \tag{13}$$

e forms the basis of Euler's other key achievement, namely $e^{i\pi} + 1 = 0$ where $i^2 = -1$. This was derived via his limiting function:

$$e^z = \lim_{n \to \infty} \left(1 + \frac{z}{n}\right)^n \tag{14}$$

$e^{i\pi} = -1$ is of significance to zero-dimensional mathematics, as per section 4 equation 10, namely $e^{i\pi}(t_B) + 1(t_N 1) = 0(t_A)$. There the golden ratio 2d plane equates to -1, mathematically as a complex grid expressed as $e^{i\pi} = -1$.

Euler also defined number relationships with functions as associated to a complex number plane, as per $e^{i\pi} = -1$.

The next achievement of Euler is the Euler-Riemann zeta function [6]:

$$\zeta(s) = \sum_{n-1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots \tag{15}$$

Equation 15 applies for $Re(s) > 1$ as a mathematical function of a complex variable $s = \sigma + it$.

When $Re(s) > 1$ the function can be written where $\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx$ as follows:

$$\zeta(s) = \sum_{n-1}^{\infty} \frac{1}{n^s} = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1} \, dx \tag{16}$$

Euler then made the breakthrough step with his product formula in making a connection between the zeta function and prime numbers:

$$\sum_{n-1}^{\infty} \frac{1}{n^s} = \prod_{p\ prime} \frac{1}{1-p^{-s}}$$

(17)

Here, $\zeta(s)$ as the left hand side of equation 17 equates to the infinite product as the right hand side of the equation for all prime numbers p termed as $Euler\ products$.

Here, both sides of the Euler product converge for $Re(s) > 1$.

When $Re(s) = 1$ the harmonic series diverges for infinitely many primes:

$$\prod_{p\ prime} \frac{p}{p-1}$$

(18)

The question then became one of how to predict the location of prime numbers from 0 to ϖ, and thus *the infinite number of primes in the overall* ϖ*-prime set*.

Thus, equation 17 is a way of relating the right hand side of the equation as the zeta function $\zeta(s)$ with an infinite series of all the prime numbers from 0 to ϖ. The issue though is finding where the primes appear as a progression from 0 to ϖ.

One application is the Gauss prime-counting function [3][4], namely the function that counts the number of primes less than or equal to a real number x, denoted by $\pi(x)$:

$$\pi(x) \cong \frac{x}{\log{(x)}}$$

(19)

This estimate (≅) is held with the limiting function:

$$\lim_{n\to\infty} \frac{\pi(x)}{x/\log{(x)}} = 1$$

(20)

Riemann thus applied the prime counting function as a log scale to a <u>*complex number plane*</u> in attempting to extract the position of primes on right hand side of equation 17 to a complex number grid for $\zeta(s)$ to thence find a correlation with the estimate (≅) for $\zeta(s)$, and thus to event a more exact distribution locale of the primes on a number grid:

$$\pi(x) = \mathrm{Li}(x) + O\left(\sqrt{x}\ \log x\right)$$

(21)

In parallel with that proposal, Riemann with his *analytic continuation* method for the zeta function showed that from the right hand side of equation 27 ($\prod_{p\ prime} \frac{1}{1-p^{-s}}$) the complex case can hold:

$$\left(1 - \frac{1}{2^{s-1}}\right) \zeta(s) = \sum_{n-1}^{\infty} \frac{(-1)^{n+1}}{n^s} = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \cdots$$

(22)

Equation 22 is shown to converge where s has a positive real part. This is associated with equation 23 where the condition exists for a non-positive real part:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-\mathrm{s}) \zeta(1-s)$$

(23)

There, $\zeta(s) = 0$ is shown for negative even integers as $\sin(\pi s/2) = 0$ for those values.

Riemann showed that $\zeta(s) = 0$ for s being equivalent to the negative even integers $(-2, -4, -6 \ldots.)$, known as the ***trivial zeros***.

The $\zeta(s) = 0$ result for all other values of s were considered as the ***non-trivial zeros***.

The interesting feature about the non-trivial zeros is that they lay on the complex numbers with their real part value of $s = \frac{1}{2}$.

What Riemann then proposed was that all these non-trivial zeros for $s = \frac{1}{2}$ represent prime numbers, as was the intention in using a complex plane with $\pi(\mathrm{x}) \cong \frac{\mathrm{x}}{\log(\mathrm{x})}$ (eq.17) as per his version of that equation as $\pi(x) = \mathrm{Li}(x) + O\left(\sqrt{x}\ \log x\right)$.

Although equation 17 as $\pi(x) \cong \frac{x}{\log(x)}$ is an estimation ($\cong$), the specificity of using a complex number approach to equation 21 as $\pi(x) = \mathrm{Li}(x) + O\left(\sqrt{x}\ \log x\right)$ is granted given that this approach is directly related to the specificity and ω-prime context of equations 8 and 10, namely $\varphi \cdot \frac{-1}{\varphi}(t_B) + 1(t_N 1) = 0(t_A)$ and $e^{i\pi}(t_B) + 1(t_N 1) = 0(t_A)$ respectively.

Such can now be presented in this Riemann hypothesis context.

The proposal *for the golden ratio result* of this Riemann hypothesis is in considering that the Riemann hypothesis should have analogous results for the golden ratio aspects of the mathematics of zero-dimensionality as what the Riemann hypothesis presents as zero results (both trivial and non-trivial).

Here, zero-dimensionality mathematics and the resultant golden ratio equation (time-equation) presents 8 basic conditions for the golden ratio analogue of the Riemann equations and associated hypothesis:

(i) Each golden ratio value φ and $\frac{-1}{\varphi}$ pertains to a prime number position.

(ii) There is the real number golden ratio position: $\varphi + \frac{-1}{\varphi} = 1$.

(iii) There is the complex plane golden ratio position: $\varphi \cdot \frac{-1}{\varphi} = -1$.

(iv) There is the zero-dimensional prime number reference as the addition of conditions (ii)-(iii): $1 + (-1) = 0$.

(v) If each golden ratio value pertains to a prime number position as per condition (i) then the addition of these primes (including the value 1) results in a positive even integer as per equations 11-12.

(vi) The overall context of the golden ratio values as per equation 8 requires condition (iii) as a complex plane 2d factor, and thus condition (v) is factored with -1, thus resulting in a negative even integer value.

(vii) The overall ϖ-prime condition must be considered as per equation 8, and thus when applied to condition (iii) the individual prime result there requires $\sqrt{\varphi \cdot \frac{-1}{\varphi}} = i = e^{\frac{i\pi}{2}}$

(viii) An overall limiting feature of $0 \to \varpi$ is required to position a prime zero-dimensional (zero result) result given the $\odot_0^{\varpi}$ model/set condition being a prime value (ϖ-prime).

These conditions and associated equations form the basis for how the complex plane of equation 9 as $\varphi \cdot \frac{-1}{\varphi} = e^{i\pi}$ comes into effect. Simply, the mathematical deduction here is that the Riemann equation results should be explainable by conditions (i)-(vii). Or rather, the features of the mathematics of zero-dimensionality should be apparent for the Riemann zero results (both trivial and non-trivial).

What therefore can be extrapolated from the Riemann equations and associated hypothesis regarding the trivial and non-trivial zero results in regard to conditions (i)-(viii)?

Fundamentally, the zero results of the Riemann hypothesis pertain to the zero-dimensional reference of the golden ratio equations, as the same basis of numbers is being used for 0 between the Riemann equations and the mathematics of zero-dimensionality.

With those zero results, the additive feature of condition (ii) begets condition (vi), thence ordaining a list of negative even integers (as s) for a zero result, as what Riemann considers as the trivial zeroes. The implication though here is that the addition of two primes (including the value $t_N 1$) as implicit in condition (ii) results in an even number, and thence according to condition (iii) a negative even integer as prescribed by condition (vi), namely in regard to the Riemann hypothesis as the value s.

Essentially, the mathematics of zero-dimensionality shows that Goldbach's conjecture is a feature of the Riemann hypothesis, a feature that has yet to be noted by contemporary mathematics, despite the negative even integer feature for s being self-evident, namely $\zeta(s) = 0$ being shown for negative even integers as $\sin(\pi s/2) = 0$ for those values.

To note is that condition (vii) presents the case that $e^{i\pi}$ is raised to the power of ½, and thus an exponentiation value of ½ where s is that exponentiation factor in its portrayal in the Riemann equations. Therefore, there would exist another set of zero results for $s = \frac{1}{2}$ that represent the intended designed capture of the positioning of the primes *and only primes*.

The significance for $\zeta(s) = 0$ for the values where $s = \frac{1}{2}$ (the real part of the function) represents the idea that a prime location (0d) must be a square root (√) value of $\varphi \cdot \frac{-1}{\varphi}$ as per condition (vii), namely the real-part exponentiation of ½ as per the use of s for equations 15-17, and thus the use of a 0d locale for the golden ratio values of equation 5, namely condition (vii) where $s = \frac{1}{2}$.

Importantly, condition (viii) specifies that for absolute specificity of the Riemann $s = \frac{1}{2}$ zero result the plots need to embark to an infinite value, hence the issue with Riemann's hypothesis and the requirement to refine the resonance of the $s = \frac{1}{2}$ prime values in approaching ϖ.

Thus, both sides of equation 17 can be considered as absolute functions given the values of the golden ratio (φ and $\frac{-1}{\varphi}$) are derived from an absolute basis of zero-dimensionality *in the context of an infinite scale* as ϖ-prime.

In short, the question for Riemann was finding the absolute basis of the complex number plane and associated use of a logarithm for equations 20 and 21, namely the absolute basis of the Riemann hypothesis process of complex numbers and log scale for the Gauss counting function, such in understanding the approximation (≅) of the Gauss function as per equation 19. This approximation though is resolved in using the stricter Riemann mathematics (in highlighting the primes of the right hand side of equation 17) of the Riemann equations, however his functions required the calculations of primes approaching ϖ, and thus the requirement for a description of ϖ as a *prime* in the context of the $\zeta(s) = 0$ results for $s = \frac{1}{2}$.

In summary, with the Euler-Riemann zeta function:

- Euler's primes were embedded in fractions as a product driven log scale, namely the right hand side of equation 17.
- Those fractions then became related to one another by Riemann using complex analysis, namely in using complex numbers and the prime number counting function logarithm scale with Euler's zeta function, allowing s to be a complex function.
- This logically created plots for the complex plane relevant to the distribution of primes.
- There, when negative even integers are inputted for s they give a $\zeta(s) = 0$ result (as the zeta zeros) considered as the *trivial zeros*.

- The other zeros are considered as the *non-trivial zeros* where $s = \frac{1}{2}$, and it is there where the proposal/conjecture is that all these non-trivial zeros at $s = \frac{1}{2}$ represent the distribution of primes as per the signature use of equations 19-23.
- Riemann then needed to prove that every critical non-trivial zero lies on the $s = \frac{1}{2}$ line, the issue there being that either an endless and exhaustive computer-driven computational process generating primes to ϖ is required to resolve the Riemann hypothesis, or an accompanying mathematical proof is required, namely an associated equation backdrop to show the infinite prime number value scope of the Riemann equations.
- Thus, only a *rigorous fundamental proof* for the Riemann hypothesis is considered to uphold the absoluteness of the link between the two sides of equation 17 in using complex numbers and log scales, namely in using the Riemann equations, or an accompanying mathematical proof for the prime values approaching if not including ϖ is required.

Essentially, Riemann 's hypothesis adapts a complex number plane to a log scale to map the proposal by Euler for the connection between the zeta function (left side of equation 17) and that of the product of the primes in that function (right side of equation 17). Such was a logical thing for Riemann to consider, namely using a complex analytical system and then applying a logarithm (as per the prime number counting function) to pattern the primes of the right side of equation 17.

Here, the mathematics of zero-dimensionality finds the Riemann hypothesis self-evident as a theorem, as from the basis of equations 8 and 10 and that context of the proposed $\odot_0^{\varpi}$ realm. The associated proof and utility of that mathematics and associated $\odot_0^{\varpi}$ realm is found with the accurate derivation of physical phenomena for 3d *timespace*. Indeed, if the self-evident nature of the 2d Riemann hypothesis is not sufficient in the context of equations 8-10 and associated and conditions (i)-(viii), the task is to understand how primes are linked with one another on the 3d *timespace* grid.

7. Zero-dimensional *processes* as 3d *timespace:* Temporal Mechanics

The previous sections form the *basis* of Temporal Mechanics [7], specifically papers 1-2 [8][9] and papers 42-43 [10][11] as the description of 3d *timespace*.

Paper 1 [8] derived the time-equation (derived here as equations 1-6) as a mechanism of temporal *logic*. There, the time-equation was applied to the basic equations of Newtonian gravity and Coulomb's charge force to determine any consistencies. Following such, the time-equation was applied to the electron shell structure of a basic atom, using the golden ratio features of the time-equation to derive the electron shell structure central to the electron number value for each derived shell.

Temporal Mechanics then moved to developing how the time-equation could construct 3d *timespace* as a *temporal wave function* analogous to an EM wave function, as per paper 2 [9]. There, from paper 2 page 6 such an analogue derived an approximation for the fine structure constant ([9]: p15), a value subsequently refined in paper 39 ([12]: p46-52), thence derived more fundamentally in paper 41 ([13]: p33). Also in paper 2 ([9]: p15-16) is derived the value of c in using the known scales of the Bohr radius (a^0) and electron charge (e_c).

From paper 2 [9] the process was one of taking those findings and comparing them to known features of physics theory, to thence develop a core underlying time-equation basis for the particles, field forces, and their associated phenomenal activities, deriving all the essential equations and constants using the time-equation basis.

Through this entire process, *timespace* became integral to the physical constants and their associated dynamic equations of force and location. This *integration* process utilized the concept of "1" for *time-now* as $t_N 1$ as a factor that can apply to any mathematical object phenomenon in time, namely *time-before* or *time-after*, as though there is that intrinsic loop of *time-now* to any potential event that has happened (*time-before*) or will happen (*time-after*). Paper 40 ([14]: p9-19) explained this process in comparison to the Lagrangian process for time, and those solutions for special and general relativity.

To note therefore with the *timespace* model is that in this process both t_B and t_A as non-localities are used together according to Pythagorean algebra in setting a 0-reference (0d) locality for 3d space, thence determining a locality for the time-domain of $t_N 1$, as 0d points in *timespace*. This was considered to form the basis of approaching and resolving Bell's theorem in paper 29 [15].

What therefore of the *scaling* of these points in *timespace*? Are there numbers with particular features in reference to the overall $\odot_0^\infty$ set of points that are derived to determine patterns in physical phenomena?

As proposed, the $\odot_0^\infty$ realm is *indivisible*, as much as 0d space is indivisible by definition, thus making the mathematical value for infinity ***as a prime*** (∞-prime) in being divisible only by itself or $t_N 1$. The implication of this is that there would exist a particular association of prime numbers with each other and thence with *timespace*, *demonstrating prime number patterns in timespace* as scales associated to the temporal wave function, particles, and associated field force effects.

Recent research has indeed found such a phenomenon of primes in being statistically correlated with crystallographic Bragg-like peaks of atoms [16]. Temporal Mechanics proposes that such is a fundamental structuring quality of 3d space regarding the $\odot_0^\infty$ realm.

Furthermore, Temporal Mechanics proposes that this "prime number" *timespace* phenomenon is primarily associated to the EM_X^{DIR} field, the proposed $0\ K$ (zero point energy) field [11], and how this field interacts with EM and mass/gravity thence *timespace* shaping/arranging such phenomena [16].

Also of note through the Temporal Mechanics derivation process is the proposed prime-based space-factor S_0 which is facilitated in deriving the mass of the lightest particle pairs (neutrino and antineutrino) from the Planck length lP. The prime feature of S_0 represents the addition of the first three primes (cubed), and then divided by 3, namely equations 1-2 from paper 35 ([17]: p27-28, eq1-2):

$$S_0 = \frac{2^3 + 3^3 + 5^3}{3} = 53.\dot{3}$$

([17]: p27, eq.1)

$$\frac{lP}{S_0} = 3.03048 \cdot 10^{-37}\ kg$$

([17]: p28, eq.2)

It thus seems there is an ***exclusivity*** with the prime numbers regarding 3d *timespace*, namely how the prime numbers are represented by physical phenomena in time and space for any 0-dimensional reference for 3d space.

To further such, given the primes 2, 3, and 5 are annexed in an algorithm for space as the equation $S_0 = \frac{2^3+3^3+5^3}{3} = 53.\dot{3}$ ([17]: p27, eq.1) and its relationship to elementary mass on the Planck scale $\frac{lP}{S_0} = 3.03048 \cdot 10^{-37}\ kg$ ([17]: p28, eq.2), the proposal is that every prime number over 5 (namely 7 onwards) would be the result of the addition of any 3 of all the primes:

$1 + 1 = 2$ (at fault in requiring 1)

$1 + 1 + 1 = 3$ (at fault in requiring 1)

$1 + 2 + 2 = 5$ (at fault in requiring 1)

$2 + 2 + 3 = 7$

$3 + 3 + 5 = 11$

$3 + 3 + 7 = 13$

$3 + 3 + 13 = 19$

….

The implication here is the uniqueness of the first three primes arbitrating by default the phenomenal consequence of $S_0 = \frac{2^3+3^3+5^3}{3} = 53.\dot{3}$ regarding the most fundamental feature of an elementary particle's mass (and thus gravity), the Planck length (and thus EM), *and space*, which is an entirely logical thing to consider, namely the relationship of primes guiding ultimately the fundamental relationship of physical phenomena (gravity and EM) *in space*.

The other implication here is that every prime over 5 would be the result of the addition of any 3 primes, thence leading to an infinite number of primes. This then proposes by default that any ϖ-prime would logically be the result of three primes, noting that "3" here is the proposed basis reference for ϖ as the addition of 3 prime set values of $t_N 1$, as per the derivation of equation 7.

Here therefore are key instances of prime numbers being essential to the numerical phenomenal feature of space and how such relates to both EM and mass/gravity as per the proposed EM_X^{DIR} field [42], as based on the premise of the $\odot_0^{\varpi}$ realm being indivisible *and thus a prime* (ϖ-prime).

In furthering this proposal of primes, given that space is derived to be 3d (eq.7) in being intrinsic to the $\odot_0^{\varpi}$ realm being a ϖ-prime value, here the proposal is that the addition of any three primes (each as sets of $t_N 1$) will lead to another prime $t_N 1$ set value *or a number that is the product of primes*. For instance:

$2+3+3=8$ (divisible by 2, 2, and 2)

$3+3+5=11$ (a prime)

$5+7+7=19$ (a prime)

$17+19+23=59$ (a prime)

$19+23+27=69$ (divisible by 3 and 23, primes)

$23+27+31=81$ (divisible by 3 and 27, primes)

$27+31+37=95$ (divisible by 5 and 19, primes)

$31+37+41=109$ (a prime)

$257+277+293=827$ (a prime)

$983+991+997=2971$ (a prime)

....

The conjecture presented here is that *the 3d timespace distribution of primes can only be considered in an infinite set which is also a prime* while also in taking in consideration the zero-dimensional position factor of "3", namely $\left(\frac{-1}{\varphi}\right)^2(t_A)+\ \varphi^2(t_A)\cong 3(t_N 1)$. In other words, presented here is a case of deriving how primes can be patterned from 0 to ϖ in a 3d *timespace* context. Of course such is a basic proposal for primes, as such is already known in prime number theory. The point here though is to introduce the new objective definitions for zero-dimensionality to prime number theory for this 3d realm.

In short, the Riemann hypothesis and those associated mathematical functions are a part of the proposed mathematics of 2d *timespace*. The 3d *timespace* work and associated mathematics is the core of Temporal Mechanics through its 43 papers thus far [7] in accounting for all the basics of physical phenomena, namely particle phenomena and associated field forces, confirmed in matching with all the available data and associated constants in physics. Although Euler's mathematics and associated complex number planes were essential for Einstein's formulation of special and general relativity, Temporal Mechanics has found that the key underlying issue with special and general relativity and thence contemporary physics in not accounting for the mathematics of zero-dimensional space, leading to improper understanding of time in regard to space, thence leading to anomalies in cosmology theory as per the ΛCDM model ([11]: p1-4).

8. Conclusion

Here has been presented a case for the natural numbers and their associated primes to be a part of an overall $\odot_0^{\varpi}$ set of natural numbers where ϖ is proposed to be a prime value (ϖ-prime) containing all the natural numbers. Such was possible by considering an ultimate prime-indivisible set that itself represented an overall *time-now=1* ($t_N 1$) condition for zero-dimensional space, the problem there being the scale and thus potential references in that zero-dimensional space, considered as the 0-ϖ paradox.

In resolving this 0-ϖ paradox, the localities of *time-after* and *time-before* in reference to *time-now* within that proposed zero-infinity ($\odot_0^{\varpi}$) set were proposed, thence deriving a time-equation. From that time-equation became the two results of the golden ratio, φ and $\frac{-1}{\varphi}$. This golden ratio was then applied to Pythagorean algebraic space to derive 3d space in the context of this universal *time-now=1* ($t_N 1$) and $\odot_0^{\varpi}$ (infinite-prime) proposal. Together with 3d space was derived the 2d complex number plane and 1d line. From there, it was possible to establish how primes can be identified in the ϖ-prime set and their proposed relationship to one another in that set as a 1d, complex 2d, and 3d relationship. From such, it was then proposed that the Reimann hypothesis must be upheld in resolving the 2d complex realm of primes as a valid realm to ϖ for the non-trivial results of 0 for values of $s = \frac{1}{2}$ as an infinite-prime set ($\odot_0^{\varpi}$), as a set that is itself a prime. Accompanying such was a solution for Goldbach's conjecture and the twin-prime problem.

Ultimately, it is shown here how numbers are best objectified on a *timespace* grid, namely on 1d, 2d, and 3d as shown here from the mathematics of zero-dimensionality, and that such a grid can be demonstrated as *being compatible with physical phenomena* in its various dimensional aspects. The significance therefore of deriving a solution for Goldbach's conjecture and the Riemann hypothesis from the proposed mathematics of zero-dimensionality is proposed to open to a new avenue of research and discovery.

Conflicts of Interest

The author declares no conflicts of interest; this has been an entirely self-funded independent project.

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