

# On 2D and 3D Shapes Based On Primality Tests

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## Abstract

Based on primality property change for two consecutive terms of infinite sequence of natural numbers it is possible to establish rules which allow to draw a figure. This work is devoted to studies on various sequences of natural numbers which produced interesting 2D and 3D outputs.

## 1 Introduction

Each natural number  $n$  belongs to either prime (P) or non-prime (NP) set. If we arrange  $N$  natural numbers  $n_1, n_2, \dots, n_N$  in an ordered sequence  $S$  ( $S(i) = n_i$ ), then we can create a corresponding ordered sequence  $P$  of  $N$  boolean values ( $P(i)$  is either *True* or *False*,  $1 \leq i \leq N$ ) based on primality of  $n_i$ .  $P(i)$  is *True* if  $S(i)$  is prime, otherwise is *False*. For instance, if  $S = 1, 2, 3, 4, 5, 6$  then  $P = False, True, True, False, True, False$ .

Observation of changes in  $P$  provides information about changeability of primality property in  $S$ . If  $S(i)$  is prime and  $S(i+1)$  is not prime, then  $P(i) = True$  and  $P(i+1) = False$  (change from *True* to *False*). If  $S(i)$  is not prime and  $S(i+1)$  is prime, then  $P(i) = False$  and  $P(i+1) = True$  (change from *False* to *True*). We can establish yet another sequence  $Q$  which indicates primality property change between two consecutive terms of  $S$ . If  $P(i) \text{ XOR } P(i+1) = True$ , then  $Q(i+1) = True$ , otherwise  $Q(i+1) = False$ . For instance, if  $S = 1, 2, 3, 4, 5, 6$  then  $P = False, True, True, False, True, False$  and  $Q = N/A, True, False, True, True, True$ . For simplification in calculations we can assume that  $Q(0)$  will be always *False*.

## 2 Basic plotting rules

Values recorded in  $Q$  may be used to draw 2D shape.  $\#Q = \#S$ , thus shape would have as many points as terms in  $S$ . We start plotting from point  $W = (0,0)$  and with vector  $D = (+1,0)$ , which is indicating current direction for next point to be drawn. If  $Q(i) \text{ XOR } Q(i+1) = False$ , then we continue plotting in already established direction. If  $Q(i) \text{ XOR } Q(i+1) = True$ , then we change direction (for instance, clockwise: Table 1) before plotting the next point.

Table 1: Clockwise change of two-dimensional  $D$

Initial $D$ (x,y)	New $D$ (x,y)
(0,1)	(-1,0)
(1,0)	(0,1)
(0,-1)	(1,0)
(-1,0)	(0,-1)

Plotting algorithm  $A$  for 2D figures is depicted in Listing 1. Figure drawing starts from point  $(0,0)$  with direction  $(+1,0)$ . In its main part  $A$  is testing primality of elements in  $S$ , one-by-one, and stores results in  $P$ . Then (based on difference between two consecutive terms of  $P$ )  $A$  decides on the next point location and draws a single point. Entire operation is repeated until end of  $S$  is reached.

Listing 1: Plotting algorithm for 2D shape

```
# initial values
W = (0, 0)
D = (+1,0)
P[0] = False

# assuming that S contains N elements
# is_prime(n) returns True iff n is prime
i = 1
for n in S:
    P[i] = is_prime(n)
    # change direction if Q is changing
    if P[i-1] XOR P[i]:
        D = change_direction (D)
    W += D
    plot_point (W)
    i += 1

def change_direction (D)
    D = rotate_vector_clockwise (D)
    return D

def plot_point (W)
    plot (W.x, W.y, color)
```

In case of 3D plotting, third dimension is added which is changing delta of  $z$  (table 2 shows rules for such change).

Table 2: Change of dimension  $z$  in three-dimensional  $D$

Initial $z$	New $z$
0	1
1	-1
-1	0

## 3 Advanced plotting rules and metrics

Basic plotting rules may be supplemented with more advanced rules like points aging, more demanding rules for changing  $D$ , including changeable length of  $D$  and using more than one color to plot the points.

Point aging may be defined in a way that each visible point  $P_i$  is associated with watchdog  $W_i$  which is started

when  $P_i$  is created.  $W_i$  initial value is set to value  $AGE_{max}$  ( $AGE_{max} > 0$ ) and is decreased by 1 with every iteration if actual point  $P_j$  is different than  $P_i$ .  $W_i$  is reset to  $AGE_{max}$  if  $P_j = P_i$ .  $P_i$  is removed (aged out) if  $W_i = AGE_{min} = 0$ .

If point aging is in use, point  $P_i$  color selection may be based on  $W_i$  value (the bigger value of  $W_i$ , the hotter color). Color may be also based on total number of hits (the more hits, the hotter color) which is ideal for case without point aging.

Each shape can be accompanied with a set of metrics which can be used to compare them and quantify properties in time. Table 3 shows such sample metrics.

Table 3: Metrics

Metric	Description
$M_1$ : % of primes	primes / all $\times 100\%$
$M_2$ : % of non-primes	non-primes / all $\times 100\%$
$M_3$ : Max diff in p.X	max(p.X) - min(p.X)
$M_4$ : Max diff in p.Y	max(p.Y) - min(p.Y)
$M_5$ : Max diff in p.Z	max(p.Z) - min(p.Z)
$M_6$ : 2D fill in %	pts / ( $M_3 \times M_4$ ) $\times 100\%$
$M_7$ : 3D fill in %	pts / ( $M_3 \times M_4 \times M_5$ ) $\times 100\%$

## 4 Results of experiments

During experiments (which were targeted to find interesting relations and visualisations between primes and composite numbers) various sequences  $S_i$  were evaluated (Table 4) using framework [1].

Table 4: Sequences  $S_i$  subjected to experiments

i	Formula for $S_i(n)$
1	$2 \times n + 1$
2	$6 \times n + 1$
3	$6 \times n - 1$
4	$6 \times n + 1$ for n even $6 \times n - 1$ for n odd
5	n
6	$30 \times n + 1$
7	$30 \times n - 1$
8	$30 \times n + 1$ for n even $30 \times n - 1$ for n odd
9	sum of decimal digits of n
10	1 for n even 2 for n odd
11	$\lfloor 10 \times \sin(n) \rfloor$
12	random (2, 3, 4, 5, 6, 7, 8, 9)

Figures 1 and 2 present 2D shapes for  $S_i$  (Table 4) and figures 3 and 4 - corresponding 3D shapes. Experiments were run for the first  $10^6$  terms of  $S_i$ .

## 5 Summary and future work

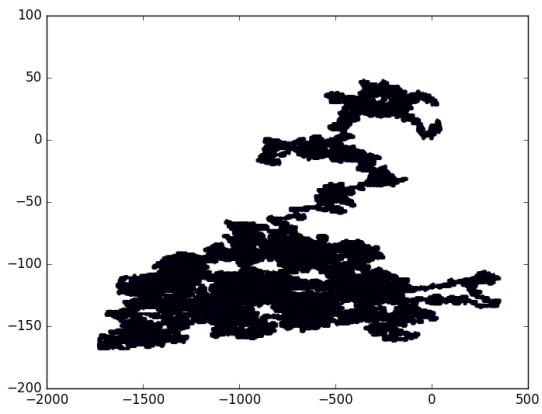
Executed experiments showed interesting outputs, in some cases we can find some additional meaning in the picture (for instance,  $S_1$  looks like a dragon and  $S_{11}$  like

a bridge). The more complex output, the bigger confirmation about the randomness of the sequence to be plotted - in other words - existence of primes in the sequence  $S$  is chaotic unless  $S$  by definition is not obvious (see:  $S_5$  or  $S_{10}$ ). It is also possible to visualize periodicity of the function building  $S - S_{11}$  is such an example.

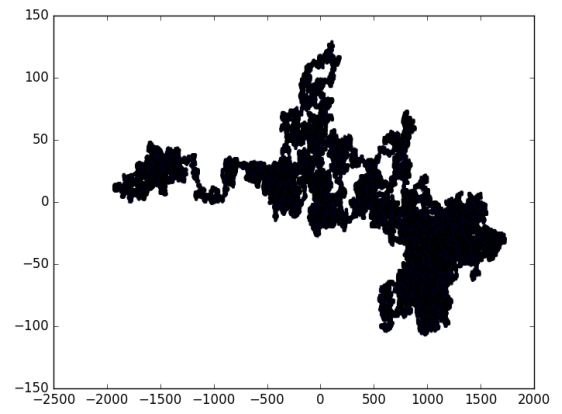
One of the open question is if it is possible to build non-trivial sequence  $S$  for which  $Q$  will have  $M_6 = 100\%$  - in other words if all points of the limited 2D surface that is visualisation of  $Q$  are visited at least once (it would be visualised in 2D space as a solid-fill rectangle).

## References

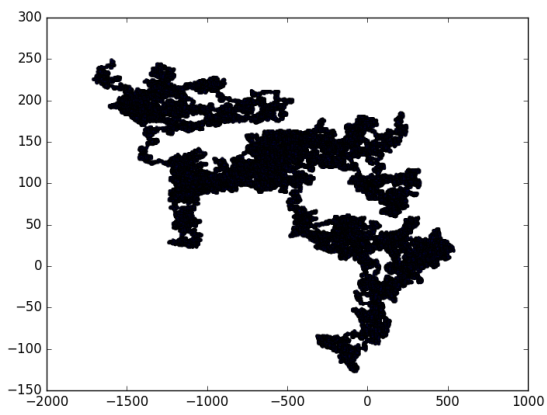
- [1] *Prime shapes automated generation framework.*  
<https://github.com/mbarylsk/prime-shapes/>



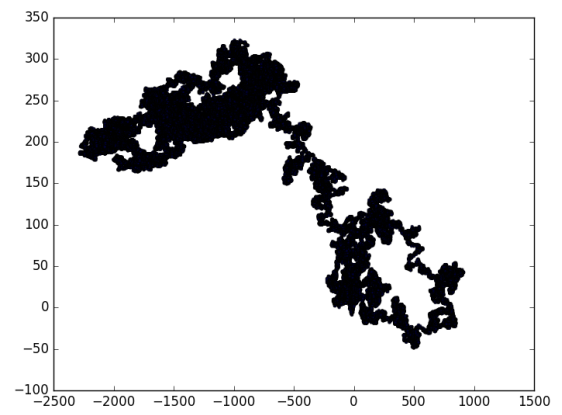
$S_1$



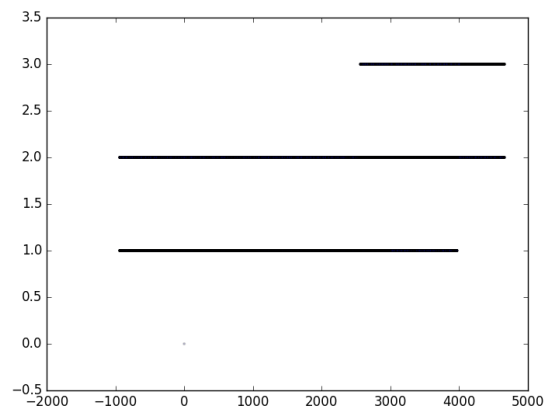
$S_2$



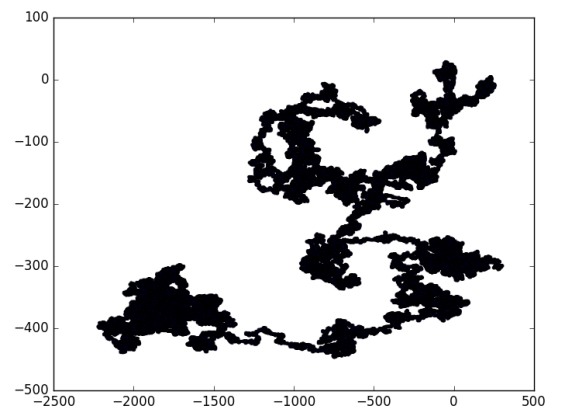
$S_3$



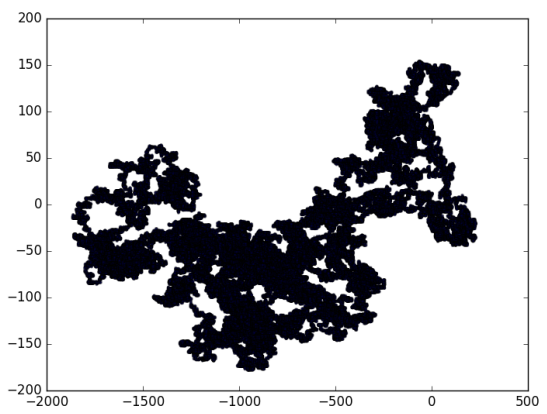
$S_4$



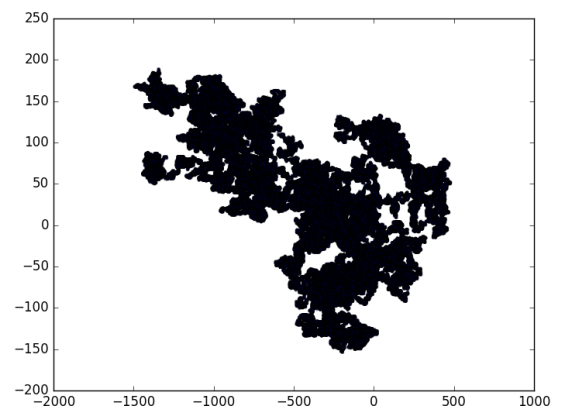
$S_5$



$S_6$

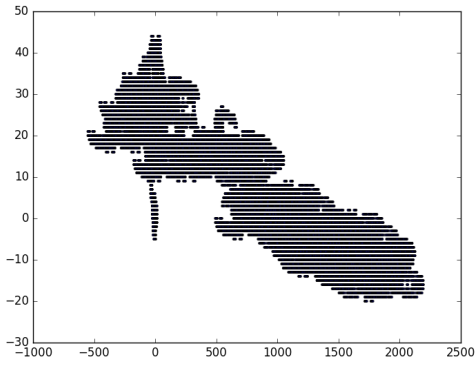


$S_7$

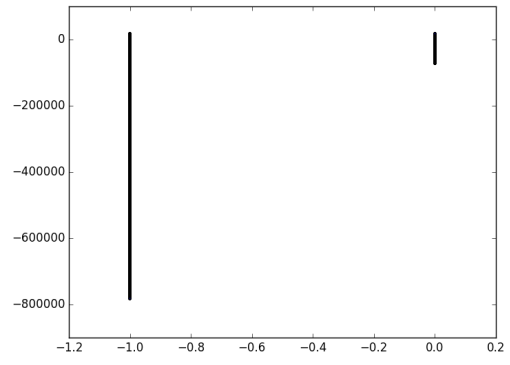


$S_8$

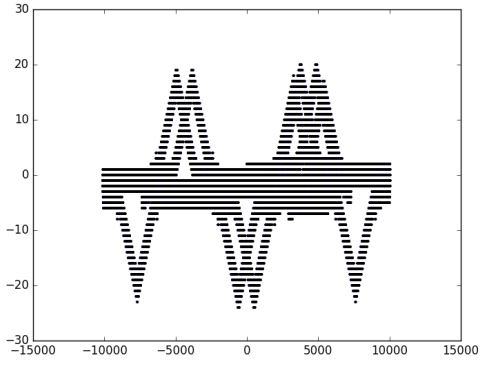
Figure 1: 2D plots - sequences 1-8



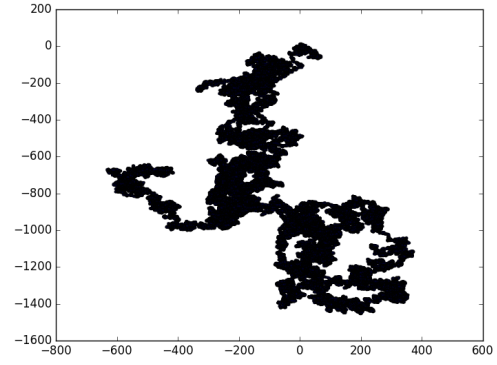
$S_9$



$S_{10}$

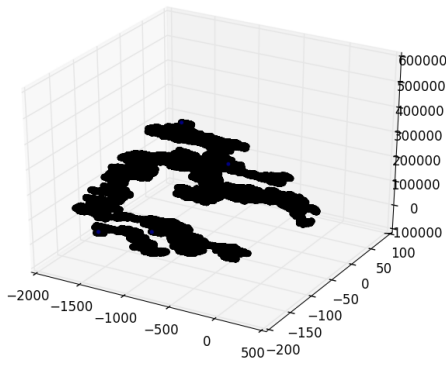


$S_{11}$

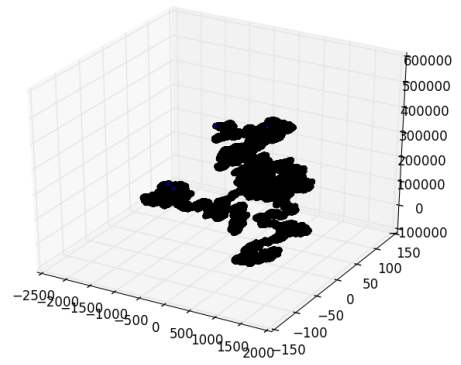


$S_{12}$

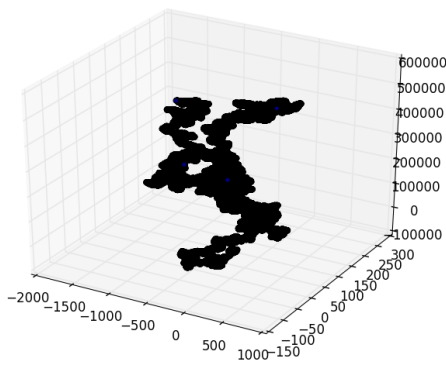
Figure 2: 2D plots - sequences 9-12



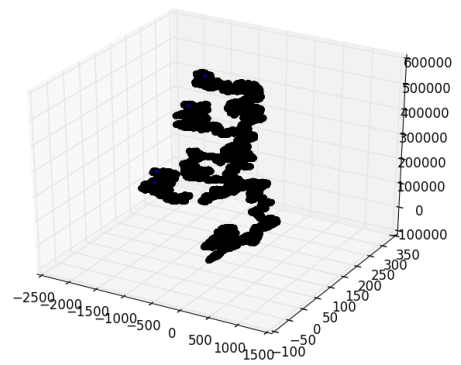
$S_1$



$S_2$

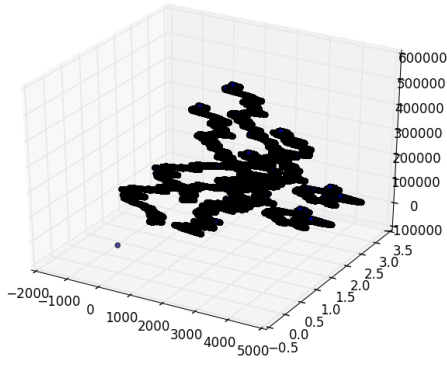


$S_3$

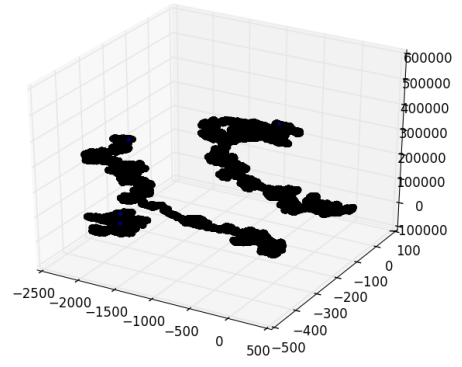


$S_4$

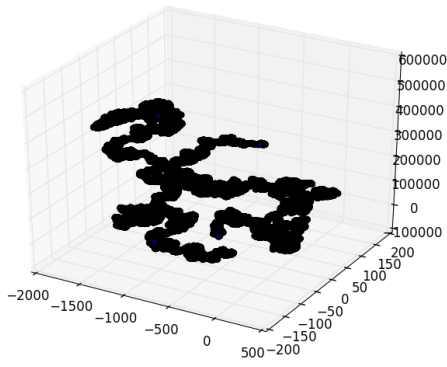
Figure 3: 3D plots - sequences 1-4



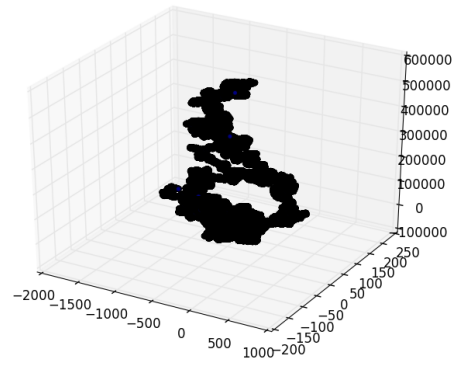
$S_5$



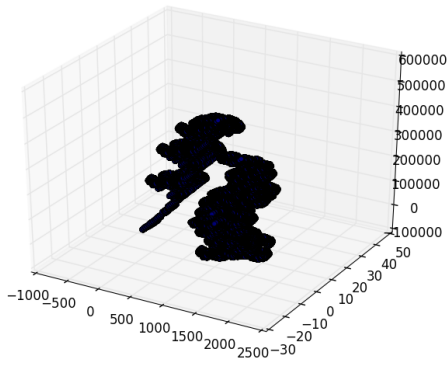
$S_6$



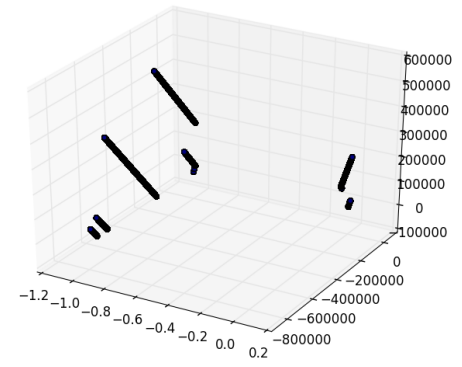
$S_7$



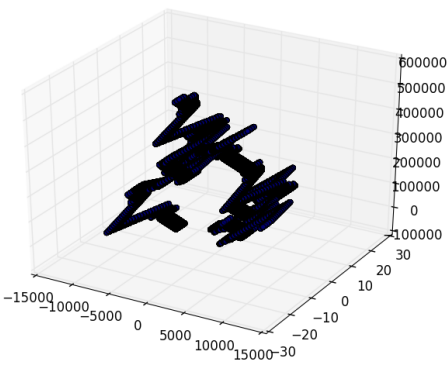
$S_8$



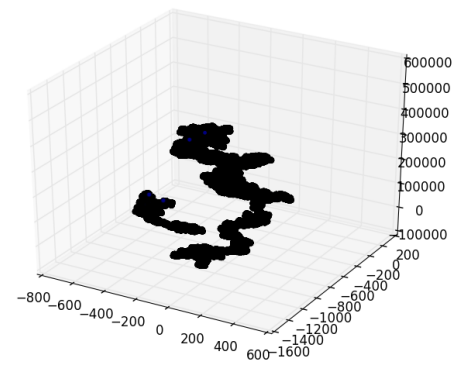
$S_9$



$S_{10}$



$S_{11}$



$S_{12}$

Figure 4: 3D plots - sequences 5-12