Does the spacetime have its own energy density?

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Can simple mathematical operation change the physical interpretation of Einstein's equation.

1. Introduction

Until the early twentieth century the space and time were considered separate beings. In 1907 H. Minkowski connected together the three-dimensional space and one-dimensional time into single idea, creating a new Minkowski's four-dimensional spacetime [1].

The Minkowski a differentiable manifold (M, η), equipped with the Minkowski tensor η, was succeed in the *Special Relativity* (SR). In *General Relativity* (GR), the spacetime is modelled by a four-dimensional Lorenz manifold *(M, g)* satisfying the Einstein equation, which relates the curvature of the Lorenz metric g to non-gravitational fields.

Is so defined the spacetime have its own energy density? The following simple mathematical operation give the Einstein equation a completely new physical interpretation.

1. The Einstein–Hilbert action

We start from the Einstein–Hilbert action:

$$S = \frac{c^4}{16\pi G}\int (R - 2\Lambda + \mathcal{L}_m)\sqrt{g}\, d^4x \tag{1}$$

and we get:

2. The Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} \tag{2}$$

where:

$R_{\mu\nu}$ is the Ricci tensor of the spacetime,
R is the Ricci scalar of the spacetime,
c is the speed of light,

$g = det(g_{\mu\nu})$ is the determinant of the metric tensor matrix,
Λ is the cosmological constant,
$\mathcal{L}_m$ describing any matter fields,
$T_{\mu\nu}$ is the energy – momentum tensor.

3. The Einstein equation – a new point of view

Let's multiply both sides of the equation (2) by the term $\frac{c^4}{G}$. This expression is called Planck's force and equation (2) takes the form:

$$\frac{c^4}{G}R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\frac{c^4}{G}R + \frac{c^4\Lambda}{G}g_{\mu\nu} = -8\pi T_{\mu\nu} \quad (3)$$

where:

$\frac{c^4}{G}R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\frac{c^4}{G}R$ describes the energy density of the spacetime.

This equation shows the influence of the energy – momentum tensor $T_{\mu\nu}$ on the energy density of the spacetime.

Reference

1. Minkowski H., *Raum und Zeit,* Physikalische Zeitschrift, **10**, 1909, pp. 104–111. English translation *Space and Time: Minkowski's Papers on Relativity*, ed. V. Petkov, Minkowski Institute Press, Montréal – Québec, 2012.