Alice meets Bob! or: The association of infinity and finiteness within the Schwarzschild metric

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The Schwarzschild metric is the basic description of a gravitational field, but it is more than that: It provides us with some hints about the way how the universe is working. One main feature of the Schwarzschild metric is the association of finite and infinite time structures, and it includes even proposals for the solution of the so-called "information paradox" of black holes and the supposed "breakdown of general relativity" near singularities.

1. The association of infinity and finiteness

The Schwarzschild metric is the basic description of a gravity field, but it is more than that: It provides us with some precious information and with some hints about the structure of the universe.

At the event horizon of a black hole, infinity is meeting finiteness, due to gravitational time dilation. An external observer will come to the conclusion that nothing will ever reach the event horizon, whereas an infalling observer is reaching it within finite time. For this reason, the Schwarzschild coordinates are supposed to be uncomplete, and alternative coordinates have been developed which are going beyond the event horizon, such as the Kruskal-Szekeres coordinates which we will use here.

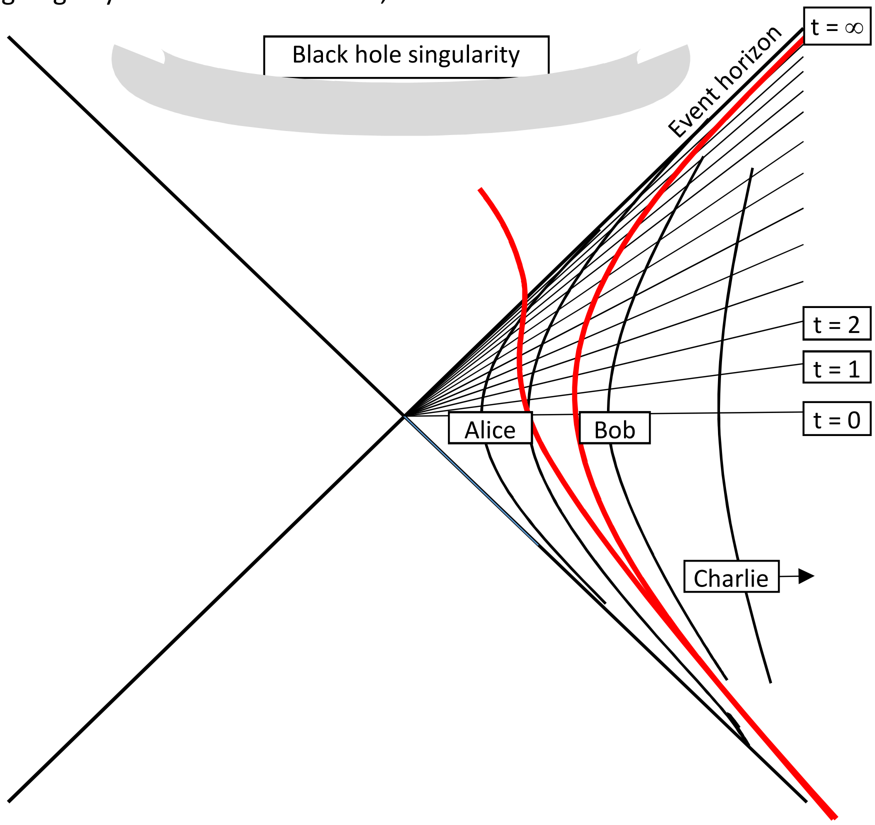

Fig. 1: The Kruskal-Szekeres coordinates

In the Kruskal diagram, the particle A (Alice) is an infalling particle, the particle B (Bob) is hovering at a constant distance from the event horizon, and we may add Charlie as a far-away observer at a place

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without any gravitational potential. The lines of simultaneity are counting time from zero to infinity, and they are directed towards the center of the diagram, whereas the lines of equal distance from the event horizon are hyperbolic lines.

If we follow the logic of the Kruskal coordinates with their simultaneity lines, the universe is getting older, approaching infinity step by step, but it is never ending because whatever is the coordinate time we chose, we may always find a finite coordinate time which is happening later. The event horizon is the limit, the ceiling which is never reached.

The Kruskal coordinates correspond to the view of a far-away observer which is outside of the gravitational potential, but the main principle of the infinity at the event horizon applies to any external observer and to the whole external universe: External observers are subject to time dilation, but nevertheless they will agree that the event horizon represents the simultaneity line of infinity and the end of our time.

So, if the whole universe agrees on infinity at the event horizon, what does that mean for the infalling observer Alice?

First of all, we must be aware of the fact that an infalling observer is also an external observer as long as she does not reach the event horizon, and as such Alice will agree that the event horizon represents infinity, until the very moment when she is reaching it. Near the event horizon, the time dilation factor of her clock is extremely high, but a high time dilation factor does not make infinity finite. From her point of view, the universal time is still infinitely far from its end, even when she is located at a place which is infinitesimally near to the event horizon.

While Alice is approaching the event horizon, she is crossing every single simultaneity line of the Kruskal coordinates, but according to her clock, due to time dilation, the universe is running quicker, exponentially accelerating. The arrival at the event horizon happens suddenly and very abruptly because time dilation has become infinitely high. When crossing the event horizon, she is going beyond our infinite time coordinate, and therefore her time coordinates do no longer correspond to any of our time coordinates, that means: We don't know where she arrives, but for sure she has left our universe of spacetime.

That means that the Schwarzschild coordinates are <u>not</u> uncomplete: They are representing our spacetime which is ending at the event horizon, and it is completely useless to look for a continuation of spacetime inside the event horizon, and the infalling observer is not a "proof" of the contrary because she is leaving our universe. This is documented very precisely by the Kruskal coordinates where our spacetime is limited to the right quarter of the diagram, and the event horizon is the limit to which spacetime is converging. In the upper quarter of the diagram, beyond the event horizon, there might be something after the end of our time, but it is not part of our spacetime.
The Schwarzschild metric introduces the possibility of a "universe after our universe", but we must accept that our spacetime is referring only to our universe, and it is a mistake to think that we can describe what happens after spacetime by the means of spacetime.

There is no "symmetry" between the points of view of Alice and Bob. Bob represents the point of view of an arbitrary observer within our spacetime. In contrast, when Alice reaches the event horizon, she is adopting a sort of "supra-universal" reference frame which is creating a sort of link between the outside and the inside of the event horizon. On the one hand we have the intrinsic evolution of a particle within our universe until infinity, on the other hand we have the "supra-universal" evolution of the particle within a cyclic cosmology, from one universe to the next.

2. Alice meets Bob

We saw that the event horizon is a simultaneity line - that implies that all objects and all particles that will ever fall into the black hole will reach this infinity simultaneously. If one later day, Bob should fall into the black hole, he becomes an infalling observer, after Alice. But the surprising result of the Schwarzschild metric is, that, even if he starts to follow Alice 1000 years later, he will join Alice at the simultaneity line of the event horizon.

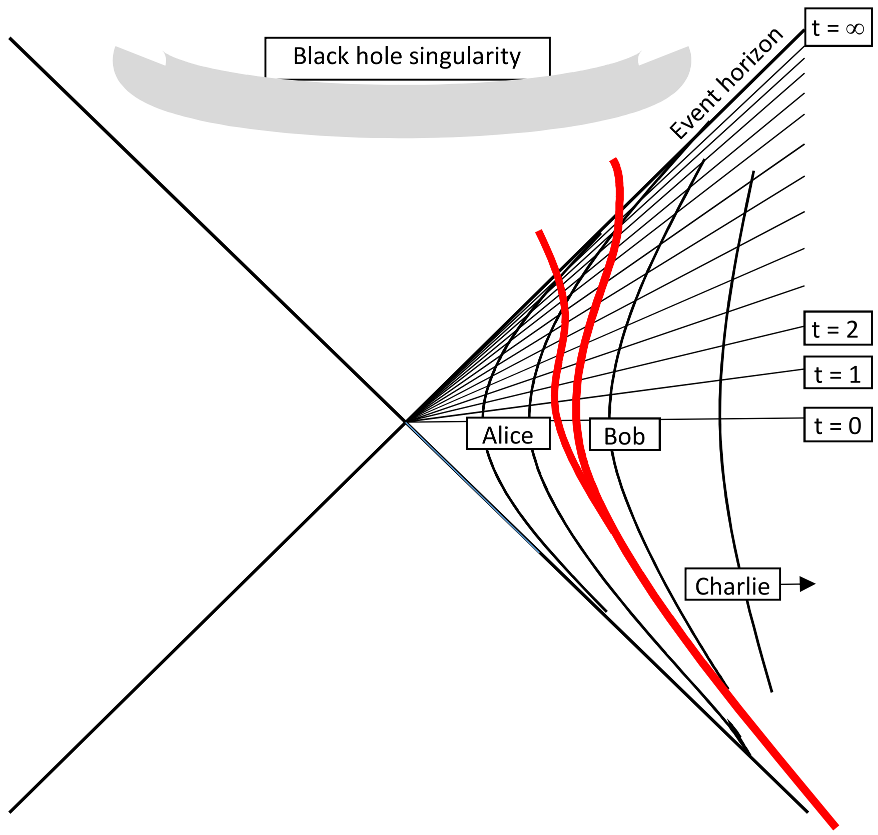

Fig. 2: Alice meets Bob

This is an effect of time dilation, and we will check this with an example, we presume that one simultaneity line corresponds to 250 years. So Alice is always ahead of Bob by 4 simultaneity lines, that is 4 time units. However, on her way to the event horizon, the distance between these simultaneity lines in the diagram is shrinking to zero, and even 1000 years are melting to nothing. As there is an infinite amount of time units, their spatial representation in the diagram is getting infinitely small near the event horizon, converging towards zero.

And this result is perfectly confirmed by the diagram: The Kruskal diagram is showing that both of them are crossing every single simultaneity line, one after the other, until infinity. She is always 4 time units ahead, but at infinity this is compensated by the fact that $\infty + 4 = \infty$.

Moreover, we must remember that Alice when she is reaching the event horizon, she does this abruptly. One infinitesimal time unit before, she was outside the event horizon, and by consequence, from her point of view, the event horizon represented still infinity, until the very last moment. That means that the last step of Alice before reaching the event horizon is equivalent to an infinite time interval, but dilated by infinite time dilation! Obviously, the time interval between Alice and Bob of 1000 years is nothing against this infinite time interval.

How can it be that Bob who started later than Alice is reaching the event horizon within the same time? The answer is the time dilation: As Alice is always ahead by 4 time units, she is always nearer to the source of gravity, and she is exposed to a stronger time dilation than Bob. It is this time dilation difference which sums up to 1000 years.

But how is it possible that there is one single event "Alice meets Bob" if in the diagram, Alice and Bob are not reaching the event horizon at the same point?
Answer: The Kruskal diagram is in no way isometric, it does not preserve lengths. The event horizon is an extreme limit case, in two different aspects:
- On the one hand, it is the limit case of the simultaneity lines where $t = \infty$, so time is the same on the whole event horizon.
- On the other hand, it is the limit of the hyperbolic lines of spatial equidistance where the radial distance equals the Schwarzschild radius, so also the radial distance is the same on the whole event horizon, and if time is the same and space is the same, that means that the event horizon in the diagram represents only one point. - We may not forget that in the Kruskal coordinates, the two other space coordinates are not represented, and they are not needed because we are considering radially infalling movements here.

As Alice and Bob are meeting in one reference frame, they are meeting in any frame, also in the respective reference frames of Alice and Bob.

3. Alice and Bob are not alone…

We saw, that Bob meets Alice at the event, and we may say that all infalling particles that will ever fall into this black hole will reach the event horizon simultaneously - the event horizon is a line of simultaneity. But that is not all - we may also suppose that all infalling particles which will ever fall into any black hole of the universe will do this simultaneously, at the end of our time, and it seems that this is a moment where something special is happening.

4. The emptiness of black holes and the holographic principle

We saw that the whole universe agrees that nothing may ever cross the event horizon of black holes within the time of our spacetime. The inside of event horizons is not part of our spacetime of general relativity, black holes are empty, they are hollow spheres, their inside may be considered as the "future after our universe".

That implies that black holes have no mass! Instead, all the mass of a black hole is consisting of the infalling matter which is approaching the event horizon.

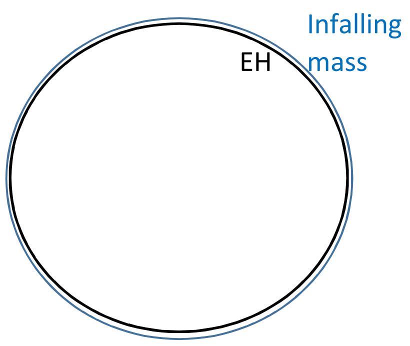

Fig. 3: Black holes are empty

The closer a zone is to the event horizon, the longer the time a particle will be found in this zone. That means that an important amount of the infalling matter is found at an infinitesimal distance to the event horizon, and still steadily approaching.
This concept corresponds to another important concept on black holes which is the holographic principle. The holographic principle supposes that the information inside the spatial, three-dimensional volume of a black hole could be encoded within the two-dimensional event horizon.

But no holographic principle is required here! According to the Schwarzschild metric, all the mass of the black hole is found outside near the event horizon: The infalling particles are remaining during an infinite time near the outside as a shell, and they are forming a nearly twodimensional surface which is not located on the surface of the event horizon, but infinitesimally near to it.

5. The merging of black holes

How are black holes forming and merging? At the encounter of two black holes, the two shells of matter around both event horizons (in blue) are merging, whatever is the size of the black hole:

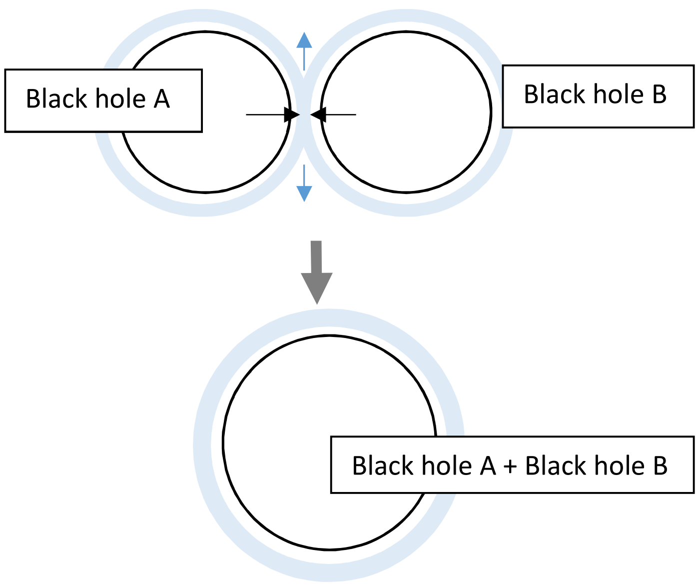

Fig. 4: The merging of two black holes

The merging process is driven by the attraction of gravity (the horizontal black arrows) and by the repulsive effect of gravitational time dilation (the vertical blue arrows) near the event horizon.

Both event horizons are merging and forming one big event horizon, and mass is remaining outside.

6. What is happening at "the end of the universe"?

Now I come to cosmology:

Physical theories predict the "Big Rip" or the "Big Crunch" at the end of the time of our universe. The Schwarzschild metric is a good basis for the Big Crunch theory: All black holes would have merged to one cosmic black hole, all particles and radiation would reach simultaneously the event horizon, crossing it and entering possibly into a new universe.

Such assumptions are speculation, but they would be a coherent interpretation of the asymptotical logic and geometry of the Schwarzschild metric. However, astronomical observations go rather in the direction of the Big Rip. For the Big Rip, the application of the Schwarzschild metric is clearly less self-explanatory, but we could imagine that the Big Rip is approaching some sort of external event horizon, at the end of the time of our spacetime.

Of course, the Schwarzschild metric should also be taken into account with respect to the creation of our universe, the "Big Bang": Currently it is not clear which physical laws might have governed the beginning of a universe emerging from one point of singularity, and how this could harmonize with general relativity. The assumption of an event horizon of a white hole avoids the concept of a single point and would facilitate this problem.

We do not know what is happening with an infalling observer Alice at the other side of the event horizon, but there is one candidate: The event horizon could be the separation between a black hole on the outside and a white hole on the inside. We could suppose that a new big bang is happening, introducing a new infinite universe after the end of our infinite universe.

At this point, there is one important question arising: If the final black hole after the Big Crunch is generating a white hole, in which direction does the white hole go? Towards the inside or towards the outside of the event horizon?
The argument in favor of the inside direction of the Big Bang is the dynamics of the infalling observer Alice. From her point of view, she is continually accelerating towards the central singularity, in accordance with Newton's law, and it seems coherent to refer for this question to her "supra-universal" reference frame. However, her momentum towards the center could be stopped at the event horizon by the environment, because we must remember that the matter of the whole universe is reaching the event horizon exactly simultaneously with her! The horizontal pressure could inhibit the entry of matter through the event horizon and push it back outside. We will not try to predict the outcome of such high-energy phenomena which could happen at this final moment of our universe, but nevertheless we want to mention this open question.

7. The second law of thermodynamics

What about entropy at the end of our universe, at the Big Crunch, at the final black hole and in the new universe after our universe? According to the second law of thermodynamics, entropy is steadily increasing, but the question arises here if the second law of thermodynamics is a law of our spacetime which is limited to spacetime, or if it is a supra-universal law which would apply also to the next universe.

Indeed, it would not be very romantic if the new universe after our universe would begin with heat death, it would mean that after our universe, nothing would happen anymore in later universes.

But there are good reasons to think that the Big Crunch could be a sort of "reset" of entropy. According to the model of the Schwarzschild metric, at the end of the universe, the whole matter is reaching simultaneously the final event horizon - and the universe has the form of a perfect hollow sphere, a shell which is exactly positioned on the event horizon. By this, the universe would adopt by itself a perfectly ordered, homogeneous structure, without any information left, with entropy close to zero, comparable with the reinitialization of a vast game.

8. Outlook

In this paper, I tried to show by the means of the Kruskal-Szekeres coordinates the conclusions we may draw from the structure of the Schwarzschild metric, in particular:

Spacetime is not "breaking down" at black holes because their event horizons are natural limits of spacetime. There is "cosmic censorship" because the inside of black holes is not defined within our spacetime.

It is a crucial mistake to think that the view of Alice is contradicting the view of external observers. The event horizons are the limit for all observes of the whole universe, even including Alice while she is outside the event horizon. Infalling observers are leaving our spacetime after the end of our universe and may be considered as "supra-universal" reference frames, going beyond our spacetime.

The current concept of black holes is clearly incompatible with the Schwarzschild metric: the inside of an event horizon must be empty because, by time dilation, all approaching matter is eternally slowed down.

All mass which will ever enter any black hole of the universe will do this simultaneously, at the end of the time of our universe.

All these conclusions are following directly from the Schwarzschild metric. Based on these conclusions, we may speculate about a cyclic cosmological model: One infinite universe is followed by the next infinite universe, and the final Big Crunch in one single black hole corresponds to the white hole of the Big Bang of the next universe.