SIMPLE PROOF OF FLT FOR PRIMES $n > 2$ (V. $\beth$)

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Abstract. We parametrize the Fermat equation, $x^n + y^n = z^n$, with a form that explicitly shows no integral solution for odd $n > 1$.

1. Proof of FLT By Contradiction for Odd $n > 1$

Fermat's last theorem (FLT) says, with integral *triple* $(x, y, z) > 0$, integral $n > 0$: Equation $x^n + y^n = z^n$ does not hold for $n > 2$.

$$y^n + x^n = ((y^n + x^n)^{\frac{1}{n}})^n. \, Notate \, (y^n + x^n)^{\frac{1}{n}} \, as \, z. \tag{1}$$

We consider, for odd $n \geq 1$, that (1), parametrized by x, y, holds for *all* integral (x, y) such that $z \geq y \geq x \geq 0$. We use 0 for completeness.

We define the *general condition* as : If $(A, B, C) > 0$ are real numbers satisfying $A^n + B^n = C^n$, and *any given* two elements of the *associated* (abbreviated as Asso.) triple are integers ≥ 0, then, the 3rd element of this triple, e.g., $(y^n + x^n)^{\frac{1}{n}}$ in (1), is an *(hypothetical)* integer ≥ 0. The result is the Asso. (*(hypothetical)* integral triple $(x, y, z) \geq 0$.

$$x^n + ((y^n - x^n)^{\frac{1}{n}})^n = y^n. \, Notate \, (y^n - x^n)^{\frac{1}{n}} \, as \, w. \tag{2}$$

We consider, for odd $n \geq 1$, that (2), parametrized by x, y, holds for the same, *all* integrals (x, y) we use in (1) such that $y \geq w, x \geq 0$. *General condition* : The Asso. integral triple is $(x, (y^n - x^n)^{\frac{1}{n}}, y) \geq 0$.

For odd $n \geq 1$, with all integral $x, y \geq 0$, all (hypothetical) Fermat triples ≥ 0 are in (1), and in (2): For different (x, y), every (x, y, z) can be found in $\{x, w, y\}$, and every (x, w, y) can be found in $\{x, y, z\}$.

The *intersection* of the associated sets of (hypothetical) integral triples ≥ 0 in (1),(2) gives the Asso. set of (hypothetical) integral triples ≥ 0 in (3), below, per simultaneous solution of Eqns. (1), (2).

Below : Eqn. (3) has the same $A^n + B^n = C^n$ form as with (1), (2); in concert, (1), (2) *implies* (3); *it is a true conditional statement* that "If the *general condition* should be met by (1), (2), then, the *general condition* would be met by (3)". Below, for odd $n > 1$: The *general condition* is not met for *any given* Asso. triple in (3), so, such condition is not met for the *corresponding*, respective Asso. triple in (1), (2).

Factually, Fermat triples exist for $n = 1, 2$. However, (3), below, shows, for integral x, that the Asso. integral triples in (1), (2) are held in common for $n = 1$, but not for $n = 2$, the main reason we consider only odd $n \geq 1$ with (1), (2), making $n \geq 1$ only odd with (3), below.

$$((y^n - x^n)^{\frac{1}{n}} K)^n + (2^{\frac{1}{n}} x K)^n = ((y^n + x^n)^{\frac{1}{n}} K)^n. \tag{3}$$

For *odd* $n \geq 1$: Equation (3) holds for *all integral* (x, y); $y \geq x \geq 0$, such that $K = \frac{1}{2}$ for same parity x, y; $K = 1$ for opposite parity x, y, with the associated set of triples $((y^n - x^n)^{\frac{1}{n}} K, 2^{\frac{1}{n}} x K, (y^n + x^n)^{\frac{1}{n}} K)$.

As defined, *algebraic identity* (3) is *useful*, so, *is not trivial*, similar to *Euclid's identity*, a well-known parametrization of $x^2 + y^2 = z^2$.

Per the *general condition* : Elements $(y^n - x^n)^{\frac{1}{n}}$ in (1), $(y^n + x^n)^{\frac{1}{n}}$ in (2) each is (hypothetically) integral; hence, in (3), the two parts of the triple in (3), $(y^n - x^n)^{\frac{1}{n}} K, (y^n + x^n)^{\frac{1}{n}} K$, each is also (hypothetically) integral; thus, the 3rd part, $2^{\frac{1}{n}} x K$ in (3), is (hypothetically) integral.

If there exists an integral $(x, y) \geq 0$ satisfying (1), (2) with resp. Asso. sets of (hypothetical) integral $(x, y, (y^n + x^n)^{\frac{1}{n}})$; $(x, (y^n - x^n)^{\frac{1}{n}}, y)$, then, there exists an integral $(x, y) \geq 0$ satisfying (3) with the resp., Asso. sets of (hypothetical) integral $((y^n - x^n)^{\frac{1}{n}} K, 2^{\frac{1}{n}} x K, (y^n + x^n)^{\frac{1}{n}} K)$.

Thus, for odd $n \geq 0$, Eqns. (1), (2) *taken together* imply Eqn. (3).

For the subset of odd $n > 1$, and integral $y > x > 0$: Any given element $2^{\frac{1}{n}} x K$ of the Asso. (hypothetical) integral triple in (3) *is in fact irrational*; hence, in fact, *any given* Asso. (hypothetical) integral triple $((y^n - x^n)^{\frac{1}{n}} K, 2^{\frac{1}{n}} x K, (y^n + x^n)^{\frac{1}{n}})K)$ in (3) is *not positive integral.*

2. Results and Conclusions

For odd all $n > 1$ with all integrals $y \geq x \geq 0; K = 1$ or $1/2$:

1) Equation (1) in concert with (2), having, resp., the Asso. (hypothetical) integrals $(x, y, (x^n + y^n)^{\frac{1}{n}})$; $(x, (y^n - x^n)^{\frac{1}{n}})$, implies (3), with the Asso. (hypothetical) integral $(y^n - x^n)^{\frac{1}{n}} K, 2^{\frac{1}{n}} x K, (y^n + x^n)^{\frac{1}{n}} K)$;

2) *Any given* $2^{\frac{1}{n}} x K$ is in fact irrational; so, *any given* Asso. (hypothetical) integral $((y^n - x^n)^{\frac{1}{n}} K, 2^{\frac{1}{n}} x K, (y^n + x^n)^{\frac{1}{n}} K)$ in (3) is *not integral* in fact. Thus, *by contradiction*, the *corresponding*, Asso. (hypothetical) *integral triples* $(x, y, (x^n + y^n)^{\frac{1}{n}})$ in (1) or $(x, (y^n - x^n)^{\frac{1}{n}}, y)$ in (2) is not positive integral in fact. Since the associated (hypothetical) integrals $\{x, y, z\}$; $\{x, w, y\}$ in (1), (2), resp., have every value in common, the *corresponding*, associated (hypothetical) integrals (x, y, z) in (1) *and* (x, w, y) in (2) *each* can not be positive integral *in fact.*

So, for primes $n > 2$: $x^n + y^n = z^n$ has no integral solution. Q.E.D.