Theoretical value for strong coupling constant

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Abstract

In this paper we will study the strong coupling constant. The recommended theoretical value for the strong coupling constant is as=Euler's number/Gelfond's constant. It will turn out that this value is the key that solves many problems of Physics. They will be found a total of ten formulas for the strong coupling constant. First we will find the beautiful unity formulas that connect the strong coupling constant and the Fine-structure constant. It will be presented the mathematical formulas that connect dimensionless physical constants with the strong coupling constant. Also we will find the formula for the Gravitational constant with exact numerical value. Finally we will be presented the formula for the Cosmological constant.

Keywords

Fine-Structure Constant, Proton to Electron Mass Ratio , Dimensionless Physical Constants , Strong Coupling Constant , Gravitational Constant , Strong Interaction , Cosmological Constant

1. Introduction

In physics, the fundamental interactions, also known as fundamental forces, are the interactions that do not appear to be reducible to more basic interactions. There are four fundamental interactions known to exist: the gravitational and electromagnetic interactions, which produce significant long-range forces whose effects can be seen directly in everyday life, and the strong and weak interactions, which produce forces at minuscule, subatomic distances and govern nuclear interactions. Some scientists hypothesize that a fifth force might exist, but these hypotheses remain speculative. Each of the known fundamental interactions can be described mathematically as a field. The gravitational force is attributed to the curvature of spacetime, described by Einstein's general theory of relativity. The other three are discrete quantum fields, and their interactions are mediated by elementary particles described by the Standard Model of particle physics.

A coupling constant is a parameter in field theory, which determines the relative strength of interaction between particles or fields. In the quantum field theory the coupling constants are associated with the vertices of the corresponding Feynman diagrams. Dimensionless parameters are used as coupling constants, as well as the quantities associated with them that characterize the interaction and have dimensions.

In nuclear physics and particle physics,the strong interaction is one of the four known fundamental interactions, with the others being electromagnetism, the weak interaction, and gravitation. Strong force involves the exchange of huge particles and therefore has a very small range. It is clear that strong force is much stronger simply than the fact that the nuclear magnitude (dominant strong force) is about 10^{-15} m while the atom (dominant electromagnetic force) has a size of about 10^{-10} m. At the range of 10^{-15} m, the strong force is approximately 137 times as strong as electromagnetism, 10^6 times as strong as the weak interaction, and 10^{38} times as strong as gravitation. The strong coupling constant a_8 is one of the fundamental parameters of the typical model of particle physics.

2. Euler's number e and Gelfond's constant eⁿ

The number e is an important mathematical constant, which is the base of the natural logarithm. All five of these numbers play important and repetitive roles in mathematics and these five constants appear in a formulation of Euler's identity. Euler's number has many practical uses, especially in higher level mathematics

such as calculus, differential equations, trigonometry, complex analysis, statistics, etc. From Euler's identity the following relation of the mathematical constant e can emerge:

$$e=i^{-2i/n}\approx 2,718281828459045....$$
 (1)

Gelfond's constant,in mathematics,is the number e^{π} ,e raised to the power π . Like e and π ,this constant is a transcendental number. It was named after the Soviet mathematician Aleksandr Gelfond. Gelfond's constant were singled out in Hilbert's 7th problem as an example of numbers whose excess was an open problem. This was first established by Gelfond and may now be considered as an application of the Gelfond–Schneider theorem,noting that:

$$e^{\pi} = (e^{i\pi})^{-i} = (-1)^{-i} = i^{-2i} = 23,140692632779269...$$
 (2)

3. Measurement of the Strong Coupling Constant

The strong coupling constant as is one of the fundamental parameters of the typical model of particle physics. The strong nuclear force confines quarks into hadron particles such as the proton and neutron. In addition, the strong force binds these neutrons and protons to create atomic nuclei, where it is called the nuclear force. Most of the mass of a common proton or neutron is the result of the strong force field energy; the individual quarks provide only about 1% of the mass of a proton. The electromagnetic force is infinite in range and obeys the inverse square law, while the strong force involves the exchange of massive particles and it therefore has a very short range.

The last measurement [16] in 23/11/2021 of European organization for nuclear research (CERN) is used in a comprehensive QCD analysis at next-to next-to-leading order, which results in significant improvement in the accuracy of the parton distributions in the proton. Simultaneously, the value of the strong coupling constant at the Z boson mass is extracted as:

$$as(mZ)=0,1170\pm0,0019$$
 (3)

For the first time, these data are used in a standard model effective field theory analysis at next to-leading order, where parton distributions and the QCD parameters are extracted simultaneously with imposed constraints on the Wilson coefficient c1 of 4-quark contact interactions.

4. Theoretical value for strong coupling constant

Interaction phenomena in field theory are often defined using perturbation theory,in which the functions in the equations are extended to forces of constant interaction. Usually,for all interactions except the strong one, the coupling constant is much smaller than the unit. This makes the application of perturbation theory effective, as the contribution from the main terms of the extensions decreases rapidly and their calculation becomes redundant. In the case of strong interactions, perturbation theory becomes useless and other calculation methods are required. One of the predictions of quantum field theory is the so-called "floating constants" phenomenon, according to which interaction constants change slowly with the increase of energy transferred during the interaction of particles. Thus, the constant of the electromagnetic interaction increases, and the constant of the strong interaction-decreases with increasing energy. For quarks in quantum chromodynamics, a strong interaction constant is introduced:

$$as = qqg^2/4 \cdot n \cdot h \cdot c \tag{4}$$

where g_{qg} is the active color charge of a quark that emits virtual gluons to interact with another quark. By reducing the distance between the quarks, which is achieved in high-energy particle collisions, a logarithmic reduction of a_s and a weakening of the strong interaction (the effect of the asymptotic freedom of the quarks) is expected. Fine-structure constant defined as:

$$a = qe^2/4 \cdot \Pi \cdot \varepsilon_0 \cdot \hbar \cdot c = (qe/qp\ell)^2$$
 (5)

So from (4) and (5):

$$as = gqg^2 \cdot \epsilon 0 \cdot \alpha / qe^2$$
 (6)

$$as = \epsilon 0 \cdot gqg^2 / qp\ell^2 \tag{7}$$

The recommended value for the strong coupling constant is:

as=Euler's number/Gelfond's constant

$$as=e/e^{\Pi}$$
 (8)

with numerical value:

Also for the value of the strong coupling constant we have the equivalent expressions:

$$as = e^{1-n} = e \cdot e^{-n} = e \cdot i^{2i} = i^{-2i/n} \cdot i^{2i} = i^{2i-(2i/n)} = i^{2i(n-1)/n}$$
(10)

Although it remains a possibility that it is a coincidence this value fits perfectly in the measurement of the strong coupling constant.

5. Fine-structure constant and the Proton to electron mass ratio

One of the most important numbers in physics is the Fine-structure constant a which defines the strength of the electro-magnetic field. It is a dimensionless number independent of how we define our units of mass, length, time or electric charge. A change in these units of measurement leaves the dimensionless constant unchanged. The number can be seen as the chance that an electron emits or absorbs a photon. It's a pure number that shapes the universe to an astonishing degree. Paul Dirac considered the origin of the number «the most fundamental unsolved problem of physics». The constant is everywhere because it characterizes the strength of the electromagnetic force affecting charged particles such as electrons and protons.

Many eminent physicists and philosophers of science have pondered why α itself has the value that it does, because the value shows up in so many important scenarios and aspects of physics. Nobody has come up with any ideas that are even remotely convincing. The 2.018 CODATA recommended value of α is:

a=0.0072973525693(11)

With standard uncertainty $0.0000000011 \times 10^{-3}$ and relative standard uncertainty 1.5×10^{10} . We propose [8] the exact formula for the Fine-structure constant a with the Golden Angle, Relativity factor and the Fifth Power of the Golden Mean:

$$\sigma^{-1} = 360 \cdot \varphi^{-2} - 2 \cdot \varphi^{-3} + (3 \cdot \varphi)^{-5}$$
 (11)

with numerical value:

Also we propose in [11] a simple and accurate expression for the Fine-structure constant $\mathfrak a$ in terms of the Archimedes constant $\mathfrak n$:

$$a^{-1} = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot \ln 2 \cdot \Pi \tag{12}$$

with numerical value:

 a^{-1} =137,03599907817552.....=1/0,007297352569594.....

In Physics, the ratio of the mass of a proton to an electron is simply the remainder of the mass of the proton divided by that of the electron, from the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious ratio of mass between a proton and an electron. The values of me and mp, and the equilibrium between them, govern nuclear reactions such as the decay of protons and the nuclear synthesis of stars, leading to the formation of basic biochemical elements, including carbon. The space where stars and planets form and support life and molecular structures can appear. The mass ratio of protons to electrons, two constant particles that make up about 95% of the visible Universe, may be related to the total computational value of the Universe. Thus, as pure numbers they are supposed to be associated with prime numbers, entropy, binary and complexity.

The proton-to-electron mass ratio μ is a ratio of like-dimensioned physical quantities, it is a dimensionless quantity, a function of the dimensionless physical constants, and has numerical value independent of the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious mass ratio between the proton and electron. The numerical challenge of the mass ratio of proton to electron in the field of elementary particle physics began with the discovery of the electron by JJ Thomson in 1.897, and with the identification of the point nature of the proton by E. Rutherford in 1.911. These two particles have electric charges that are identical in size but opposite charges. The 2.018 CODATA recommended value of the proton to electron mass ratio μ is:

 $\mu = m_p/m_e = 1.836, 15267343$

With standard uncertainty 0,00000011 and relative standard uncertainty 6.0×10^{-11} . We propose [9] the exact equivalent mathematical expressions for the proton to electron mass ratio:

$$\mu^{32} = \phi^{-42} \cdot F_5^{160} \cdot L_5^{47} \cdot L_{19}^{40/19} \tag{13}$$

with exact numerical value:

 μ =1.836,15267343......

Also we propose in [9] the exact simply mathematical expression for the proton to electron mass ratio:

$$\mu^{3} = 7^{-1} \cdot (5 \cdot 13)^{3} \cdot [\ln(2 \cdot 5)]^{11}$$
(14)

with numerical value:

 μ =1836,152673929077......

It was explained [13] that the $\mu \cdot a^{-1}$ is one of the roots of the following trigonometric equation:

$$2 \cdot 10^{2} \cdot \cos(\mu \cdot a^{-1}) + 13^{2} = 0 \tag{15}$$

The exponential form of this equation is:

$$10^{2} \cdot (e^{i\mu/a} + e^{-i\mu/a}) + 13^{2} = 0$$
 (16)

This exponential form can also be written with the beautiful form:

$$10^{2} \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) = 13^{2} \cdot e^{i\pi}$$
 (17)

Also this unity formula can also be written in the form:

$$10 \cdot (e^{i\mu/a} + e^{-i\mu/a})^{1/2} = 13 \cdot i \tag{18}$$

So other beautiful formula that connect the Fine-structure constant, the proton to electron mass ratio and the Fifth Power of the Golden Mean is:

$$5^{2} \cdot (5 \cdot \varphi^{-2} + \varphi^{-5})^{2} \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + (5 \cdot \varphi^{2} - \varphi^{-5})^{2} = 0$$
 (19)

Also the formula that connect the Fine-structure constant, the proton to electron mass ratio and the mathematical constants π , φ , e, i is:

$$10^{2} \cdot (e^{i\mu/a} + e^{-i\mu/a}) = (5 \cdot \varphi^{2} - \varphi^{-5})^{2} \cdot e^{i\pi}$$
 (20)

The expressions are very important because they show us that the Fine-structure constant a depends on the proton to electron mass ratio μ . All these equations are simple, elegant and symmetrical in a great physical meaning.

6. Unity formulas that connect the strong coupling constant and the Fine-structure constant

Jesús Sánchez in the paper [13] explained that the Fine-structure constant is one of the roots of the following trigonometric equation:

$$\cos(a^{-1}) = e^{-1}$$
 (21)

Another elegant expression is the following exponential form equation:

$$e^{i/a} - e^{-1} = -e^{-i/a} + e^{-1}$$
 (22)

From (9) and (21) resulting the beautiful formula that connects the strong coupling constant a_s and the Fine-structure constant a_s :

$$e^{\Pi} \cdot as \cdot cos(a^{-1}) = 1 \tag{23}$$

So the beautiful formula for strong coupling constant as is:

$$as=[e^{n} \cdot cos(a^{-1})]^{-1}$$
 (24)

From (9) and (22) resulting the beautiful formulas that connects the strong coupling constant as and the Fine-structure constant a:

$$e^{i/a} + e^{-i/a} = 2 \cdot (e^{n} \cdot as)^{-1}$$
 (25)

$$e^{i/a} - (e^{n} \cdot as)^{-1} = -e^{-i/a} + (e^{n} \cdot as)^{-1}$$
 (26)

So the beautiful formula for strong coupling constant as is:

$$as=2 \cdot [e^{n} \cdot (e^{i/a} + e^{-i/a})]^{-1}$$
 (27)

From the 2.018 CODATA recommended value of a and the equations (24),(27) the value for strong coupling constant as is:

as=0,1174758.....

7. Mathematical Formulas that connects dimensionless physical constants

In physics, a dimensionless physical constant is a physical constant that is dimensionless, a pure number having no units attached and having a numerical value that is independent of whatever system of units may be used. The term fundamental physical constant is used to refer to some universal dimensionless constants. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values.

It was presented [12] the mathematical formulas that connects the proton to electron mass ratio μ , the Fine-structure constant α , the ratio N₁ of electric force to gravitational force between electron and proton, the Avogadro number NA, the gravitational coupling constant α for the electron and the gravitational coupling constant of proton α (p):

$$aG(p) = \mu^2 \cdot aG \tag{28}$$

$$a = \mu \cdot N_1 \cdot a_G \tag{29}$$

$$a \cdot \mu = N1 \cdot aG(p) \tag{30}$$

$$a^2 = N1^2 \cdot aG \cdot aG(p) \tag{31}$$

$$2 \cdot e \cdot a \cdot NA \cdot aG^{1/2} = 1 \tag{32}$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot a^3 \cdot N_A^2 \tag{33}$$

$$4 \cdot e^2 \cdot a \cdot \mu \cdot a G^2 \cdot N A^2 \cdot N 1 = 1 \tag{34}$$

$$\mu^{3} = 4 \cdot e^{2} \cdot a \cdot aG(p)^{2} \cdot NA^{2} \cdot N1 \tag{35}$$

$$\mu = 2 \cdot e \cdot aG^{1/2} \cdot aG(p) \cdot NA \cdot N1 \tag{36}$$

$$\mu = 4 \cdot e^2 \cdot a \cdot aG \cdot aG(p) \cdot NA^2 \cdot N1 \tag{37}$$

From these expressions and (9) resulting the mathematical formulas that connects the strong coupling constant a_s , the proton to electron mass ratio μ , the Fine-structure constant a, the ratio N₁ of electric force to gravitational force between electron and proton, the Avogadro number NA, the gravitational coupling constant a_s for the electron and the gravitational coupling constant of proton $a_s(p)$:

$$2 \cdot e^{\Pi} \cdot as \cdot a \cdot NA \cdot aG^{1/2} = 1 \tag{38}$$

$$\mu \cdot N1 = 4 \cdot e^{2\pi} \cdot as^2 \cdot a^3 \cdot NA^2 \tag{39}$$

$$4 \cdot e^{2\pi} \cdot as^2 \cdot a \cdot \mu \cdot ag^2 \cdot NA^2 \cdot N1 = 1 \tag{40}$$

$$\mu^{3} = 4 \cdot e^{2\pi} \cdot as^{2} \cdot a \cdot ag(p)^{2} \cdot NA^{2} \cdot N1 \tag{41}$$

$$\mu = 2 \cdot e^{\Pi} \cdot as \cdot aG^{1/2} \cdot aG(p) \cdot NA \cdot N1$$
 (42)

$$\mu = 4 \cdot e^{2\pi} \cdot as^2 \cdot a \cdot aG \cdot aG(p) \cdot NA^2 \cdot N1$$
 (43)

From these expressions resulting the mathematical formulas for the strong coupling constant as:

$$as = (2 \cdot e^{\Pi} \cdot a \cdot NA \cdot aG^{1/2})^{-1}$$
(44)

$$as = (2 \cdot e^{\Pi} \cdot a^{3/2} \cdot NA)^{-1} \cdot (\mu \cdot N1)^{1/2}$$
(45)

$$as = (2 \cdot e^{\pi} \cdot aG \cdot NA)^{-1} \cdot (a \cdot \mu \cdot N1)^{-1/2}$$
(46)

$$as = [2 \cdot e^{\Pi} \cdot aG(p) \cdot NA)^{-1} \cdot (a \cdot \mu^{-3} \cdot N1)^{-1/2}$$
(47)

$$as = [2 \cdot e^{\Pi} \cdot aG^{1/2} \cdot aG(p) \cdot NA \cdot N1)^{-1}$$
(48)

$$as = (2 \cdot e^{\Pi} \cdot NA)^{-1} \cdot (a \cdot \mu^{-1} \cdot aG \cdot aG(p) \cdot N1)^{-1/2}$$
(49)

From these expressions the value for strong coupling constant as is:

as=0,1174718.....

8. Gravitational Constant G

The gravitational constant G is an empirical physical constant that participates in the calculation of gravitational force between two bodies and is denoted by the letter G. It usually appears in Isaac Newton's law of universal gravitation and Albert Einstein's general theory of relativity. The physicist Sir Isaac Newton in 1.687 published his book "Philosophiae Naturalis Principia Mathematica" where he presented the law of universal gravity to describe and calculate the mutual attraction of particles and huge objects in the universe. In this paper, Isaac Newton concluded that the attraction between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance separating them. However,these must be adjusted by introducing the gravity constant G. The 2.018 CODATA recommended value of gravitational constant G is:

$$G=6,67430\times10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$$

With standard uncertainty 0.00015×10^{-11} m³/kg·s² and relative standard uncertainty 2.2×10^{-5} . The gravitational coupling constant a_G is defined as:

$$aG=G\cdot me^2/\hbar\cdot c=(me/mp\ell)^2$$

From this expression and (38) resulting the formula for gravitational constant G with exact value:

$$G = (2 \cdot e^{\Pi} \cdot as \cdot a \cdot NA)]^{-2} \cdot (\hbar \cdot c/me^{2})$$
(50)

From this expression resulting the mathematical formula for the strong coupling constant as:

$$as = (2 \cdot e^{\pi} \cdot a \cdot NA)^{-1} \cdot (\hbar \cdot c/G \cdot me^{2})^{1/2}$$
(51)

with numerical value:

as=0.1174718.....

9. Cosmological constant Λ

The cosmological constant Λ is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly, but it is required in cosmological equations in order to reconcile theory with our observations of the universe.

One potential explanation for the cosmological constant lies in the realm of modern particle physics. Experiments have verified that empty space is permeated by countless virtual particles constantly popping in and out of existence. This ceaseless action creates what is known as a "vacuum energy," or a force arising from empty space, inherent in the fabric of space-time that could drive apart the universe.

Laurent Nottale [15] assumed that the cosmological constant Λ is the sum of a general-relativistic term and of the quantum, scale-varying, gravitational self-energy of virtual pairs. A renormalization group approach is used to describe its scale-dependence. We argue that the large scale value of Λ is reached at the classical electron

scale. This reasoning provides with a large-number relation:

$$a \cdot (\mathsf{mpl/me}) = (\ell \mathsf{pl} \cdot \mathsf{\Lambda}^{1/2})^{-1/3} \tag{52}$$

This unity formula is a simple analogy between atomic physics and cosmology. From (52) and (38) resulting the expressions:

$$(aG/\alpha^{2})^{3/2} = (\ell_{p\ell} \cdot \Lambda^{1/2}) = (G \cdot \hbar \cdot \Lambda/c^{3})^{1/2} = \ell_{p\ell}^{3}/re^{3} = (2 \cdot e^{\pi} \cdot as \cdot a^{2} \cdot NA)^{-3}$$
(53)

$$\Lambda^{1/2} = (2 \cdot e^{\Pi} \cdot as \cdot a^2 \cdot NA)^{-3} \cdot \ell_{p\ell} \cdot {}^{-1}$$
(54)

So the formula for cosmological constant Λ is:

$$\Lambda = (2 \cdot e^{\Pi} \cdot as \cdot a^2 \cdot NA)^{-6} \cdot \ell p \ell^{-2}$$
(55)

10. Conclusions

We presented the recommended theoretical value for the strong coupling constant:

as=Euler's number/Gelfond's constantas= e/e^{π}

Although it remains a possibility of being a coincidence, this value is the key that solves many problems of Physics. We found the beautiful unity formulas that connect the strong coupling constant a_s and the Fine-structure constant a_s :

$$as=[e^{\pi} \cdot cos(a^{-1})]^{-1}$$

$$as=2 \cdot [e^{\pi} \cdot (e^{i/a} + e^{-i/a})]^{-1}$$

It was presented the mathematical formulas that connect dimensionless physical constants with the strong coupling constant:

$$as = (2 \cdot e^{\Pi} \cdot a \cdot NA \cdot aG^{1/2})^{-1}$$

$$as = (2 \cdot e^{\Pi} \cdot a^{3/2} \cdot NA)^{-1} \cdot (\mu \cdot N1)^{1/2}$$

$$as = (2 \cdot e^{\Pi} \cdot aG \cdot NA)^{-1} \cdot (a \cdot \mu \cdot N1)^{-1/2}$$

$$as = [2 \cdot e^{\Pi} \cdot aG(p) \cdot NA)^{-1} \cdot (a \cdot \mu^{-3} \cdot N1)^{-1/2}$$

$$as = [2 \cdot e^{\Pi} \cdot aG^{1/2} \cdot aG(p) \cdot NA \cdot N1)^{-1}$$

$$as = (2 \cdot e^{\Pi} \cdot NA)^{-1} \cdot (a \cdot \mu^{-1} \cdot aG \cdot aG(p) \cdot N1)^{-1/2}$$

Also the formula for the Gravitational constant G with exact numerical value:

$$G=(2 \cdot e^{\Pi} \cdot as \cdot a \cdot NA)]^{-2} \cdot (\hbar \cdot c/me^2)$$

From this expression resulting the mathematical formula for the strong coupling constant as:

$$as=(2 \cdot e^{\pi} \cdot a \cdot NA)^{-1} \cdot (\hbar \cdot c/G \cdot me^2)^{1/2}$$

Finally the formula for the Cosmological constant Λ is:

$$\Lambda = (2 \cdot e^{\Pi} \cdot as \cdot a^2 \cdot NA)^{-6} \cdot \ell_{p\ell} \cdot e^{-2}$$

All these equations are simple, elegant and symmetrical in a great physical meaning.

References

- [1] www.en.wikipedia.org/
- [2] https://en.wikiversity.org/
- [3] www.wolframalpha.com/
- [4] www.math.stackexchange.com/
- [5] www.mathworld.wolfram.com/
- [6] www.numberempire.com/
- [7] http://physics.nist.gov/cuu/Constants/
- [8] Stergios Pellis Unification Archimedes Constant π,Golden Ratio φ,Euler's Number e and Imaginary Number i https://vixra.org/pdf/2110.0043v1.pdf, 2.021
- [9] Stergios Pellis Exact Formula for the Fine-structure Constant α in Terms of the Golden Ratio ϕ

https://vixra.org/pdf/2110.0053v1.pdf, 2.021

[10] Stergios Pellis Exact expressions of the Fine Structure constant a

https://vixra.org/pdf/2110.0117v1.pdf, 2.021

[11] Stergios Pellis Exact Mathematical Expressions of the Proton to Electron Mass Ratio with Symmetrical Shape and Physical Meaning

http://dx.doi.org/10.2139/ssrn.3967998, 2.021

[12] Stergios Pellis Exact mathematical Formula that connect 6 dimensionless physical constants

http://dx.doi.org/10.2139/ssrn.3963427, 2.021

[13] Stergios Pellis Unity Formula That Connect the Fine Structure Constant and the Proton to Electron Mass Ratio

http://dx.doi.org/10.2139/ssrn.3963425, 2.021

[14] Jesús Sánchez Calculation of the Fine-Structure Constant

https://vixra.org/pdf/1905.0366v1.pdf, 2.018

[15] L. Nottale, Mach's principle, Dirac's large numbers and the cosmological constant problem

http://www.luth.obspm.fr/%7Eluthier/nottale/arlambda.pdf, 1.993

[16] European organization for nuclear research (CERN)

https://arxiv.org/pdf/2111.10431.pdf, 2.021