Unconventional If(f) Convenient: Effective Structures to Construct Alternate Primes Experimentally

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ABSTRACT[1]

Dial symmetry, Complenarity, and *#-scoring* could prove productive as well as efficient means of [re]constructing primes. Seen as alternate constructs accommodating the same prime values, or otherwise pairing those implicitly involved, as *p'=p+2k*9* or *p'=2n+(2k+1)*9* (both capturing the laxer subset of p=2m+1), *complenary* values tend to follow the parity: *#p'=#p=#(2m)*. Moreover, the early primes sequence appears to fit into Fibonacci-like regularities along much the same #-scoring lines.

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Dial Symmetry

Contemplate any standard dialing interface, e.g. your calculator or handset, and you'll almost instantaneously gain an appreciation for just how much symmetry is revealed thereby, be it on the prime end or beyond (the latter, somewhat tautologically, falling outside the intended scope of the presently attempted exposition). When presented as natural couples, triples, or n-tuples depending on how many digits these may take, primes span symmetric patterns (see Appendix) with some of the values garnered being as diverse as: (11, 13, 17, 19), ((13, 31), (17, 71)), ((19, 37), (19, 73)), (71, 73, 79), (23, 47), ((23, 89), (13, 79)), (29, 61), ((53, 59), (61, 67)), ((41,47), (37, 73), (79, 97)), (43, 83), (41, 89), ((17,79), (71, 97), (13, 71), (17, 31)). This, too, comprises primes basic and further alike: (3,5,7), ((137, 173, 317), (319, 391)) (157, 571, 751), (359, 593, 953), etc. (Yet not candidates like 371=7*53, 391=17*23, 319=11*29, 913=11*83, 931=7^2*19, nor definitely 357 or 753 as reducible by a gcd within, e.g. (35,7)=7, (75,3)=3. For that matter, note (51, 57) as the [disqualified] mirror image for (53, 59).)

Values such as 1, 11, 101 and the like stand alone as special ones, indeed as poles or loops—which will surprisingly carry over into rather distant expositions as part of the forthcoming research.

Now, if one were to incorporate the missing 0 explicitly, generating more primes would be facilitated, albeit not without qualifications: say, 101, 103, 109, 409, 509, 709, 809 (but not 209=11*19, 309 or 609, the latter two clearly showing gcd reducibility, which will reappear as a caveat in line with a structure based approach).

[1] To Darya Melnikova for many happy returns of her Birthday! 30=2*3*(2+3)

Complenary Structures Based on Alternate Kernels

A rather densely distributed, productive as well as efficient prime space could be generated by attempting either or both these complenary (i.e. alternate as per arriving at the exact same primes) structures:

$$p' = p + 2k * 9, \;\; k \in \boldsymbol{N} \geq 0, \;\; \gcd(p, 9) \equiv 1 = \gcd(p, k) \qquad (A)$$

$$p' = 2m + (2k + 1) * 9, \;\; \gcd(m, 9) \equiv 1 = \gcd(m, 2k + 1), \qquad m > 0 \quad (B)$$

Subject to the constraints, it should come as no surprise that, whilst *p-kernel* equaling 3 is rejected from the outset (alongside *p* and *k* non-coprimes) as per (A), the same goes for m=0, 3, 6 etc. or otherwise *non-co*prime with respect to the (2k+1) residuale factor. Please note that we discard a *p*-kernel of 2 (not treating 2 as a prime *by convention* nor *ad hoc*), whereas one could make a provision for p=3 as long as k=0 were an option. *Inter alia*, this produces primes amid composites accruing as the rare (less controlled net of the prior cut-offs) exception: 1+0*9=1, 1+18=19, 1+36=37, 5+0=5, 5+18=23, 5+36=41, 5+54=59 (but: 1+54=55, 5+72=77), 7+0=7, 7+36=43, 7+54=61, 7+72=79 (but: 7+18=25=5^2), 11+0=11, 11+18=29, 11+36=47, 11+72=83 (but: 11+54=65), 13+0=13, 13+18=31, 13+54=67 (but: 13+36=49=7^2, 13+72=85), etc per *(A)*. Overall, the additional qualifier is nearly trivial: Based on the prime kernel, neither symmetry (e.g. 77, 55) should occur nor inner reducibility based on the latter digit (e.g. 5 or any multiple of 2).

As far as the *(B)* residuale is concerned, candidates pop up most naturally: 2+9=11, 2+27=29, 2+45=47, 2+81=83 (but: not 2+63=65). (Interestingly enough, right from the start the p=2 disqualifier could be waived by assuming a rather arcane allowance for 2k+1=0, such that 2=2+0.) Next, 4+9=13, 4+27=31, 4+63=67 (but: not 4+45=49=7^2, 4+81=85), 8+9=17, 8+45=53, 8+63=71, 8+81=89 (but: not 8+27=35), 10+9=19, 10+27=37, 10+63=73 (but: not 10+45=55, 10+81=91=7*13).

Rehashing on the [All-Pervading] Sharp Scores

Consider referring to a *hash/sharp score* as a kind of "numerological" value if you like, even though it has a well-defined as well as rigorous algebra behind it which will not for now be disclosed beyond the required setup. Other than that, say, #137=2=#(1+3+7)=#11. At this rate, a variety (indeed, subspaces or sub-infinities) of otherwise distinct and seemingly disjoint numbers could be compared, again in ways somewhat complementary vis-à-vis modulo comparison.

To illustrate a candidate usage, consider how *p'=83* alternatively emerges: 83=2*1+(2*4+1)*9=11+(2*4)*9. Since all residuales that are multiples of 9 are thrown away as part of *#-scoring* (which is how 9 really *is* special), it comes as little surprise that, #83=#2=#11. More generally, it can cautiously be postulated based on (A) and (B) that,

$$\#p' = \#p = \#(2m) \qquad (C)$$

$$p' = 2m \pm 9 = 2m' + 1 = 2(m \pm 4) + 1 \quad (D)$$

$$(D) \ni (C) \in (A) \cap (B) \leftrightarrow (A) \cap (B) \rightarrow (C) \cap (D)$$

Put another way, as long as the given prime allows for *complenarity* (i.e. can have been produced by applying both/either (A) and/or (B) frameworks), its #(2m)-characteristic may be *odd* even if the underlying 2m-kernel is *even* by definition/design. To illustrate, 37 could be one case in point: 1+36=10+27, #10=#(2*5)=1.

One might be surprised to observe how the #-scores for the early primes imply Fibonacci-like patterns:

$$p_1 + p_k + 1 = p_{k+1} \qquad (1)$$

$$p_{k-1} + p_k = p_{k+1} + 1 \quad (2)$$

$$p_{k-1} + p_{k+1} = p_{k+2} \qquad (3)$$

$$p_{k-1} + p_k + p_{k+1} = p_{k+2} + p_{k+3} \quad (4)$$

Far from being exhaustive or dust-settled, the above set of regularities could be showcased as follows: (1) 1+3+1=5, 1+5+1=7; (2) 3+5=7+1, #(5+7)=#(11)+1; (3) #(11+17)=#(19), #(13+19)=#23; (4) 1+3+5=7+2, #(3+5+7)=#(11+13) etc.

APPENDIX: Dial Symmetry Visualized

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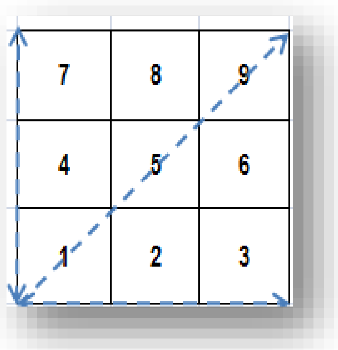
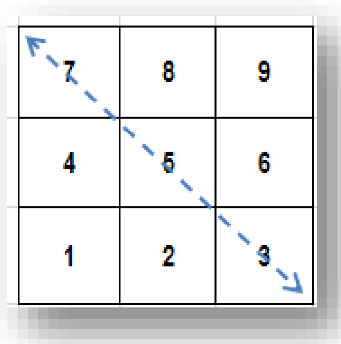
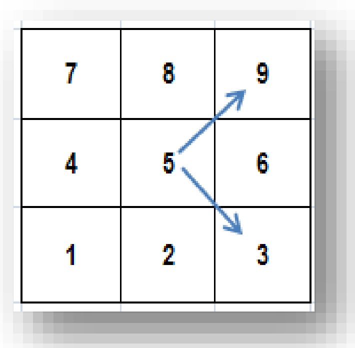
<tr><td></td><td></td><td></td></tr>
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