Solving Rubik's Cubes

V. Nardozza*

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Abstract

I present all beginner methods to solve the most common Rubik Cubes.

Contents

*Electronic Engineer (MSc). Turin, IT. Any comment to: vinardo@nardozza.eu
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1 Introduction

I present here all beginner methods to solve the most common Rubik Cubes. Following a common terminology used by cube solver, I will call sequences of moves "algorithms" although they are not technically so.

Every time I mention a symmetric algorithm I mean basically two things. For algorithms permuting three pieces, the symmetric algorithm is the one that permutes the pieces in the other way round (e.g. clockwise, anticlockwise). For other algorithm, I mean the algorithm that changes the cube configuration in a symmetric way with respect to the plane dividing your body in two symmetric parts. Often the two definitions coincide.

Symmetric algorithms will not be given and they need to be worked out by the reader. This is always an easy task. However, if you can do it, keep in mind that symmetric algorithms, although useful, are not essential for solving the cubes.

2 Classic Rubik's Cube

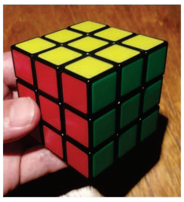

Figure 1: Classic Rubik's Cube

Notation. *A classic Rubik's Cube has three type of pieces. Centres, Edges and Corners. I will use the standard notation for algorithms where faces are denoted by:*

F: front, R: right, U: up, L: left, D: down, B: back

- Moves are intended to be a clockwise rotation. For an anti-clockwise rotation the letter of the relevant move will be followed by the ' symbol (ex. D').

- A number after a letter of a move shall indicate the number of 90deg rotations to be applied when different from 1 (ex. D2 is a 180 deg rotation of the down layer).

- When two slices have to be rotated (the external slice and the one next to it i.e. the middle slice in the case of a 3x3x3 cube), the letter relevant to the move shall have a w as a lower index (ex. D_w).

Classic Rubik's Cube can be solved by layers.

Layer 1:

Starting always from the layer relatives to a specific colors helps a lot to visualise the moves. It is custom in western countries to start always from the white one.

- Complete the white cross placing the edges so that they match the white and the other color centre piece (intuitive moves, no algorithm).

- Place the corners moving them from the 3rd layer to the 1st layer with the algorithm standard(1) and its symmetrical. It is always possible to take a 1st layer corner on layer 3 and with the right orientation (white on one side) by using algorithm standard(1) in various ways.

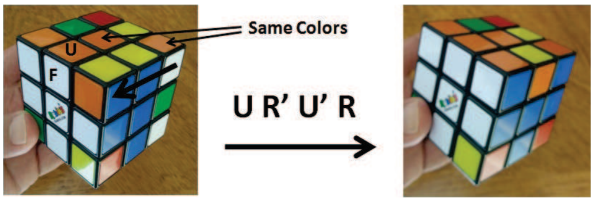

Figure 2: classic(1): $U\ R'\ U'\ R$

Layer 2:

- Place edges on the second layer. To do that match one 2nd layer edges piece (the ones without yellow on them) present on the 3rd layer with the relevant corner piece of the 1st layer with the algorithm classic(2) and its symmetrical.

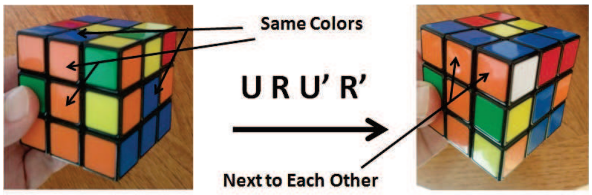

Figure 3: classic(2): $U\ R\ U'\ R'$

- now the 1st layer corner will not be in place any more. Put it in place by using algorithm classic(1). This will eventually put in place also the 2nd layer edges as desired.

It is always possible to remove a 2st layer edge which is misplaced by replacing it with any edge on layer 3 by using algorithms classic(1) and classic(2) as described above.

Layer 3:

- Make the yellow cross on the 3rd layer. This can be done using the algorithm classic(3). If only one yellow piece is present on the 3rd layer then apply algorithm classic(3) with any front face. This will make an L pattern of yellow pieces on the 3rd layer.

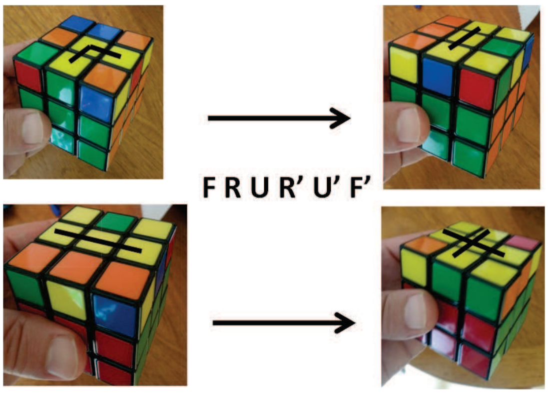

Figure 4: classic(3): $F\ R\ U\ R'\ U'\ F'$

There are two option to complete the 3rd layer.

Layer 3 Option 1:

- Place corners Layer 3 in their right position with respect to colors of the lateral faces by using algorithm classic(4) and its symmetrical. This algorithm, permutes 3 corner of 3rd layer, and swap two edges (but we do not care).

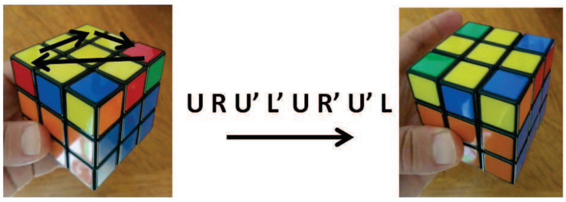

Figure 5: classic(4): $U\ R\ U'\ L'\ U\ R'\ U'\ L$

- Match the color of the edges of Layer 3 with the color of the lateral faces without moving corners from their own position by using algorithm classic(5) and its symmetrical. This algorithm, permutes 3 edges of 3rd layer, rotates corners around their axis (but we do not care) and it is known as "Sune". Note that the Sune can be performed also inverting the sense of rotation of the move before the last as $R\,U\,R'\,U\,R\,U2'\,R'$ and this is the way it is normally presented in tutorials because it is better to be performed in speed cubing. However, I prefer the other way because I do not do speed cubing and I like to turn the Up layer always in the same sense.

Figure 6: classic(5): $R\ U\ R'\ U\ R\ U2\ R'$ (Sune)

- Now all pieces are in place but corner of 3rd layer are rotated around their own axis. To rotate them in the right way we can use algorithm classic(6) which is twice algorithm classic(1) in disguise. Algorithm classic(6) rotate one corner of the 3rd layer without changing the other pieces of the layer. Moreover if applied 3 times it makes the cube to go to the original configuration. To rotate a corner, apply algorithm classic(6) with the corner to rotate in the upper-right part of the front face till the corner is rotated in the correct way. Rotate the up face in order to put the next corner to rotate in the upper-right part of the front face. Keep applying algorithm classic(6) for total number of time multiple of 3 till the cube is solved.

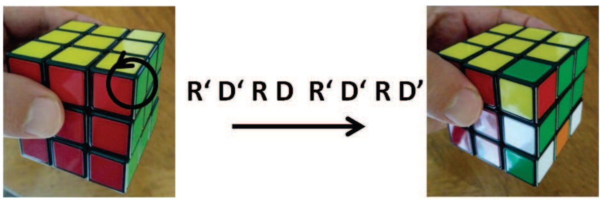

Figure 7: classic(6): $R'\ D'\ R\ D\ \ R'\ D'\ R\ D$

Layer 3 Option 2:

- Place corners in the right position even if not rotated in the correct way by permuting them using algorithm classic(4) and its symmetric.
- Orient Last Layer (OLL), i.e. make the up face of the cube completely yellow by using algorithm classic(5) and its symmetric (Sune). It should be possible to do it using algorithm classic(5) and/or its symmetric at most twice.

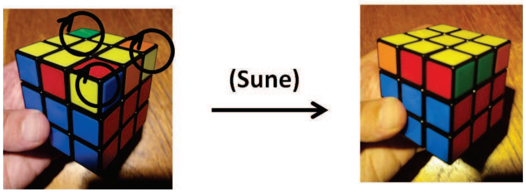

Figure 8: Orient Last Layer (OLL)

- Turn the up layer till the 4 corners go in the right position. The edges will generally not be in the right position.

- Permute the edges of layer 3 using algorithm classic(7). This is called a T-permutation (T-perm).

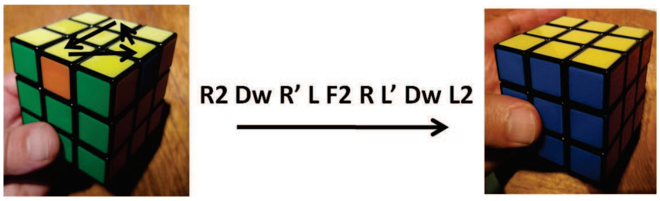

Figure 9: classic(7): $R2\ D_w\ R'\ L\ F2\ R\ L'\ D_w\ L2$

Note that a T-perm can be also performed by applying a Sune (algorithm classic(5)) and its symmetric in sequence. I let the reader to find out how to do it by his own.

3 Rubik's Revenge Et Al

This type of Rubik's cubes (apart from the 2x2x2 one) can be solved by reducing them to a classic 3x3x3 Rubik Cube. In many cases some parity cases arises that have to be solved with specific algorithms.

These days, commercial Rubik's cube exist till the size 17x17x17 or even more. After the 12x12x12 they usually exist in odd sizes only which are mechanically more stable.

New algorithms are required to solve cubes up to the 5x5x5. After that all cubes of these type can be solved without additional algorithms.

Notation. *We need some additional notation.*

- A lower index on a move, if different from 1, will indicate that an internal layer shall be rotated. If different from 1 the index is normally omitted. For example, in a 4x4x4 cube $R_1 = R$ *and* $R_4 = L'$*. Moreover, as additional examples,* R_2 *and* R_3 *are rotation of internal slices,* $R\ R'_w = R'_2$ *and* $R2\ R_w2 = R_22$*.*

These cubes (apart from the 2x2x2 one) are solved in 4 phases:

- **Phase 1**, make the centres by putting all centre of the same color together. Centres mast be in the correct position to each other. If the yellow is the up face, there mast be a edges where red is on the left and green on the right. Moreover, blue must be opposite to green, orange to red and yellow to white.

- **Phase 2**, make the edges by putting all edges with same colors together.

- **Phase 3**, solve the cube as a 3x3x3 classic cube.

- **Phase 4**, solve parities.

It is important to remember that external layers can be always turned without messing up edges and centres.

3.1 2x2x2

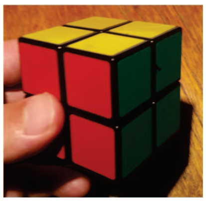

Figure 10: 2x2x2

This cube can be solved as a classic 3x3x3 cube just taking into account that the middle layers do not exist as follows:

- Using algorithm classic(1), place the 1st layer corners in the right place in such a way colours matches.

- Using algorithms classic(5) (Sune), Orient Last Last layer (OLL), i.e. make the top of last layer completely yellow.

3.2 4x4x4 - Rubik's Revenge

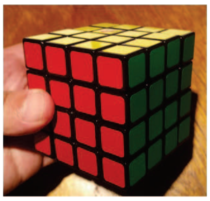

Figure 11: 4x4x4 - Rubik's Revenge

Phase 1:

With a little bit of practice to build your intuition, centres can be solved using the algorithm revenge(1): $R_w\, Un\, R'_w$ and its symmetric where n is any integer. This move can be used to make cluster of two centres as well as to put 2 clusters of two in a clusters of 4. In particular for n=2, revenge(1) take two clusters of two centres on the 3rd layer (longitudinally) on the front and up face and put them together to form a centre of 4.

Figure 12: revenge(1): $R_w\, Un\, R'_w$

It may happen that to complete the last two centres there is one last piece to swap. This can be done with algorithm revenge(2) which is a commutator that can be used to swap pieces on centre's diagonals.

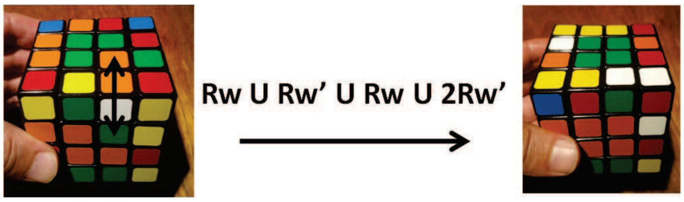

Figure 13: revenge(2): $R_w\, U\, R'_w\, U\, R_w\, U2\, R'_w$

Phase 2:

To complete each edge use algorithms revenge(3).

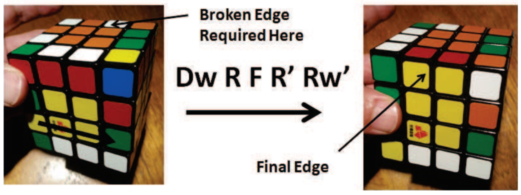

Figure 14: revenge(3): $D_w\, R\, F\, R'\, D'_w$

By rotating external layers it is always possible to place the edges to which apply the algorithm in the correct position. For example we can turn the edge on the 4th column of the front face upside down using the sequence $R'\, D\, B\, R'2$, which is so intuitive that I do not even give it as an algorithm.

To use algorithm revenge(3) we need at least a 3rd broken edge (i.e. not completed) to be put in the back of the up layer. It may happen that only two edges are left to be completed. In this case we can use algorithm revenge(4).

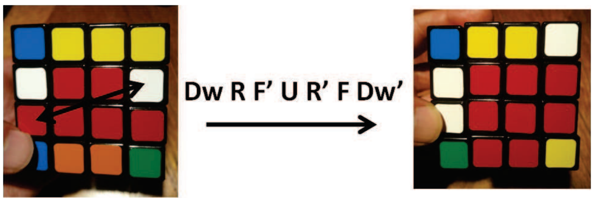

Figure 15: revenge(4): $D_w\, R\, F'\, U\, R'\, F\, D'_w$

Phase 3:

Solve the cube as a classic 3x3x3 cube. Be aware that due to the presence of a parity a complete cross may not be possible to be achieved and one of the yellow edges may be missing making the cross to look like a T instead. With a bit of experience this cases will be easy to be recognised.

Phase 4:

For this cube there are two parity cases possible. Many tutorials describe 3 parity cases. However, when solving the cube in Phase 3, it is always possible to permute edges of the yellow layer by using algorithm classic(7) in order to reduce the cube to one of the two parity cases (or both at the same time) described below.

Parity case 1 (OLL parity):

This parity case is when one edge of the cube is flipped with respect to its correct orientation. This parity has a very long and awful algorithm to be solved. In order to help memorizing this algorithm we will use a special notation:

$$\begin{cases} R\downarrow \;=\; R'_w R U2 \;=\; R'_2 U2; & R\uparrow \;=\; R_w R' U2 \;=\; R_2 U2 \\ R\updownarrow \;=\; R_w 2 R2 U2 \;=\; R_2 2 U2 & \\ L\downarrow \;=\; L_w L' F2 \;=\; L_2 F2; & L\uparrow \;=\; L'_w L F2 \;=\; L'_2 F2 \end{cases} \tag{1}$$

To solve this parity apply algorithm revenge(5).

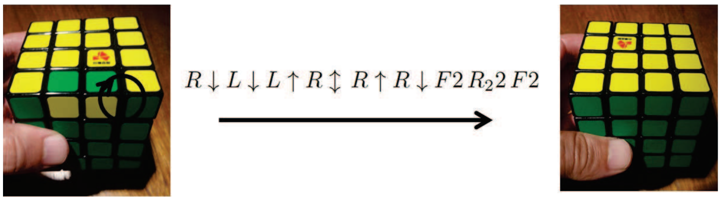

Figure 16: revenge(5): $R\downarrow\ L\downarrow\ L\uparrow\ R\updownarrow\ R\uparrow\ R\downarrow\ F2\, R_2 2\, F2$

Parity case 2 (PLL parity): This parity case is when two opposite edges on a face are swapped. To solve this parity apply algorithm revenge(6).

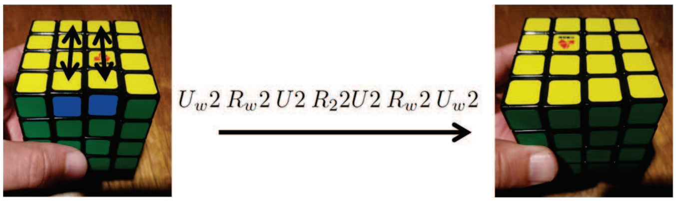

Figure 17: revenge(6): $U_w 2\, R_w 2\, U2\, R_2 2U2\, R_w 2\, U_w 2$

3.3 Bigger Sizes

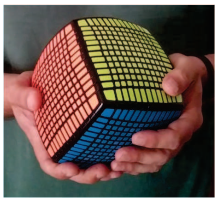

Figure 18: 13x13x13

Once you are able to solve the 5x5x5 cube, you have all the algorithm required to solve any size. In this section I will describe all the algorithm required.

Phase 1:

Complete the first 4 centres using algorithm revenge(1) leaving two contiguous centres undone. With a bit of practice this can be done intuitively. Complete one line at a time in a free (not yet done) centres and then insert the line in the centre you are working at the moment. For even sizes, be sure the colour of the centres are in the right position with respect to each other. For odd sizes. the central peace of each face cannot be moved and it will ensure the position of the colors is correct.

To complete the last two centres you need centres commutator. Before using the commutator play a bit with algorithm revenge(1) in order to complete as much as possible the two faces and in particular trying to complete the little crosses if the size is odd. This because commutators are boring and the Less you use them, the better.

Up to size 5x5x5 a modified version (but only in the notation, it is actually the same algorithm) of algorithm revenge(2) is enough.

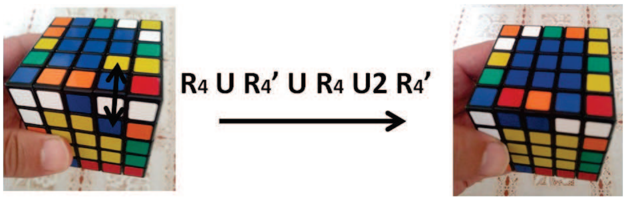

Figure 19: revenge(7): $R_iUR_i'UR_iU2R_i'$

Which swaps pieces on centre's diagonals of cubes of any size. However, for bigger sizes you need a commutator which swaps any pieces in a (i,j) position and that therefore does the job also of the previous algorithm.

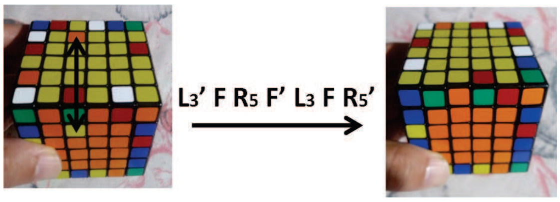

Figure 20: revenge(8): $L_i'FR_jF'L_iFR_j'$

Phase 2:

Edges can be completed with algorithm revenge(9).

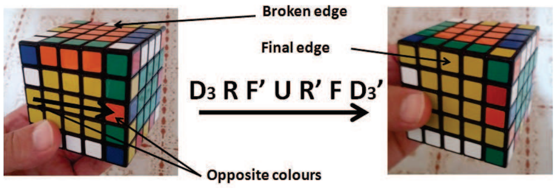

Figure 21: revenge(9): $D_{ijk}RF'UR'FD_{ijk}'$

Where more then one pieces at a time can be inserted if necessary.

When only two undone edges are left, place the two renaming edges on the front face. The two edges on can be fixed using algorithm revenge(10). When placing the pieces in the edges do not worry about orientation, this will be fixed later.

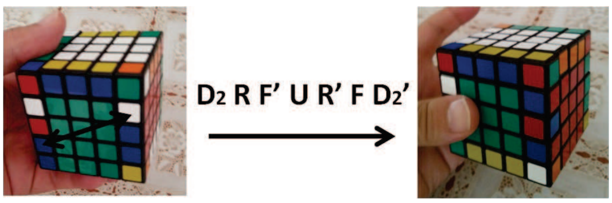

Figure 22: revenge(10): $D_{ijk}RF'UR'FD_{ijk}'$

Where more then one pieces at a time can be inserted if necessary.

Now we need to orient the pieces of the two last edges. With algorithm revenge(11) it is possible to fix the edge on the right of the front face but the other edge will in general not be fixable. When this happen is because there is an OLL parity (actually, in general, a multiple parity depending on the size of the cube). This can be fixed now or at the end as described in phase 4. The layer in the algorithm are the ones relevant to pieces that we do not want to flip. If we turn a layer, we have also to turn the layer symmetric with respect to the middle horizontal layer (the ones with the "*"). The middle horizontal layer is the symmetric to itself.

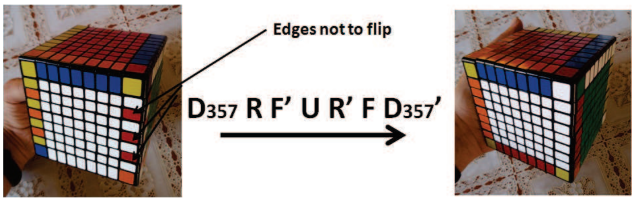

Figure 23: revenge(11): $D_{ii^*jj^*}RF'UR'FD'_{ii^*jj^*}$

Phase 3:
Nothing to add with respect to smaller size cubes.

Phase 4: Odd size cube can have only the the OLL parity. Even size cube can have OLL and PLL parities. OLL Parity can occur with multiplicity depending on the size of the cube.

OLL parity can be fixed using algorithm revenge(5) where, being (i,j,k) the layers of the pieces to fix (on one side only), the notation has to be interpreted as follows:

$$\left\{ \begin{array}{rcl|rcl} R\downarrow & = & R'_{ijk}U2; & R\uparrow & = & R_{ijk}U2 \\ R\updownarrow & = & R_{ijk}2U2 & & & \\ L\downarrow & = & L_{ijk}F2; & L\uparrow & = & L'_{ijk}F2 \end{array} \right. \tag{2}$$

where each index will fix a couple of symmetric pieces to be flipped (even if the layer are rotated only on one side) and pieces can be fixed all at the same time. Note that fix one couple at a time, although boring, would work as well.

PLL parity can be fixed using algorithm revenge(6) where U_w, D_w and R_2 have to be intended as all the internal layers on one side of the median plane of the big size cube replacing the 2 layer of the 4x4x4 cube.

4 Cubes Mechanically Equivalent to the classic 3x3x3 Cube

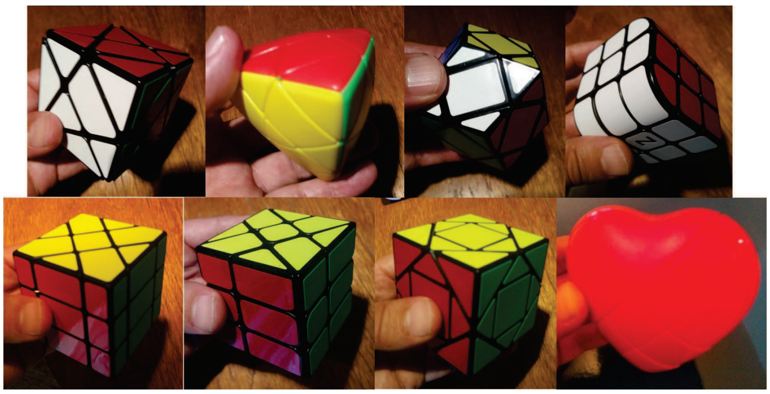

Figure 24: 3x3x3 Mechanically Equivalent to the classic 3x3x3 Cube

In the picture above, from left to right and from top to down we have: Axel or Axis cube, Mastermorphix, 3x3x3 Rhomboidal Dodecahedron, Penrose Cube, Fisher Cube, Windmill or Wheel Cube, Pandora cube and Heart shaped.

These cubes are mechanically equivalent to the classic cube and can be solved using the same method. With a bit of practice it will be possible to visualise centre pieces, edges and corners, although they may have a different shape and a number of colours different from the classic cube. I suggest to use option one for solving the last layer.

Some of these cubes also have parity cases and the reason for that will be clear in the discussion below where I give tips for some of the most common cubes belonging to this category.

A tip that it is applicable to all this cubes is that the centre pieces of the middle layer will not be in general oriented correctly although they will always be in the correct position. Before completing the second layer it is necessary to orient them.

In order to do that:

- Rotate the fist layer till the centre pieces of the second layer are in the correct position.
- Facing the centre to orient, perform moves $F2\,U2$.
- Orient the centre piece.
- Perform $U2\,F2$ to undo the moves done at the beginning and put all pierces of the fist layer in place.

There are many cubes falling in this categories. I will give some tips for the most common ones present in the picture above.

Axel or Axis Cube
TBD

Mastermorphix
TBD

3x3x3 Rhomboidal Dodecahedron
TBD

Penrose Cube
TBD

Fisher Cube
TBD

Windmill or Wheel Cube
TBD

Pandora Cube
TBD

Heart Shaped
TBD

5 Megamix Puzzle

This are very nice puzzles that, although mechanically different from classic cubes, can be solved with algorithms very similar to the classic 3x3x3 ones.

5.1 Megamix Size 2

Figure 25: Megamix Size 2

This cube can be solved as a standard Megamix just taking into account that the middle ”layers” do not exist.

5.2 Megamix

Figure 26: Megamix

The Megamix has a 5 edge star instead on the cross if compared with a classic 3x3x3 cube. Moreover, it has 5 lines of pieces (that cannot be moved with respect to each other) that have to be completed from the down face to the up face. The central line (line 3) is a zigzag line with down and up corners. Lines 2 and 4 in five points are thicker then the other lines since they include a central piece.

As a matter of fact, a Megamix can be solved using exactly (or almost exactly) the same algorithm of a classic 3x3x3 cube.

Line 1:

I suggest to use the white face as down face. Complete the white star. This can be done intuitively. Insert the corner of the down layer. this also can be done intuitively. With a bit of practice you will notice that the algorithm to insert corners is algorithm classic(1) used for the classic cube where Up and Right faces have to be interpreted in the obvious way. Once the corners are in place the first line will be completed.

Line 2:

To complete this line you need to insert edges in the right place. This is equivalent to complete the second layer of a classic cube and algorithms classic(1) plus classic(2) can be used where Up and Right faces have to be interpreted in the obvious way. With a bit of practice you will understand that if you can complete the second layer of a classic cube then you can complete the second line of a Megamix.

Line 3:

Insert the edges in the right position and with the right orientation. Insert the down corners in the zigzag line using algorithm megamix(1).

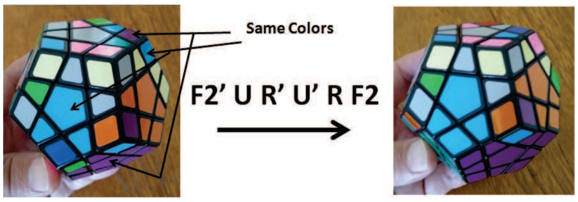

Figure 27: megamix(1): $F2'\, U\, R'\, U'\, R\, F2$

Insert the up corners. This can be done intuitively. With a bit of practice you will notice that the algorithm to be used is the same for inserting corners in the first layer of a classic cube which is algorithm classic(1).

Line 4:

Insert the up edges. This can be done intuitively. With a bit of practice you will notice that the algorithms to be used are the same for inserting edges in the second layer of a classic cube which are algorithms classic(1) plus classic(2).

Line 5:

Make the star (usually grey depending on the cube) on the top layer. This can be done with algorithm megamix(2) which is identical to algorithm classic(3). To do this you need to use the above algorithms several times.

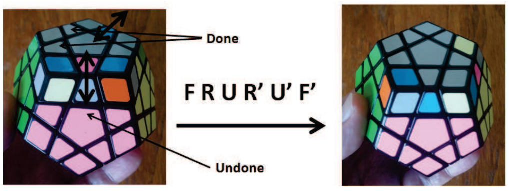

Figure 28: megamix(2): $F\, R\, U\, R'\, U'\, F'$

Permute edges till each edge is in the right position. This can be done with algorithm megamix(3) which is the equivalent of the Sune for classic cubes, which is algorithm classic(3). To do this you need to use the above algorithms several times. Note that the algorithm can be performed also inverting the sense of rotation of the move before the last as "$R\, U\, R'\, U\, R\, U2'\, R'$" and this is the way it is normally presented in tutorials because it is better to be performed in speed cubing. However, I prefer the other way because I do not do speed cubing and I like to turn the Up layer always in the same sense. When presented this way, algorithm megamix(3) it is identical to a Sune where Up and Right faces have to be interpreted in the obvious way.

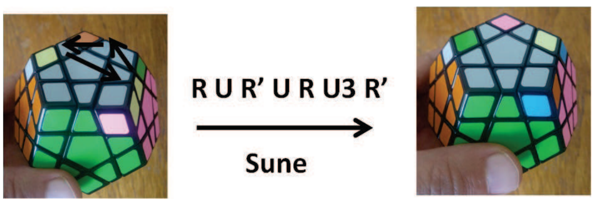

Figure 29: megamix(3): $R\, U\, R'\, U\, R\, U3\, R'$ (Sune)

Permute corners till each corner is in the right position. Do not worry about orientation This can be done with algorithm megamix(4) which is the equivalent of algorithm classic(4). To do this you need to use the above algorithms several times.

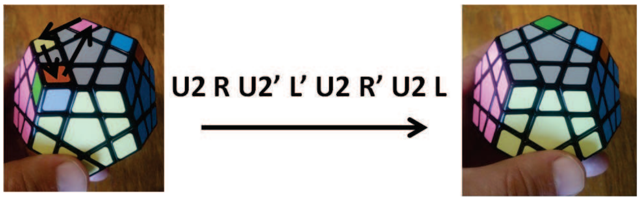

Figure 30: megamix(4): $U2\,R\,U2'\,L'\,U2\,R'\,U2\,L$

In tutorials it is often presented algorithm megamix(5) that does the same job and that it is presented here for completeness.

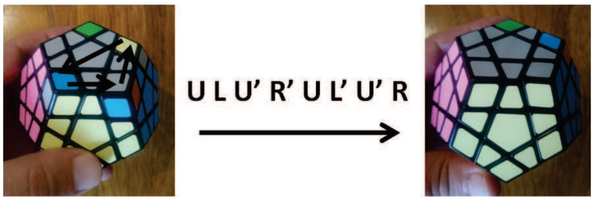

Figure 31: megamix(5): $U\ L\ U'\ R'\ U\ L'\ U'\ R$

Orient the last layer (OLL) using algorithm classic(6) where Up and Right faces have to be interpreted in the obvious way. This bit is identical to solving a classic cube using option 1 for completing the last layer as presented in the paragraphs above.

5.3 Megamix size 4

Figure 32: Megamix Size 4

TBD

6 Pyramix

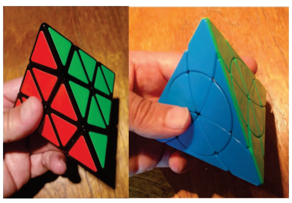

Figure 33: Pyramix and Flower/Petal Pyraminx

TBD

7 Cuboids

7.1 2x2x3

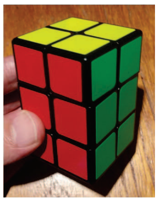

Figure 34: 2x2x3

This cuboid can be solved with the same algorithms of the 2x2x4 cuboid with the difference that there is only a single central slice (instead of two) that can be solved easily and intuitively. Once the central slice is solved, the rest of the cuboid can be solved in the same way as for a 2x2x4 cuboid.

7.2 2x2x4 - Rubik's Tower

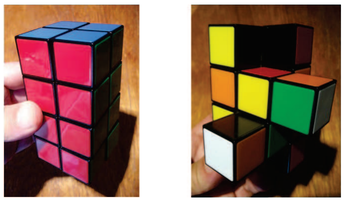

Figure 35: 2x2x3 - Tower

As a first remark, this cube should be scrambled initially in its original tower shape since, once the external layers do nor form a 2x2 cube with the next slice, it it impossible to rotate them along the tower axis and the cube may be non scrambled properly.

In the cube in the above fig the down face is white and the up face is blue. They may exist cubes with different color arrangement but I will refer to the up and down faces in the description below as the white and blue faces.

- Solve the inner 2x2x2 cube with the standard algorithms for it. At first it may be difficult to visualise it as a 2x2x2 cube but this can be done with a bit of practice. For each edge of the two inner layers there are two pieces that have the same two lateral colors, one for each of the two inner layer. This two pieces may have two white, two blue or a white and a blue piece attached to their 3rd face. All combinations are possible. It does not matter which of the two identical pieces it is used when completing the 1st layer of the inner 2x2x2 cube and which of the two identical pieces goes to the 2nd layer.

- When the inner 2x2x2 cube is completed the cube will have its original tower shape. Find out which face is white and which face is blue by rotation on on the external layer till a piece matches with the color of the inner 2x2x2 cube.

- By using algorithm cuboid(1) place all the pieces of the first layer (white face) in the correct positions. This is always possible by swapping pieces between the 1st and the 4th layer. Algorithm cuboid(1) will rotate the right layer of the internal 2x2x2 cube. this is not a problem and can be fixed later as long as when applying the algorithm the rotated layer is kept always on the right.

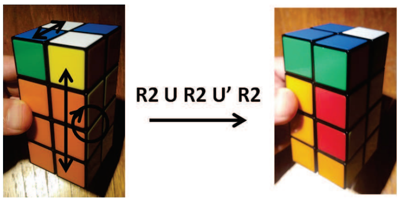

Figure 36: cuboid(1): $R2\,U\ R2\,U'\ R2$

- When the 1st layer is completed, if the right layer of the inner 2x2x2 cube is rotated, rotate it back by using algorithm cuboid(2).

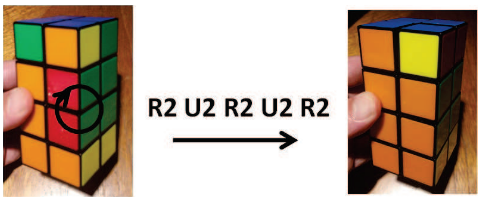

Figure 37: cuboid(2): $R2\,U2\ R2\,U2\ R2$

- Now the solve the last layer applying algorithm cuboid(3) to swap two pieces of 4th layer. It is always possible to sove the last layer applying algoritm cuboid(3) at most twice.

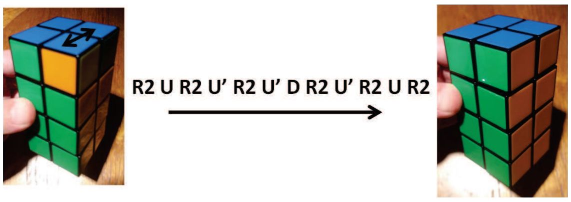

Figure 38: cuboid(3): $R2\,U\ R2\,U'\ R2\,U'\ D\ R2\,U'\ R2\,U\ R2$

7.3 Barrel

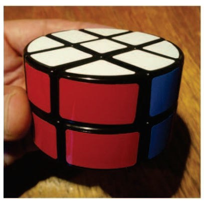

Figure 39: Barrel

TBD

8 Square-one

A Square-one Cube has three type of pieces. Edges, Corners and pieces of the middle layer. The standard notation used for the square-one is very different from the notation for other cubes.

Figure 40: Square-one

Notation. *We will use the standard notation for the Square-one cube. Up and down layer can be rotated of a multiple of the angle of an edge piece (360/12 degs). Note that the angle of a corner piece is 2*360/12 degs. Only 3 moves are possible:*

- The up layer can be rotated by a multiple m of 360/12 Degs clockwise (m positive) or anti-clockwise (m negative).

- The down layer can be rotated by a multiple n of 360/12 Degs clockwise (n positive) or anti-clockwise (n negative).

- The right part of the cube can be rotated by 180 degs and it is notated with a slash.

- Moves are grouped in three moves at a time and notated as $m, n/$ *meaning: Up rotation by n, Down rotation by m, right part of the cube rotation for the slash.*

- If the firs up and down layer rotations are both zero, these are not written down and the algorithm start with a slash. Same applies for the last slash of the notation that will be omitted if not required.

For example with the notation "/-3,0/0,3" we mean a rotation of the right part of the cube followed by a rotation anti-clockwise by $3 * 360/12$ degs of the up layer only, another rotation of the right part of the cube, etc..

Note that apart from algorithm square-one(1) (where there is no proper square in the top layer), in all other algorithms the slash move is to be intended along the plane that goes from 1/3 on the left on the front edge of the top layer to 1/3 on the right on the back edge of the top layer.

Square-one Cube can be solved in 7 steps:

- **Step 1**: Make a Cube Shape.
- **Step 2**: Corner Orientation.
- **Step 3**: Edges Orientation.
- **Step 4**: Corner Swap.
- **Step 5**: Edges Permutation.
- **Step 6**: Parity.
- **Step 7**: Fix the Middle Layer.

Before we start, I would like to introduce a simple algorithm which will be needed only at the very end in step 7 but that it can be used to familiarise with the notation. This algorithm can be used to flip the right central layer piece by 180 degs and, of course, it can be used to flip both central layer pieces just by turning the cube by 180 degs around the z axis.

Although not really needed till the end, especially at the beginning, we may use this algorithm to flip central pieces and give to the central layer a shape we like, for example, for better grabbing the cube.

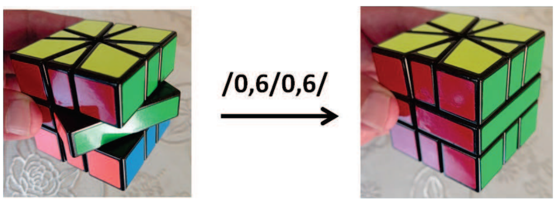

Figure 41: square-one(1): /0,6/0,6/

Step 1: Make a cube shape

This step consists in giving the right cube shape to the Square-one. This applies to the up and down layer while we do not care for the moment to the shape of the middle layer. Do do so, we group all the edges together on one layer as shown in the left hand picture of the fig. below. Although not completely trivial, this can be done intuitively and I will not give algorithms for it at least in this revision of this tutorial. The first time you do it, it may take 10 or 20 minutes. As long as you get familiar with the cube it will take much less.

Once you have grouped all edges in one layer, facing the centre of the edges on the up layer, this step can be completed by applying algorithm square-one(2).

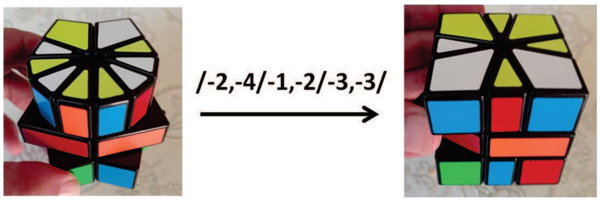

Figure 42: square-one(2): /-2,-4/-1,2/-3,-3/

Step 2: Corners orientation

This step consists in putting all the corner with the same color on the same face. This step can be performed intuitively. The only need to be careful not to break the square shape of up and down layers. In order to do so, we misalign the up and down face by 360/12 degs as in the left hand picture of the figure below. Once we do that, using only 90 degs rotations of up and down layers and the slash move we can easily orient all the corners without the need for any algorithm.

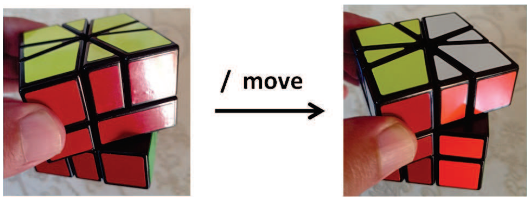

Figure 43: Never break the up and down layers square shape

Step 3 - Edge Orientation: For this step we need two algorithms:

$$\begin{cases} 1) & \text{sqare-one}(3): \quad 1{,}0/\text{-}1{,}\text{-}1/ \\ 2) & \text{sqare-one}(4): \quad /3{,}0/\text{-}3{,}\text{-}3/0{,}3/ \end{cases} \tag{3}$$

The figure below shows the edges of the wrong color with respect to the face where they are placed. Algorithms 1 and 2 above can be used to cycle among the various configuration and you can join the chain of configurations at any point according to the configuration you have and you need to fix.

In the picture below, configurations with 3 or 4 edges on the wrong face are missing. However, by applying one of the two algorithms above to these configuration in a more or less obvious way, you can always get to one of the configurations shown in the figure below.

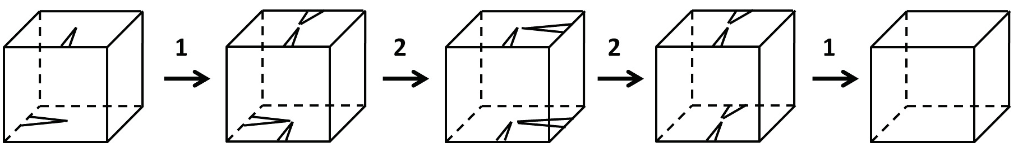

Figure 44: Edge Orientation

Step 4: Corners Swap

Aim of this step is to put the corners in their correct position. To do that, place two corner with matching colors on the left and use algorithm square-one(5). If you do not have two pieces with matching colors, perform the algorithm once and you will have them.

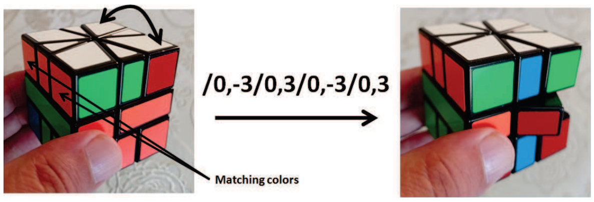

Figure 45: square-one(5): /0,-3/0,3/0,-3/0,3

Once you have completed one layer, turn the cube upside down and do the other one. Please bear in mind that algorithm square-one(5) will mess up the down layer. However, if you perform the algorithm a second time without turning the down layer, this will fix it. This means that the second layer has to be done using the algorithm an even number of time.

To do that, when working on the second layer, place the pieces with the matching colors at the back. Perform the algorithm once. This will not solve the up layer and will mess up the down layer. Turn the up layer in order to have the matching colors on the left. Apply the algorithm again.

Step 5: Edges Permutation

Aim of this step is to permute edges and complete the up and down layer. To permute the edges we use algorithm square-one(5) as we did in previous step. This time we perform the algorithm, we do the set-up move 1,0, we perform the algorithm again and we undo the set-up move. This will turn the edges on the right clockwise. by performing algorithm square-one(5) twice, we do not mess up corners.

If you cannot complete the task (i.e. two edges swapped) it means you have a parity.

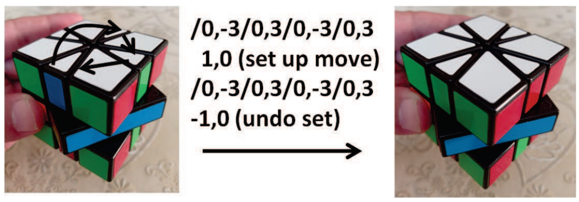

Figure 46: square-one(6): square-one (5) 1,0 (set-up move) square-one(5)

Finally, the symmetric algorithm turning the edges anticlockwise is as follows: we perform the set-up move first, we perform algorithm square-one(5), we undo the set-up move with the move -1,0, we perform the algorithm square-one(5) again.

Step 6: Parity

You have parity when two edges of the same face are swapped. This can happen to one or both faces. To fix the party there is a long and nasty algorithm that swaps the two edges.

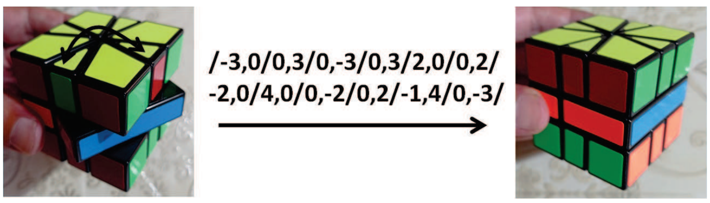

Figure 47: square-one(7): /-3,0/0,3/0,-3/0,3/2,0/0,2/-2,0/4,0/0,-2/0,2/-1,4/0,-3/

Step 7: Fix the Middle Layer

In this last step we fix the middle layer in terms of shape and color matching. This can be done using one or twice algorithm square-one(1).

9 Diagonal Axis Turning Cubes

This section includes but it is not limited to Dinos and Skewbs.

9.1 Dino

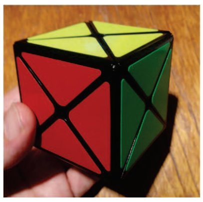

Figure 48: Dino

TBD

9.2 Skewbs

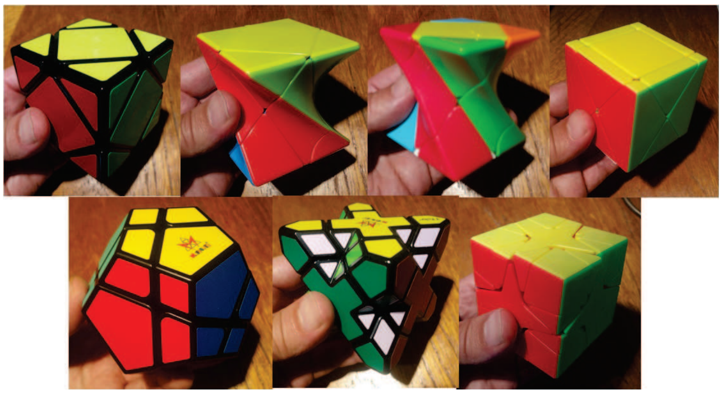

Figure 49: Skewb

In the picture above, from left to right and from top to down we have: Skewb, Twisted Skewb, Twisted Skewb (again but this is a second variant), Fisher Skewb, Skewb Ultimate, Skewb Xtreme and Polaris Cube.

All the above cubes are mechanically equivalent and they can be solved with the same method. We will describe how to solve the most common ones present in the picture above.

9.2.1 Skewb

Notation. *For the Skewb we need a new notation. The Skewb can be solved (beginner method) just rotating along the two axis in the figure below. As usual, R and L will be used for clockwise rotations, R' and L' will be used for anti-clockwise rotations. In the figure I show also the z axis since we often need to rotate the cube by 180 degs around this axis. This move will be notated as $z2$.*

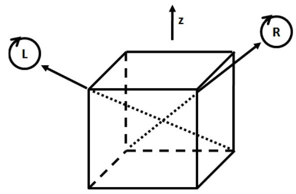

Figure 50: Skewb Notation

Skewbs have centre pieces and corners. For some Skewbs, as the standard Skewb, centre pieces are symmetric in shape and have one colour. For such pieces we do not need to bother rotating them to solve the Skewb. For some other Skewbs, as the twisted Skewb, this is not true. For the above reason, the steps to solve the standard Skewb is a subset of the steps required for the twisted Skewb and will not be presented here. For the solution of this cube the reader can refer to the section for the twisted Skewb.

9.2.2 Twisted Skewb

To solve the Skewb we basically need only one algorithm, called Sledgehammer, reported in the figure below. Some tutorial give a Sledgehammer different from the one below performed rotating different axis or with a different order of Left and Right rotation. Hoverer, given the due symmetries all algorithms are equivalent.

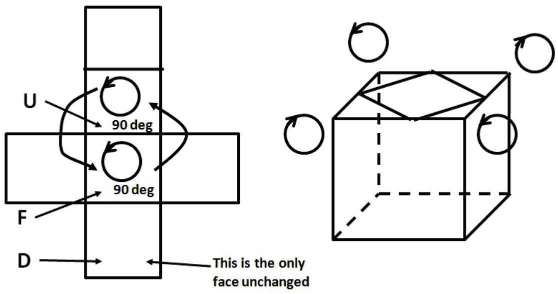

Figure 51: skewb(1): $R'\,L\,R\,L'$ (Sledghammer)

In addition to the Sledgehammer, we need two more algorithms which are reported in the figure below and are Sledghammer performed in sequence with additional 180 degs rotations around the z axis.

- skewb(2): $(R'\,L\,R\,L')\,z2\,(R'\,L\,R\,L') \;=\; SH\,z2\,SH$
- skewb(3): $(R'\,L\,R\,L')\,z2\,(R'\,L\,R\,L')\,(R'\,L\,R\,L')\,z2\,(R'\,L\,R\,L') \;=\; SH\,z2\,SH\,SH\,z2\,SH$

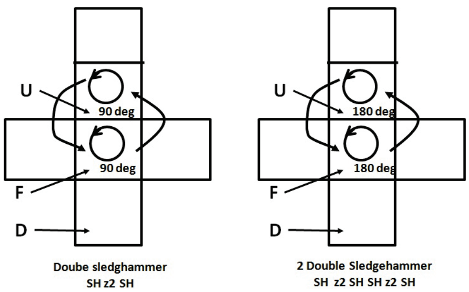

Figure 52: skewb(2): Double Sledghamme and skewb(3): Two Double Sledghammer

To solve the Skewb, perform the following steps.

Step 1:
Complete the white face (or whatever face with a symmetric and one coloured centre piece you have on your cube). This is intuitive. Take the corner to the white face in the correct position. To rotate a corner without rotating the other ones on the same face use algorithm skewb(4) where A and B are the axis in the fugures below. This is, as a matter of fact and once again a Sledghammer.

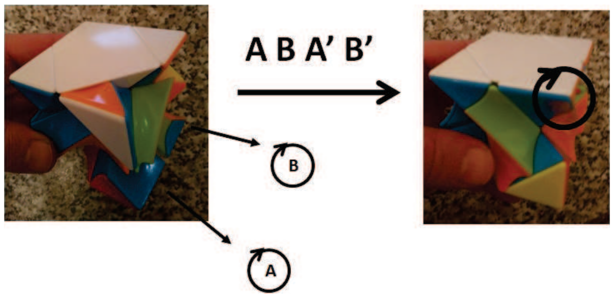

Figure 53: skewb(4): $A\,B\,A'\,B'$

Step 2:
Orient Yellow corners. this should be possible to be done at most using twice algorithm skewb(1) (i.e Sledgehammer) with the white face pointing in the down direction.

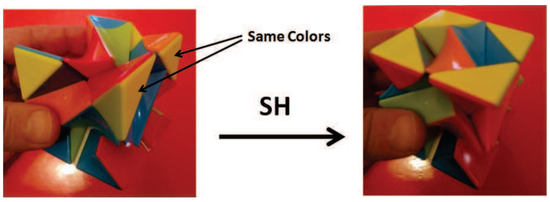

Figure 54: Orient Corners

Step 3:
Complete yellow face. To do so, with the white face pointing down and the yellow centre in the back face, use algorithm skewb(2) (i.e. $SH\ z2\ SH$) once.

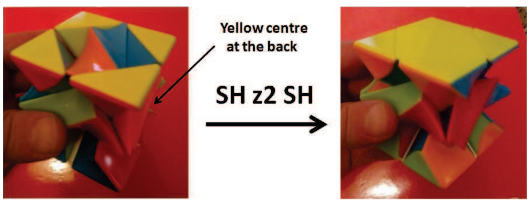

Figure 55: Complete Yellow Face

Step 4:
Give it the right shape and put centre pieces in place. In order to do that, find a centre piece that is already in the right position. If the piece is turned by 180 degs and color do not match, it is not important. Pieces can be turned by 180 degs at the end. If you do not find such a piece,

apply algorithm skewb(2) (i.e. $SH\ z2\ SH$) with white and yellow face on the right and left face (or vice versa). By applying algorithms skewb(2) with the piece already in place at the bottom and the pieces to be turned by 90 degs on the top and back face, you will be able to put each piece in place and correctly oriented (apart the colors). If you have done all correctly, this will give to the skewb the right shape. If you do not do it correctly you may get the skewb with the correct shape but with centre pieces swapped for opposite faces.

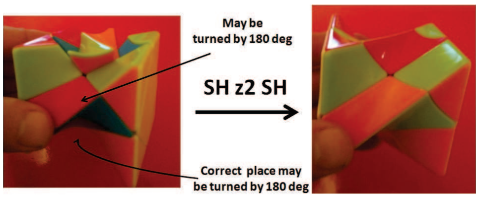

Figure 56: Give it the Right Shape

Step 5:

Rotate the centre pieces by 180 degs to make colors match. Do that by using algorithm Skewb(3) (i.e. $SH\ z2\ SH\ SH\ z2\ SH$) with pieces to rotate on the top and back faces. The algorithm will rotate two pieces at a time. If the pieced to rotate are odd, this can be fixed by rotating also the white or yellow piece that are psychometric and it does not matter if they are rotated by 180 deg. this can be done by applying the algorithm with the white and yellow pieces on the front and back faces or vice versa.

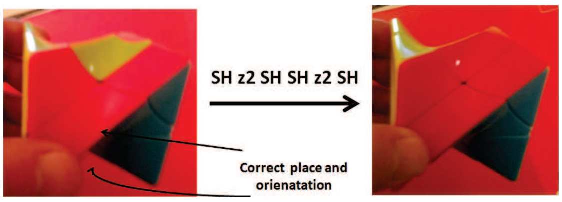

Figure 57: Rotate Centres Faces

9.2.3 Polaris Cube

TBD

9.3 Master Skewb

Figure 58: Master Skewb

Piece Names. *We have four kind of pieces in the Master Skewb. Corners, Edges, Centre Pieces, Outer Centre Pieces. You can easily tell which one is which.*

In the notation, rotations are around axis that go trough the corners of the relevant letter. The ’ sign, as usual indicates anticlockwise rotation. Master Skewb can be solved in 6 steps:

- **Step 1**: Avoid Parity.
- **Step 2**: Edges Permutation.
- **Step 3**: Centres Permutation.
- **Step 4**: Outer Centre Pieces Swap.
- **Step 5**: Corners Swap.
- **Step 6**: Corners Orientation.

Step 1: Avoid Parity

The fist step consist to put corner in the correct position. Orientation of the corners is not important. When putting corner in place it is important to respect color of faces which are the same of the classic 3x3x3 cube. White opposite to yellow, red to orange blue to green with white green and red in the going anticlockwise around the common corner.

This step may seems useless since corner will be messed up by further move. However, the Master Skewb has a parity on corner permutation. Although further moves proposed in this paper will mess up the corner positions, they will not break the parity that will therefore not show up at the end.

Step 2: Edges Permutation

This step consists in moving all edges in their correct position. Once again, as per previous step, face colors have to respect the position of colors on the standard 3x3x3 cube as explained above. This step can be performed easily using the three algorithms below to make the edges travel around the cube till they are in place and choosing the correct algorithm that does not perturb the pieces already in place. Moreover, using the symmetric algorithms that permute edges in the opposite way, may help.

This algorithms will nor scramble the corners. However, corner will be scrambled in further steps. It is convenient to start always from the same face (e.g. white), however everyone can do the way more natural to him.

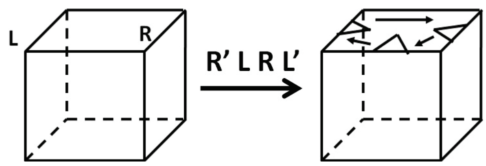

Figure 59: master-skewb(1): $R'\ L\ R\ L'$

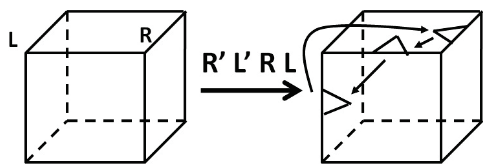

Figure 60: master-skewb(2): $R'\ L'\ R\ L$

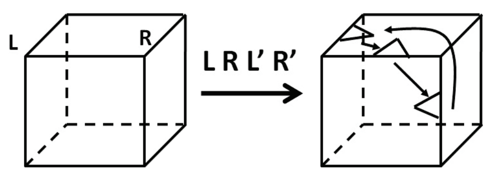

Figure 61: master-skewb(3): $L\ R\ L'\ R'$

At the end of the this step the cube will look as follows:

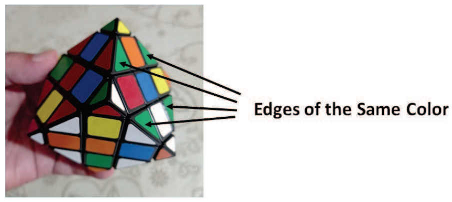

Figure 62: End of Step 2

Step 3: Centres Permutation

Aim of this step is to place the centre pieces in place. This can be easily done with algorithm master-skewb(4) which permute centres anticlockwise and its symmetric that permutes centres clockwise.

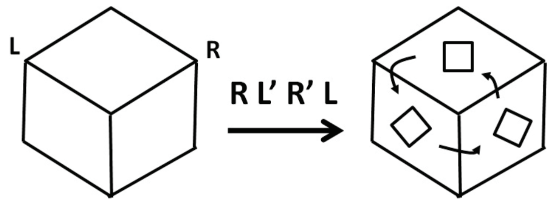

Figure 63: masters-kewb(4): $R\,L'\,R'\,L$

Step 4: Outer Centre Pieces Swap

Aim of this step is to put the outer centre pieces in their correct position. This can be done with algorithm master-skewb(5) (and its symmetric) which swaps outer centre pieces on the same face and for two faces that are next to each other. Note that master-skewb(5) is composed by two parts. The first one is a rotation anticlockwise of the centre pieces (i.e. algorithm master-skewb(5)). The second part is its antisymmetric with a clockwise rotation. The 120 deg Rotation between the two parts will let to a final effect of swapping outer centre pieces.

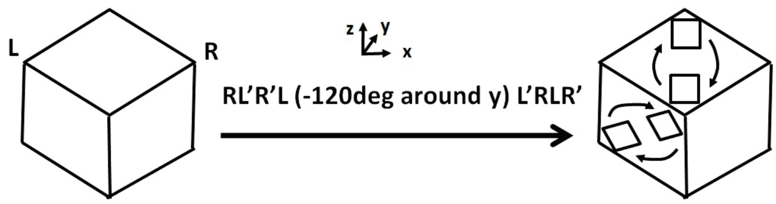

Figure 64: master-skewb(5): $R\,L'\,R'\,L$ **(-120deg [y])** $L'\,R\,L\,R'$

Since to perform our task we need to swap pieces on two different faces, the trick to use is to do a set-up move that puts the two pieces to be swapped on the same face, to perform algorithm master-skewb(5) and to undo the set-up move at the end.

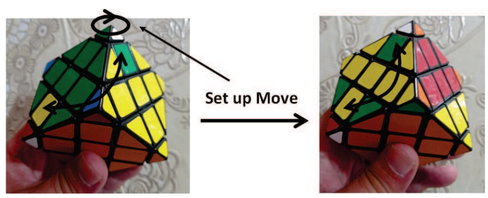

Figure 65: Set-up Move

Remember that the move will swap pieces on two different faces. For the other face use a face with colors that have not been addressed yet or swap pieces of the same color.

Sometimes the piece you want to move to the face you are completing is not on the correct side of the centrepiece. In this case you can use algorithm master-skewb(5) without a set-up move to move it on its face.

It may happen that the piece that you need is not on a face next to the one you are completing. In this case you can move to the correct face in two steps applying twice the algorithm. Another way to do it is to use two set-up moves and move the piece on the correct face in one go. This is a bit more advanced but I am sure you can figure out how to do it.

Step 5: Corners Swap

In the last step we want to put each corner in the correct position. This can be done with algorithm master-skewb(6). Note that this algorithm is simply algorithm master-skewb(4) repeated 3 times with a net effect of leaving the centre pieces unchanged but with an effect o the corners.

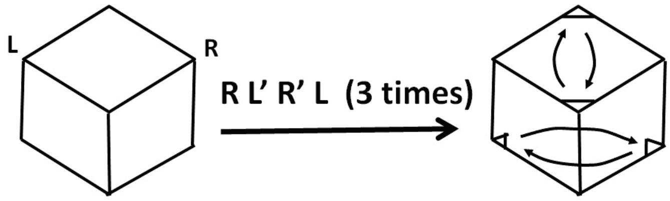

Figure 66: master-skewb(6): $R\,L'\,R'\,L$ **(3 times)**

When you do this, you want your corner (the one closer to you) to go in the correct position and with the correct orientation. In order to do that you may need to perform a set-up move to make the up color of the corner match with the color of the centre piece on the up face (see fig below). Once you have performed algorithm master-skewb(6), remember to undo the set-up move.

Remember that algorithm master-skewb(6) swaps two couple of corners and this may be confusing. What I usually do to complete this step is to move the corners belonging to the white face in their place and this will fix the other one but you can use the method you like more.

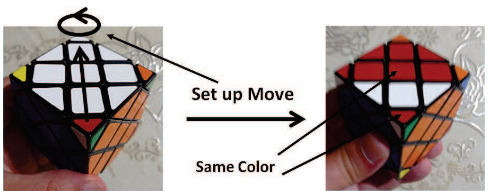

Figure 67: Set-up Move

If you performed correctly step 1, you should be able to complete this step easily. Otherwise you will incur in a parity. I have no algorithm to fix this so go back to step 1, perform it to fix the parity trying to mess up the cube as little as possible, and go on from there.

Step 6: Corners Orientation

Once you have placed all corners in the correct position, some of them may still need to be orientated correctly. In order to do this, you can rotate the cube in order to have the corner you want to rotate on the top face further away from you. Then swap twice the two corners on the top face using algorithm master-skewb(6) twice and before the second swap you use a set-up move to put the corner closer to you (i.e. the one you wanted to rotate) back to its position with the correct orientation. After a few attempts, this will eventually make you able to complete the cube.

9.4 Face Turning Octahedron

Figure 68: Face Turning Octahedron

Face Turning Octahedron has not to be confused with Corner Turning Octahedron with is identical when you see it but that has a complete different mechanics. This puzzle can be solved with a method which reminds a lot the one used for the Master Skewb.

In the notation, rotations are around axis that go trough the corners of the relevant letter. The ' sign, as usual indicates anticlockwise rotation. We have three kind of pieces in this puzzle. Corners, Edges and Centre Pieces Pieces. You can easily tell which one is which.

Face Turning Octahedron can be solved in 3 steps:

- **Step 1**: Solve Corners.
- **Step 2**: Centres Swap.
- **Step 3**: Edges Permutation.

Step 1: Solve Corners

Aim of this step is to place corner in their correct position and orientation. Place two corners is straight forward. The third corner can be placed with the move in the figure below which is very similar to the move we use to place corners for the Skewb.

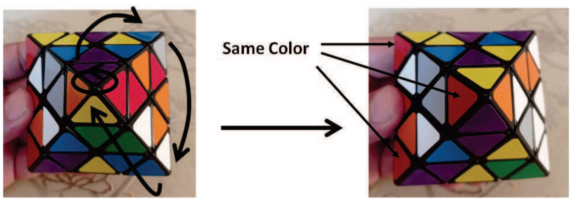

Figure 69: Turn 3rd Corner

At this point we can be in two different cases. The other three corners are already solved or two corners need to be rotated. The second case can be solved using algorithm FTO(1).

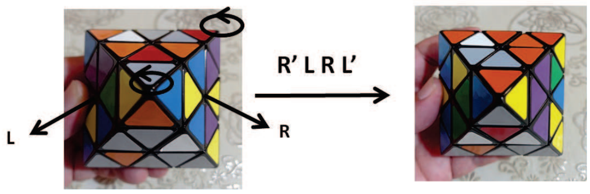

Figure 70: FTO(1): $R'\ L\ R\ L'$

Step 2: Centres Swap

Aim of this step is to place centres in their correct position. This can be done using algorithm FTO(2), which rotates all pieces on a face leaving the rest of the cube unaffected.

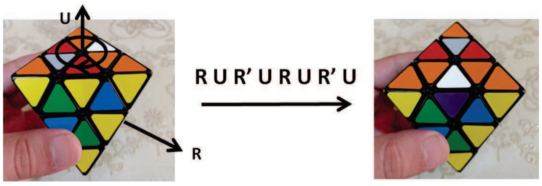

Figure 71: FTO(2): $R\,U\,R'\,U\,R\,U\,R'\,U$

Since to complete the task we need to swap two pieces on two faces, in order to do that we need to use a set-up move, apply algorithm FTO(2), and then undo the set-up move. This is shown in the figure below. Sometimes, before apply the set up move, we need to rotate one of the two faces using algorithm FTO(2).

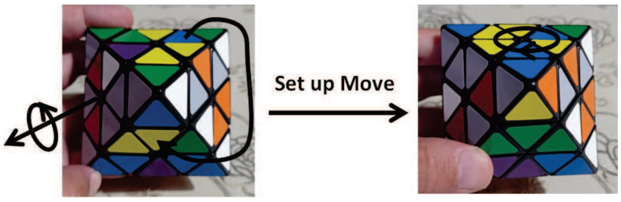

Figure 72: Set-up Move

This is very similar to what we did to solve Outer centre pieces of the Mastercube.

Step 3: Edges Permutation

Aim of this step is to solve the edges. This will complete the cube. We can use algorithm FTO(2) again and make edges travelling around the cube till they are in place. This is very similar to what we did for solving edges of the Mastercube. Once we solve four faces on one side of the cube we may end-up in three cases on the other four faces: the cube is solved. Three pieces are out of place, four pieces are out of place.

If three pieces are out of place, we can permute them using algorithm FTO(3) and its symmetric.

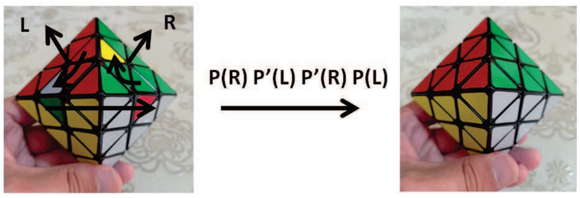

Figure 73: FTO(3): $P(R)\,P'(L)\,P'(R)\,P(L)$

Were with $P(A)$ I mean algorithm FTO(2) applied to face A (P for permutation) and with P' I mean the symmetric (i.e. rotating face A anticlockwise) of algorithm FTO(2). The reader can work out by himself the symmetric algorithm to permute edges clockwise.

If four pieces are out of place, apply algorithm FTO(3) to three of them. One of the 3 pieces will go to the right position and we will be left with only three pieces out o place as per case before.

10 Miscellanea

In This section I present some cube that do not fall under other specific categories.

10.1 Ivy Cube

Figure 74: Ivy Cube

TBD

10.2 Copter and Curvy Copter

Figure 75: Curvy Copter

TBD

10.3 Redimix

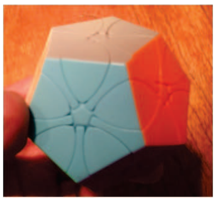

Figure 76: Redimix

TBD

11 Mirror and Ghost Puzzles

TBD