Addition Tensor and Goldbach's Conjecture

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Abstract In this paper, an addition tensor, or A-tensor is going to be presented. This is done by analogy to the recently introduced multiplication tensor or M-tensor. By comparing sub-tensors of A-tensor and M-tensor it is going to be proved that (strong) Goldbach's conjecture can not hold.

1 Introduction

In elementary mathematics, a number line is a picture of the graduated straight line that serves as an abstraction to real numbers. Idea of number line was first introduced by John Napier [1], and later, John Wallis [2] used this graphical representation to explain operations of addition and subtraction in terms of moving backward and forward under the metaphor of a person walking. However, that type of graphical interpretation is not particularly suitable in other contexts of interest. In order to obtain another useful representation of natural numbers, a multiplication tensor or $\mathbf{M}_N$-tensor has been recently introduced [3]. Idea came from the fundamental theorem of arithmetic [4].

In this paper, an addition tensor or $\mathbf{A}_N$-tensor is going to be introduced. The addition tensor is going to be created from $\mathbf{M}_N$-tensor by using analogy and replacing the operation of multiplication with addition. In order to show usefulness of this presentation of natural numbers it is going to be shown, in an elementary way, that famous (strong) Goldbach's conjecture [5] can not hold.

2 Multiplication tensor

The fundamental theorem of arithmetic states that every integer greater than 1 can be uniquely represented by a product of powers of prime numbers, up to the order of the factors [4]. Having that in mind, an infinite dimensional tensor $\mathbf{M}_{\mathbf{N}}$ that contains all natural numbers only once, is going to be constructed [3]. In order to do that we are going to mark vector that contains all prime numbers with **p**. So, $p(1) = 2$, $p(2) = 3$, $p(3) = 5$, and so on. Tensor $\mathbf{M}_{\mathbf{N}}$ with elements $m_{i1\ i2\ i3\ \ldots}$ is defined by the following equation (i_1, i_2, i_3, … are natural numbers):

$$m_{i_1 i_2 i_3 \ldots} = p(1)^{i_1 - 1}\, p(2)^{i_2 - 1}\, p(3)^{i_3 - 1} \ldots \quad .$$

The alternative definition is also possible. Now, the following notation is going to be assumed for some infinite size vectors

$$\mathbf{2} = [2^0\ 2^1\ 2^2\ 2^3\ \ldots],\ \mathbf{3} = [3^0\ 3^1\ 3^2\ 3^3\ \ldots],\ \ \mathbf{5} = [5^0\ 5^1\ 5^2\ 5^3\ \ldots]\ \ldots$$

It is simple to be seen that every vector is marked by bold number that is equal to some prime number and that components of the vector are defined as powers of that prime number, including power zero (it can be seen that every vector represents infinite cyclic semi group defined by a primitive that is one of the prime numbers). Now, the $\mathbf{M}_{\mathbf{N}}$-tensor can be defined as

$$\mathbf{M}_{\mathbf{N}} = \mathbf{2} \circ \mathbf{3} \circ \mathbf{5} \circ \mathbf{7} \circ \ldots,$$

where ○ stands for outer product.

The tensor $\mathbf{M}_{\mathbf{N}}$ is of infinite dimension (equal to number of prime numbers) and size, and contains all natural numbers exactly ones. It is easy to understand why it is so, having in mind the fundamental theorem of arithmetic. This type of infinite tensor is called a half infinite tensor [3].

The tensor that represents all odd numbers, $\mathbf{M}_{\mathbf{NO}}$, contains elements defined as

$$m_{i_1 i_2 \ldots} = p(2)^{i_1 - 1} p(3)^{i_2 - 1} \ldots \quad ,$$

or

$$\mathbf{M}_{\mathbf{NO}} = \mathbf{3} \circ \mathbf{5} \circ \mathbf{7} \circ \ldots,$$

where ○ stands for outer product.

3 Introduction of $\mathbf{A_N}$-tensor

Now, the tensor $\mathbf{A_N}$ is going to be defined. The tensor $\mathbf{A_N}$ with elements $\boldsymbol{a}_{i1\ i2\ i3\ \ldots}$ is defined by the following equation (i_1, i_2, i_3, … are natural numbers):

$$a_{i_1 i_2 i_3 \ldots} = (i_1 - 1)\, p(1) + (i_2 - 1)\, p(2) + (i_3 - 1)\, p(3) + \ldots \quad .$$

The edges of that tensor will contain the following vectors

$$\mathbf{2a} = [0\ 2\ 4\ 6\ \ldots],\ \mathbf{3a} = [0\ 3\ 6\ 9\ \ldots],\ \mathbf{5a} = [0\ 5\ 10\ 15\ \ldots]\ \ldots$$

It is simple to be seen that every vector is defined by some prime number and that components of the vector represent all non-negative integer multiples of that prime number. Now, the $\mathbf{A_N}$-tensor can be defined as

$$\mathbf{A_N} = \mathbf{2a} \circ^{+} \mathbf{3a} \circ^{+} \mathbf{5a} \circ^{+} \mathbf{7a} \circ^{+} \ldots,$$

where $\circ^{+}$ stands for outer sum, which is analogous to outer product where operation of interest is addition..

It is interesting to notice that the tensor $\mathbf{M_N}$ does not contain number 0 that is neutral element for addition, while, on the other hand, the tensor $\mathbf{A_N}$ does not contain number 1 that is neutral number for multiplication.

Here we will present an additional addition tensor $\mathbf{A}_{\mathbf{NO}}$ that is created by odd prime numbers, where elements of that tensor are defined as

$$a_{i_1 i_2 i_3 \ldots} = (i_1 - 1)\, p(2) + (i_2 - 1)\, p(3) + (i_3 - 1)\, p(4) + \ldots \quad .$$

or

$$\mathbf{A}_{\mathbf{NO}} = \mathbf{3a} \circ^{+} \mathbf{5a} \circ^{+} \mathbf{7a} \circ^{+} \ldots,$$

where $\circ^{+}$ stands for outer sum.

4 A proof that (strong) Goldbach's conjecture cannot hold

Goldbach's conjecture (strong version) states that every even natural number bigger than 4 can be expressed as a sum of two odd prime numbers [5]. Here we will show in an elementary way that this conjecture cannot hold. In order to do that we will analyze tensors $\mathbf{A}_{\mathbf{NO}}$ and $\mathbf{M}_{\mathbf{NO}}$. More precisely speaking, sub-tensors $\mathbf{A}_{\mathbf{NO}}$(1:2, 1:2, 1:2, …**)** and $\mathbf{M}_{\mathbf{NO}}$(1:2, 1:2, 1:2, …**)** are going to be compared. Those subtensors contain all sums and all products that are produced by two different odd prime numbers, respectively. They also contain some additional elements that are composed by three different primes, but this does not affect the final conclusion. The additional numbers that are composed of two prime numbers are on the positions (3 1 1 1 …) (1 3 1 1 1…), (1 1 3 1 1 1…) and so on, and their number is equal in both tensors - in the case of M-tensor those numbers represent squares of odd primes and in the case of A-tensor they represent doubles of odd primes. Since we know that tensor $\mathbf{M}_{\mathbf{NO}}$ contains all odd numbers exactly once, we know that the number of numbers in the tensor is equal to the number of even numbers. Since the subtensor $\mathbf{M}_{\mathbf{NO}}$(1:2, 1:2, 1:2, …**)** together with additional elements that represent squares of odd prime numbers obvioulsy contains smaller number of numbers than the tensor $\mathbf{M}_{NO}$, we can clearly see that the number of sums created by two odd prime numbers is smaller than the number of even numbers, which means that (strong) Goldbach's conjecture can not hold. That completes the proof.

References

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