

Theoretical value for the strong coupling constant

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Abstract

In this paper we will study the strong coupling constant. The recommended theoretical value for the strong coupling constant is $\alpha_s = \text{Euler's number} / \text{Gelfond's constant}$. It will turn out that this value is the key that solves many problems of Physics. They will be found a total of ten formulas for the strong coupling constant. All these equations prove that the value of the strong coupling constant depends on the energy scale. First we will find the beautiful unity formulas that connect the strong coupling constant and the fine-structure constant. This equation is the simple unification of the strong nuclear and electromagnetic force. It will be presented the mathematical formulas that connect the dimensionless physical constants with the strong coupling constant. From these equations we reached the formula of the unification of the strong nuclear, electromagnetic and the gravitational force. Also we will find the formula for the Gravitational constant with exact numerical value. Finally we will be presented the formula for the cosmological constant. This unity formula is a simple analogy between atomic physics and cosmology.

Keywords

Fine-Structure Constant, Proton to Electron Mass Ratio , Dimensionless Physical Constants , Strong Coupling Constant , Gravitational Constant , Strong Interaction , Cosmological Constant

1. Introduction

In physics, the fundamental interactions, also known as fundamental forces, are the interactions that do not appear to be reducible to more basic interactions. There are four fundamental interactions known to exist: the gravitational and electromagnetic interactions, which produce significant long-range forces whose effects can be seen directly in everyday life, and the strong and weak interactions, which produce forces at minuscule, subatomic distances and govern nuclear interactions. Some scientists hypothesize that a fifth force might exist, but these hypotheses remain speculative. Each of the known fundamental interactions can be described mathematically as a field. The gravitational force is attributed to the curvature of spacetime, described by Einstein's general theory of relativity. The other three are discrete quantum fields, and their interactions are mediated by elementary particles described by the Standard Model of particle physics. The Standard Model of particle physics was developed throughout the latter half of the 20th century. In the Standard Model, the electromagnetic, strong, and weak interactions are associated with elementary particles, whose behaviors are modeled in quantum mechanics (QM). For predictive success with QM's probabilistic outcomes, particle physics conventionally models QM events across a field set to special relativity, altogether relativistic quantum field theory (QFT).

A coupling constant is a parameter in field theory, which determines the relative strength of interaction between particles or fields. In the quantum field theory the coupling constants are associated with the vertices of the corresponding Feynman diagrams. Dimensionless parameters are used as coupling constants, as well as the quantities associated with them that characterize the interaction and have dimensions. A coupling constant is a number that determines the strength of an interaction. Usually the Lagrangian or the Hamiltonian of a system can be separated into a kinetic part and an interaction part. The coupling constant determines the strength of the interaction part with respect to the kinetic part, or between two sectors of the interaction part. For example, the electric charge of a particle is a coupling constant. A coupling constant plays an important role in dynamics. For example, one often sets up hierarchies of approximation based on the importance of various coupling constants. In the motion of a large lump of magnetized iron, the magnetic forces are more important than the gravitational force.

In nuclear physics and particle physics, the strong interaction is one of the four known fundamental interactions, with the others being electromagnetism, the weak interaction, and gravitation. Strong force involves the exchange of huge particles and therefore has a very small range. It is clear that strong force is much stronger simply than the fact that the nuclear magnitude (dominant strong force) is about 10^{-15} m while the

atom (dominant electromagnetic force) has a size of about 10^{-10} m. At the range of 10^{-15} m, the strong force is approximately 137 times as strong as electromagnetism, 10^6 times as strong as the weak interaction, and 10^{38} times as strong as gravitation. The strong coupling constant α_s is one of the fundamental parameters of the typical model of particle physics.

Grand Unified Theories (GUT) aim to describe the strong interaction and the electroweak interaction as aspects of a single force, similarly to how the electromagnetic and weak interactions were unified by the Glashow–Weinberg–Salam model into electroweak interaction. The strong interaction has a property called asymptotic freedom, wherein the strength of the strong force diminishes at higher energies (or temperatures). The theorized energy where its strength becomes equal to the electroweak interaction is the grand unification energy. However, no Grand Unified Theory has yet been successfully formulated to describe this process and Grand Unification remains an unsolved problem in physics. If GUT is correct, after the Big Bang and during the electroweak epoch of the universe, the electroweak force separated from the strong force. Accordingly, a grand unification epoch is hypothesized to have existed prior to this.

2. Euler's number and Gelfond's constant

The number e is an important mathematical constant, which is the base of the natural logarithm. All five of these numbers play important and repetitive roles in mathematics and these five constants appear in a formulation of Euler's identity. Euler's number has many practical uses, especially in higher level mathematics such as calculus, differential equations, trigonometry, complex analysis, statistics, etc. Euler's number frequently appears in problems related to growth or decay, where the rate of change is determined by the present value of the number being measured. One example is in biology, where bacterial populations are expected to double at reliable intervals. Another case is radiometric dating, where the number of radioactive atoms is expected to decline over the fixed half-life of the element being measured. From Euler's identity the following relation of the mathematical constant e can emerge:

$$e = i^{-2i/n} = 2,7182818284... \quad (1)$$

Gelfond's constant, in mathematics, is the number e^n , e raised to the power n . Like e and n , this constant is a transcendental number. It was named after the Soviet mathematician Aleksandr Gelfond. Gelfond's constant were singled out in Hilbert's 7th problem as an example of numbers whose excess was an open problem. This was first established by Gelfond and may now be considered as an application of the Gelfond–Schneider theorem, noting that:

$$e^n = (e^{in})^{-i} = (-1)^{-i} = i^{-2i} = 23,1406926327... \quad (2)$$

3. Measurement of the strong coupling constant

The strong coupling constant α_s is one of the fundamental parameters of the typical model of particle physics. The strong nuclear force confines quarks into hadron particles such as the proton and neutron. In addition, the strong force binds these neutrons and protons to create atomic nuclei, where it is called the nuclear force. Most of the mass of a common proton or neutron is the result of the strong force field energy; the individual quarks provide only about 1% of the mass of a proton. The electromagnetic force is infinite in range and obeys the inverse square law, while the strong force involves the exchange of massive particles and it therefore has a very short range. The value of the strong coupling constant, like other coupling constants, depends on the energy scale. As the energy increases, this constant decreases.

The last measurement [16] in 23 November 2021 of European organization for nuclear research (CERN) is used in a comprehensive QCD analysis at next-to next-to-leading order, which results in significant improvement in the accuracy of the parton distributions in the proton. Simultaneously, the value of the strong coupling constant at the Z boson mass is extracted as:

$$\alpha_s(m_Z) = 0,1170 \pm 0,0019 \quad (3)$$

A measurement of the inclusive jet production in proton-proton collisions at the LHC at $\sqrt{s} = 13$ TeV is presented. The double-differential cross sections are measured as a function of the jet transverse momentum p_T and the absolute jet rapidity $|y|$. The anti-kT clustering algorithm is used with distance parameter of 0.4 (0.7) in a phase space region with jet p_T from 97 GeV up to 3.1 TeV and $|y| < 2.0$. Data collected with the CMS detector are used, corresponding to an integrated luminosity of 36.3 fb^{-1} (33.5 fb^{-1}). The measurement is used in a comprehensive QCD analysis at next-to next-to-leading order, which results in significant improvement in

the accuracy of the parton distributions in the proton. Simultaneously, the value of the strong coupling constant at the Z boson mass is extracted as $\alpha_s(m_Z) = 0,1170 \pm 0,0019$. For the first time, these data are used in a standard model effective field theory analysis at next to-leading order, where parton distributions and the QCD parameters are extracted simultaneously with imposed constraints on the Wilson coefficient c_1 of 4-quark contact interactions.

4. Theoretical value for the strong coupling constant

Interaction phenomena in field theory are often defined using perturbation theory, in which the functions in the equations are extended to forces of constant interaction. Usually, for all interactions except the strong one, the coupling constant is much smaller than the unit. This makes the application of perturbation theory effective, as the contribution from the main terms of the extensions decreases rapidly and their calculation becomes redundant. In the case of strong interactions, perturbation theory becomes useless and other calculation methods are required. One of the predictions of quantum field theory is the so-called "floating constants" phenomenon, according to which interaction constants change slowly with the increase of energy transferred during the interaction of particles. Thus, the constant of the electromagnetic interaction increases, and the constant of the strong interaction decreases with increasing energy. For quarks in quantum chromodynamics, a strong interaction constant is introduced:

$$\alpha_s = g_{qq}^2 / 4 \cdot \pi \cdot \hbar \cdot c \quad (4)$$

where g_{qq} is the active color charge of a quark that emits virtual gluons to interact with another quark. By reducing the distance between the quarks, which is achieved in high-energy particle collisions, a logarithmic reduction of α_s and a weakening of the strong interaction (the effect of the asymptotic freedom of the quarks) is expected. The fine-structure constant defined as:

$$\alpha = q_e^2 / 4 \cdot \pi \cdot \epsilon_0 \cdot \hbar \cdot c = (q_e / q_p t)^2 \quad (5)$$

So from (4) and (5) we have:

$$\alpha_s = g_{qq}^2 \cdot \epsilon_0 \cdot \alpha / q_e^2 \quad (6)$$

$$\alpha_s = \epsilon_0 \cdot g_{qq}^2 / q_p t^2 \quad (7)$$

The recommended value for the strong coupling constant is:

$$\alpha_s = \text{Euler's number} / \text{Gelfond's constant}$$

$$\alpha_s = e / e^\pi \quad (8)$$

$$\alpha_s = e^{1-\pi} \quad (9)$$

with numerical value:

$$\alpha_s = 0,1174676... \quad (10)$$

Also for the value of the strong coupling constant we have the equivalent expressions:

$$\alpha_s = e \cdot e^{-\pi} = e \cdot i^{2i} = i^{-2i/\pi} \cdot i^{2i} = i^{2i-(2i/\pi)} = i^{2i(n-1)/\pi} \quad (11)$$

Although it remains a possibility that it is a coincidence this value fits perfectly in the measurement of the strong coupling constant. The series representations for the strong coupling constant is:

$$e^{1-\pi} = \left(\sum_{k=0}^{\infty} \frac{1}{k!} \right)^{1-\pi}$$

$$e^{1-\pi} = \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{1-\pi}$$

$$e^{1-\pi} = e^{1-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

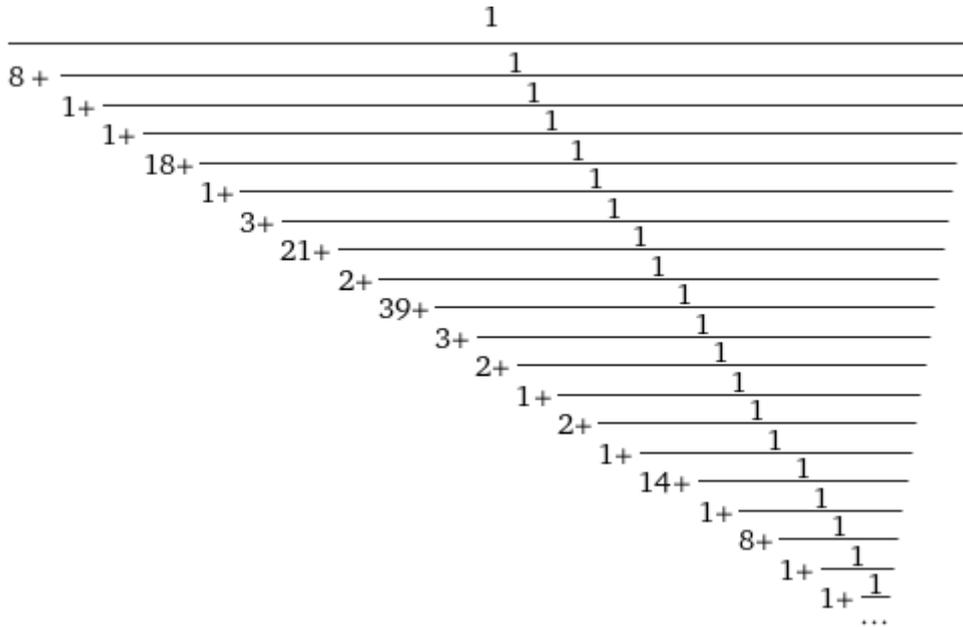
$$e^{1-\pi} = \left(\sum_{k=0}^{\infty} \frac{1}{k!} \right)^{1-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$e^{1-\pi} = \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{1-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

The pattern of the continued fraction for the strong coupling constant is:

[0; 8, 1, 1, 18, 1, 3, 21, 2, 39, 3, 2, 1, 2, 1, 14, 1, 8, 1, 1, 101, 1, 1, 1, 7, 3, 2, 2, 1, 3, 1, 1, 4, 2, 1, 1, 1, 5, 16, 2, 2, 3, 69, 1, 2, 5, 1, 4, 9, 6, 1, 1, 1, 1, 1, 9, 10, 3, 4, 5, 1, 3, 1, 1, 4, 5, 2, 1, 2, 23, 1, 1, 2, 2, 2, 2, 142, 3, 1, 1, 1, 1, 4, 1, 1, 37, 10, 2, 2, 3, 4, 1, 1, 1, 8, 2, 1, 115, 1, 4, 1, 4, 11, 1, 3, 1, 5, 1, 1, 5, 1, 1, 4, 2, 3, 1, 1, 28, 2, 7, 1, 5, 7, 2, 2, 1, 3, 1, 14, 1, 1, 4, 1, 22, 4, 6, 3, 5, 2, 2, 1, 5, 2, 8, 2, 2, 14, 9, 9, 14, 18, 10, 2, 1, 7, 3, 1, 1, 3, 3, 1, 17, 1, 1, 15, 1, 1, 1, 1, 1, 1, 7, 3, 1, 7, 2, 4750, 5, 1, 4, 1, 1, 1, 1, 12, 1, 1, 4, 4, 4, 1, 3, 9, 2, 8, 1, 2, 2, 1, 3, 1, 3, 3, 4, 1, 2, 1, 12, 1, 11, 2, 11, 4, 4, 1, 5, 8, 2, 2, 1, 1, 3, 1, 1, 2, 19, 1, 2, 3, 1, 1, 8, 2, 80, 1, 2, 1, 1, 1, 4, 3, 1, 2, 1, 4, 5, 1, 2, 1, 2, 3, 3, 1, 4, 1, 1, 3, 38, 1, 1, 3, 1, 1, 1, 1, 12, 1, 1, 10, 7, 4, 1, 3, 16, 4, 9, 1, 1, 8, 1, 1, 7, 85, 1, 2, 1, 7, 1, 3, 11, 1, 13, 4, 5, 31, 1, 2, 1, 1, 1, 1, 690, 4, 3, 2, 2, 1, 2, 5, 1, 2, 4, 2, 4, 5, 1, 1, 1, 2, 1, 1, 2, 4, 19, 2, 1, 12, 1, 1, 6, 1, 2, 5, 2, 1, 2, 1, 145, 1, 1, 1, 2, 1, 73, 2, 3, 2, 1, 1, 5, 3, 1, 2, 12, 9, 1, 1, 33, 1194, 2, 5, 1, 24, 1, 1, 26, 3, 6, 3, 11, 2, 2, 10, 118, 2, 1, 56, 2, 5, 1, 13, 18, 1, 1, 3, 1, 2, 1, 2, 2, 4, 1, 7, 1, 3, 3, 2, 8, 4, 5, 1, 7, 7, 2, 4, 1, 1, 8, 1, 1, 1, 4, 1, 1, 2, 1, 3, 1, 13, 18, 8, 20, 1, 1, 3, 2, 2, 1, 5, 1, 12, 2, 1, 16, 1, 1, 1, 6, 1, 2, 6, ...]

The continued fraction for the strong coupling constant is:



5. Fine-structure constant and the proton to electron mass ratio

One of the most important numbers in physics is the fine-structure constant α which defines the strength of the electro-magnetic field. It is a dimensionless number independent of how we define our units of mass, length, time or electric charge. A change in these units of measurement leaves the dimensionless constant unchanged. The number can be seen as the chance that an electron emits or absorbs a photon. It's a pure number that shapes the universe to an astonishing degree. Paul Dirac considered the origin of the number «the most fundamental unsolved problem of physics». The constant is everywhere because it characterizes the strength of the electromagnetic force affecting charged particles such as electrons and protons.

Many eminent physicists and philosophers of science have pondered why α itself has the value that it does, because the value shows up in so many important scenarios and aspects of physics. Nobody has come up with any ideas that are even remotely convincing. The 2.018 CODATA recommended value of the fine-structure constant α is:

$$\alpha = 0,0072973525693(11)$$

With standard uncertainty $0,0000000011 \times 10^{-3}$ and relative standard uncertainty $1,5 \times 10^{-10}$. We propose in [9] the exact formula for the fine-structure constant α with the golden angle, relativity factor and the fifth power of the golden mean:

$$\alpha^{-1} = 360 \cdot \phi^{-2} - 2 \cdot \phi^{-3} + (3 \cdot \phi)^{-5} \tag{12}$$

with numerical value:

$$\alpha^{-1} = 137,03599916476564....$$

Also we propose in [10] a simple and accurate expression for the fine-structure constant α in terms of the Archimedes constant π :

$$\alpha^{-1} = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot \ln 2 \cdot \pi \tag{13}$$

with numerical value:

$$\alpha^{-1} = 137,03599907817552.....$$

In Physics, the ratio of the mass of a proton to an electron is simply the remainder of the mass of the proton

divided by that of the electron, from the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious ratio of mass between a proton and an electron. The values of m_e and m_p , and the equilibrium between them, govern nuclear reactions such as the decay of protons and the nuclear synthesis of stars, leading to the formation of basic biochemical elements, including carbon. The space where stars and planets form and support life and molecular structures can appear. The mass ratio of protons to electrons, two constant particles that make up about 95% of the visible Universe, may be related to the total computational value of the Universe. Thus, as pure numbers they are supposed to be associated with prime numbers, entropy, binary and complexity.

The proton to electron mass ratio μ is a ratio of like-dimensioned physical quantities, it is a dimensionless quantity, a function of the dimensionless physical constants, and has numerical value independent of the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious mass ratio between the proton and electron. The numerical challenge of the mass ratio of proton to electron in the field of elementary particle physics began with the discovery of the electron by JJ Thomson in 1.897, and with the identification of the point nature of the proton by E. Rutherford in 1.911. These two particles have electric charges that are identical in size but opposite charges. The 2.018 CODATA recommended value of the proton to electron mass ratio μ is:

$$\mu = m_p/m_e = 1.836,15267343$$

With standard uncertainty 0,00000011 and relative standard uncertainty $6,0 \times 10^{-11}$. We propose in [11] the exact equivalent mathematical expressions for the proton to electron mass ratio:

$$\mu^{32} = \varphi^{-42} \cdot F_5^{160} \cdot L_5^{47} \cdot L_{19}^{40/19} \quad (14)$$

with exact numerical value:

$$\mu = 1.836,15267343\dots$$

Also we propose in [11] the exact simply mathematical expression for the proton to electron mass ratio:

$$\mu^3 = 7^{-1} \cdot (5 \cdot 13)^3 \cdot [\ln(2 \cdot 5)]^{11} \quad (15)$$

with numerical value:

$$\mu = 1836,152673929077\dots$$

It was explained in [13] that the $\mu \cdot a^{-1}$ is one of the roots of the following trigonometric equation:

$$2 \cdot 10^2 \cdot \cos(\mu \cdot a^{-1}) + 13^2 = 0 \quad (16)$$

The exponential form of this equation is:

$$10^2 \cdot (e^{i\mu/a} + e^{-i\mu/a}) + 13^2 = 0 \quad (17)$$

This exponential form can also be written with the beautiful form:

$$10^2 \cdot (e^{i\mu/a} + e^{-i\mu/a}) = 13^2 \cdot e^{in} \quad (18)$$

Also this unity formula can also be written in the form:

$$10 \cdot (e^{i\mu/a} + e^{-i\mu/a})^{1/2} = 13 \cdot i \quad (19)$$

So other beautiful formula that connect the fine-structure constant, the proton to electron mass ratio and the fifth power of the golden mean is:

$$5^2 \cdot (5 \cdot \varphi^{-2} + \varphi^{-5})^2 \cdot (e^{i\mu/a} + e^{-i\mu/a}) + (5 \cdot \varphi^2 - \varphi^{-5})^2 = 0 \quad (20)$$

Also the formula that connect the fine-structure constant, the proton to electron mass ratio and the

mathematical constants π, φ, e, i is:

$$10^2 \cdot (e^{i\mu/a} + e^{-i\mu/a}) = (5 \cdot \varphi^2 - \varphi^{-5})^2 \cdot e^{i\pi} \quad (21)$$

The expressions are very important because they show us that the fine-structure constant α depends on the proton to electron mass ratio μ .

6. Unity formulas that connect the strong coupling constant and the fine-structure constant

Jesús Sánchez in the paper [13] explained that the fine-structure constant is one of the roots of the following trigonometric equation:

$$\cos(\alpha^{-1}) = e^{-1} \quad (22)$$

Another elegant expression is the following exponential form equation:

$$e^{i/\alpha} - e^{-1} = -e^{-i/\alpha} + e^{-1} \quad (23)$$

From (8) and (22) resulting the beautiful formula that connects the strong coupling constant α_s and the fine-structure constant α :

$$e^{\pi} \cdot \alpha_s \cdot \cos(\alpha^{-1}) = 1 \quad (24)$$

$$i^{-2i} \cdot \alpha_s \cdot \cos(\alpha^{-1}) = 1 \quad (25)$$

$$\alpha_s \cdot \cos(\alpha^{-1}) = i^{2i} \quad (26)$$

So the beautiful formulas for strong coupling constant α_s are:

$$\alpha_s = [e^{\pi} \cdot \cos(\alpha^{-1})]^{-1} \quad (27)$$

$$\alpha_s = i^{2i} \cdot \cos(\alpha^{-1})^{-1} \quad (28)$$

$$\alpha_s = i^{2i} / \cos \alpha^{-1} \quad (29)$$

From (8) and (23) resulting the beautiful formulas that connects the strong coupling constant α_s and the fine-structure constant α :

$$e^{i/\alpha} + e^{-i/\alpha} = 2 \cdot (e^{\pi} \cdot \alpha_s)^{-1} \quad (30)$$

$$e^{i/\alpha} + e^{-i/\alpha} = 2 \cdot i^{2i} \cdot \alpha_s^{-1} \quad (31)$$

$$\alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot i^{2i} \quad (32)$$

$$e^{i/\alpha} - (e^{\pi} \cdot \alpha_s)^{-1} = -e^{-i/\alpha} + (e^{\pi} \cdot \alpha_s)^{-1} \quad (33)$$

$$e^{i/\alpha} - i^{2i} \cdot \alpha_s^{-1} = -e^{-i/\alpha} + i^{2i} \cdot \alpha_s^{-1} \quad (34)$$

So the beautiful formulas for strong coupling constant α_s are:

$$\alpha_s = 2 \cdot [e^{\pi} \cdot (e^{i/\alpha} + e^{-i/\alpha})]^{-1} \quad (35)$$

$$\alpha_s = 2 / e^{\pi} \cdot (e^{i/\alpha} + e^{-i/\alpha}) \quad (36)$$

$$\alpha_s = 2 \cdot i^{2i} (e^{i/\alpha} + e^{-i/\alpha})^{-1} \quad (37)$$

$$a_s = 2 \cdot i^{2i} / (e^{i/a} + e^{-i/a}) \quad (38)$$

Quantum mechanics is a theoretical framework that only focuses on the three non-gravitational forces for understanding the universe in regions of both very small scale and low mass: subatomic particles, atoms, molecules, etc. Quantum mechanics successfully implemented the Standard Model that describes the three non-gravitational forces: strong nuclear, weak nuclear, and electromagnetic force—as well as all observed elementary particles. From these educations we reached the conclusion of unification of the strong nuclear and electromagnetic force:

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i} \quad (39)$$

This equation proves that the value of the strong coupling constant depends on the energy scale.

7. Mathematical formulas that connects dimensionless physical constants

In physics, a dimensionless physical constant is a physical constant that is dimensionless, a pure number having no units attached and having a numerical value that is independent of whatever system of units may be used. The term fundamental physical constant is used to refer to some universal dimensionless constants. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values.

It was presented in [12] the mathematical formulas that connects the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro number N_A , the gravitational coupling constant α_G for the electron and the gravitational coupling constant of proton $\alpha_{G(p)}$:

$$\alpha_{G(p)} = \mu^2 \cdot \alpha_G \quad (40)$$

$$\alpha = \mu \cdot N_1 \cdot \alpha_G \quad (41)$$

$$\alpha \cdot \mu = N_1 \cdot \alpha_{G(p)} \quad (42)$$

$$\alpha^2 = N_1^2 \cdot \alpha_G \cdot \alpha_{G(p)} \quad (43)$$

$$2 \cdot e \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2} = 1 \quad (44)$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2 \quad (45)$$

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (46)$$

$$\mu^3 = 4 \cdot e^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (47)$$

$$\mu = 2 \cdot e \cdot \alpha_G^{1/2} \cdot \alpha_{G(p)} \cdot N_A \cdot N_1 \quad (48)$$

$$\mu = 4 \cdot e^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1 \quad (49)$$

From these expressions and (8) resulting the mathematical formulas that connects the strong coupling constant a_s , the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro number N_A , the gravitational coupling constant α_G for the electron and the gravitational coupling constant of proton $\alpha_{G(p)}$:

$$2 \cdot e^n \cdot a_s \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2} = 1 \quad (50)$$

$$\mu \cdot N_1 = 4 \cdot e^{2n} \cdot a_s^2 \cdot \alpha^3 \cdot N_A^2 \quad (51)$$

$$4 \cdot e^{2n} \cdot a_s^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (52)$$

$$\mu^3 = 4 \cdot e^{2n} \cdot a_s^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (53)$$

$$\mu = 2 \cdot e^n \cdot a_s \cdot a_G^{1/2} \cdot a_{G(p)} \cdot N_A \cdot N_1 \quad (54)$$

$$\mu = 4 \cdot e^{2n} \cdot a_s^2 \cdot a \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \quad (55)$$

$$2 \cdot a_s \cdot a \cdot N_A \cdot a_G^{1/2} = i^{2i} \quad (56)$$

$$i^{4i} \cdot \mu \cdot N_1 = 4 \cdot a_s^2 \cdot a^3 \cdot N_A^2 \quad (57)$$

$$4 \cdot a_s^2 \cdot a \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 = i^{4i} \quad (58)$$

$$i^{4i} \cdot \mu^3 = 4 \cdot a_s^2 \cdot a \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (59)$$

$$i^{2i} \cdot \mu = 2 \cdot a_s \cdot a_G^{1/2} \cdot a_{G(p)} \cdot N_A \cdot N_1 \quad (60)$$

$$i^{4i} \cdot \mu = 4 \cdot a_s^2 \cdot a \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \quad (61)$$

From these expressions resulting the mathematical formulas for the strong coupling constant a_s :

$$a_s = (2 \cdot e^n \cdot a \cdot N_A \cdot a_G^{1/2})^{-1} \quad (62)$$

$$a_s = (2 \cdot e^n \cdot a^{3/2} \cdot N_A)^{-1} \cdot (\mu \cdot N_1)^{1/2} \quad (63)$$

$$a_s = (2 \cdot e^n \cdot a_G \cdot N_A)^{-1} \cdot (a \cdot \mu \cdot N_1)^{-1/2} \quad (64)$$

$$a_s = (2 \cdot e^n \cdot a_{G(p)} \cdot N_A)^{-1} \cdot (a \cdot \mu^{-3} \cdot N_1)^{-1/2} \quad (65)$$

$$a_s = (2 \cdot e^n \cdot a_G^{1/2} \cdot a_{G(p)} \cdot N_A \cdot N_1)^{-1} \quad (66)$$

$$a_s = (2 \cdot e^n \cdot N_A)^{-1} \cdot (a \cdot \mu^{-1} \cdot a_G \cdot a_{G(p)} \cdot N_1)^{-1/2} \quad (67)$$

$$a_s = i^{2i} \cdot (2 \cdot a \cdot N_A \cdot a_G^{1/2})^{-1} \quad (68)$$

$$a_s = i^{2i} \cdot (2 \cdot a^{3/2} \cdot N_A)^{-1} \cdot (\mu \cdot N_1)^{1/2} \quad (69)$$

$$a_s = i^{2i} \cdot (2 \cdot a_G \cdot N_A)^{-1} \cdot (a \cdot \mu \cdot N_1)^{-1/2} \quad (70)$$

$$a_s = i^{2i} \cdot (2 \cdot a_{G(p)} \cdot N_A)^{-1} \cdot (a \cdot \mu^{-3} \cdot N_1)^{-1/2} \quad (71)$$

$$a_s = i^{2i} \cdot (2 \cdot a_G^{1/2} \cdot a_{G(p)} \cdot N_A \cdot N_1)^{-1} \quad (72)$$

$$a_s = (2 \cdot e^n \cdot N_A)^{-1} \cdot (a \cdot \mu^{-1} \cdot a_G \cdot a_{G(p)} \cdot N_1)^{-1/2} \quad (73)$$

$$a_s = i^{2i} (2 \cdot N_A)^{-1} \cdot (a \cdot \mu^{-1} \cdot a_G \cdot a_{G(p)} \cdot N_1)^{-1/2} \quad (74)$$

In his experiments of 1.849–50, Michael Faraday was the first to search for a unification of gravity with electricity and magnetism. However, he found no connection. In 1.900, David Hilbert published a famous list of mathematical problems. In Hilbert's sixth problem, he challenged researchers to find an axiomatic basis for all of physics. From these educations we reached the conclusion of the simple unification of the strong nuclear, electromagnetic and the gravitational force:

$$2 \cdot a_s \cdot a \cdot N_A \cdot a_G^{1/2} = i^{2i} \quad (75)$$

8. Gravitational Constant G

The universal gravitational constant G relates the magnitude of the gravitational attractive force between two bodies to their masses and the distance between them. It usually appears in Isaac Newton's law of universal

gravitation and Albert Einstein's general theory of relativity. The physicist Sir Isaac Newton in 1687 published his book "Philosophiæ Naturalis Principia Mathematica" where he presented the law of universal gravity to describe and calculate the mutual attraction of particles and huge objects in the universe. In this paper, Isaac Newton concluded that the attraction between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance separating them. However, these must be adjusted by introducing the gravity constant G. Its value is extremely difficult to measure experimentally. It has been suggested that G has varied with time throughout the history of the universe and that it is scale-dependent. If so, values determined in the laboratory would not be appropriate for terrestrial or astronomical problems, but there is presently no convincing evidence that this is the case. The 2018 CODATA recommended value of gravitational constant G is:

$$G=6,67430 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$$

With standard uncertainty $0,00015 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ and relative standard uncertainty $2,2 \times 10^{-5}$. The gravitational coupling constant α_G is defined as:

$$\alpha_G = G \cdot m_e^2 / \hbar \cdot c = (m_e / m_{pl})^2$$

From this expression and (44) resulting the formula for gravitational constant G with exact value:

$$G = (2 \cdot e^n \cdot a_s \cdot a \cdot NA)^{-2} \cdot (\hbar \cdot c / m_e^2) \quad (76)$$

$$G = i^{4i} \cdot (2 \cdot a_s \cdot a \cdot NA)^{-2} \cdot (\hbar \cdot c / m_e^2) \quad (77)$$

From this expression resulting the mathematical formulas for the strong coupling constant α_s :

$$\alpha_s = (2 \cdot e^n \cdot a \cdot NA)^{-1} \cdot (\hbar \cdot c / G \cdot m_e^2)^{1/2} \quad (78)$$

$$\alpha_s = i^{4i} \cdot (2 \cdot a \cdot NA)^{-1} \cdot (\hbar \cdot c / G \cdot m_e^2)^{1/2} \quad (79)$$

9. Cosmological constant Λ

In the context of cosmology the cosmological constant is a homogeneous energy density that causes the expansion of the universe to accelerate. Originally proposed early in the development of general relativity in order to allow a static universe solution it was subsequently abandoned when the universe was found to be expanding. Now the cosmological constant is invoked to explain the observed acceleration of the expansion of the universe. The cosmological constant is the simplest realization of dark energy, which is the more generic name given to the unknown cause of the acceleration of the universe. Its existence is also predicted by quantum physics, where it enters as a form of vacuum energy, although the magnitude predicted by quantum theory does not match that observed in cosmology.

The cosmological constant Λ is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly, but it is required in cosmological equations in order to reconcile theory with our observations of the universe. One potential explanation for the cosmological constant lies in the realm of modern particle physics. Experiments have verified that empty space is permeated by countless virtual particles constantly popping in and out of existence. This ceaseless action creates what is known as a "vacuum energy," or a force arising from empty space, inherent in the fabric of space-time that could drive apart the universe.

Laurent Nottale in [15] assumed that the cosmological constant Λ is the sum of a general-relativistic term and of the quantum, scale-varying, gravitational self-energy of virtual pairs. A renormalization group approach is used to describe its scale-dependence. We argue that the large scale value of Λ is reached at the classical electron scale. This reasoning provides with a large-number relation:

$$a \cdot (m_{pl} / m_e) = (\ell_{pl} \cdot \Lambda^{1/2})^{-1/3} \quad (80)$$

This unity formula is a simple analogy between atomic physics and cosmology. The gravitational fine structure constant α_g plays a fundamental role in cosmology. The gravitational fine-structure constant α_g is defined as:

$$a_g = a_G^{3/2} / a^3 \quad (81)$$

$$a_g = (a_G / a^2)^{3/2} \quad (82)$$

$$a_g^2 \cdot a^6 = a_G^3 \quad (83)$$

$$a_g = 1,886837472 \times 10^{-61} \quad (84)$$

From this equation resulting the expressions:

$$a_g = \ell_{pl} \cdot \Lambda^{1/2} \quad (85)$$

$$a_g = (G \cdot \hbar \cdot \Lambda / c^3)^{1/2} \quad (86)$$

$$a_g = \ell_{pl}^3 / r_e^3 \quad (87)$$

$$a_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot NA)^{-3} \quad (88)$$

$$\Lambda^{1/2} = (2 \cdot e^n \cdot a_s \cdot a^2 \cdot NA)^{-3} \cdot \ell_{pl}^{-1} \quad (89)$$

$$\ell_{pl} \cdot \Lambda^{1/2} = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot NA)^{-3} \quad (90)$$

$$\ell_{pl}^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot NA)^{-6} \quad (91)$$

This unity formula is a simple analogy between atomic physics and cosmology. Therefore the cosmological constant equals:

$$\Lambda = a_g^2 \cdot \ell_{pl}^{-2} \quad (92)$$

$$\Lambda = \ell_{pl}^4 / r_e^6 \quad (93)$$

$$\Lambda = (G^2 / \hbar^4) \cdot (m_e / a)^6 \quad (94)$$

$$\Lambda = a_g^2 \cdot (c^3 / G \cdot \hbar) \quad (95)$$

$$\Lambda = (2 \cdot e^n \cdot a_s \cdot a^2 \cdot NA)^{-6} \cdot \ell_{pl}^{-2} \quad (96)$$

$$\Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot NA)^{-6} \cdot (c^3 / G \cdot \hbar) \quad (97)$$

10. Conclusions

We presented the recommended theoretical value for the strong coupling constant:

$$a_s = \text{Euler's number} / \text{Gelfond's constant} = e / e^n = e^{1-n}$$

From this definition of the strong coupling constant we arrived at significant results. We found the beautiful unity formula that connect the strong coupling constant a_s and the fine-structure constant a :

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i}$$

From this equation we reached the unification of the strong nuclear and electromagnetic force. It was presented the formula of the simple unification of the strong nuclear, electromagnetic and the gravitational force:

$$2 \cdot a_s \cdot a \cdot NA \cdot a_G^{1/2} = i^{2i}$$

Also the formula for the gravitational constant G with exact numerical value is:

$$G=i^{4i} \cdot (2 \cdot a_s \cdot a \cdot NA)^{-2} \cdot (\hbar \cdot c/me^2)$$

The follow unity formula is a simple analogy between atomic physics and cosmology:

$$\hbar c^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot NA)^{-6}$$

Finally the formula for the cosmological constant Λ is:

$$\Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot NA)^{-6} \cdot (c^3/G \cdot \hbar)$$

All these equations are simple, elegant and symmetrical in a great physical meaning. Also the equations prove that the value of the strong coupling constant depends on the energy scale.

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