

X17 and the dark mark

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Recent developments in particle physics, astrophysics and cosmology seem to converge towards the recognition of the existence of the QCD axion. Moreover, a substantial equivalence between the X17 boson and the QCD axion has been proposed. A description of X17 is suggested as a particle composed of four preons, from a model described in previous articles of ours. The paper ends by pointing to a possible application of results related to braiding and fusion statistics to this model.

The QCD axion is a hypothetical elementary particle postulated by the Peccei-Quinn theory to resolve the strong CP problem [1].

In [2] it is suggested that the QCD axion, in the mass range of $O(10 \text{ MeV})$, must possess properties that correspond, remarkably, to those possessed by the vector boson X17, whose existence has been recently proposed [3].

The existence of the axion is also one of the hypothesized explanations for the presence of black holes with a mass within the black hole mass gap, prohibited by current models [4].

Moreover, X17 can offer an explanation of the 511 keV line [5].

Further references on the subject and on possibly related anomalies can be found here [6].

A particle model from nonlinear dynamical systems theory

The logistic map [7]

$$x_{n+1} = r x_n (1 - x_n) \quad x_n \in [-0.5, 1.5], r \in [-2, 4]$$

is an example of a simple dissipative nonlinear dynamical system which shows a rich dynamical behavior. In particular, in Fig. 1 is plotted the bifurcation diagram of the map as the parameter r varies.

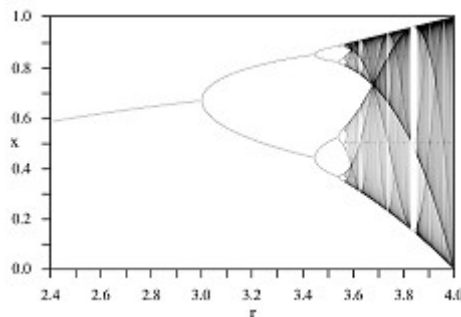


Fig. 1: Bifurcation diagram for the logistic map (from [Wikipedia](https://en.wikipedia.org/wiki/Bifurcation_diagram))

In [8] V. Manasson hypothesized a relation between the bifurcation diagram and the quantum numbers of some particles. This idea is connected with the phenomenon of the production of electron-positron pairs by high-energy photons [9].

In previous articles [10] we have proposed to extend Manasson's original idea and developed a toy model in which the bifurcations produce a hierarchy of particles, whose properties derive from the properties of the particles of the previous level of the hierarchy, recursively in a sense.

The level of the hierarchy after the electron-positron level is occupied by a particle we call “mark”.

As the electron derives from the photon that halves its spin and the electric charge appears, so the mark derives from the electron that halves its spin again and halves the electric charge. From this it follows that the mark has spin $1/4$ and electric charge $1/2$.

Furthermore, we postulate the appearance of a new quantum number, the “strong charge”, which in analogy to the spin of the photon (up and down), and to the electric charge of the electron-positron, can assume only two values, $+1$ and -1 .

Our model is somewhat reminiscent of “digital physics” [11]: let's take as an example the next level of the hierarchy, characterized by the “supermark”.

This particle has 4 quantum numbers: “weak charge” ± 1 , “strong charge” $\pm 1/2$, electric charge $\pm 1/4$, spin $\pm 1/8$.

Any quantum number can be positive or negative, and can have a fractional value (only negative powers of two).

We assume also that the bifurcation processes leading from one level to the next level of the hierarchy mimic the dynamic behavior of Falaco solitons (which can be seen as half vortex rings).

“I propose (not yet supported by the mathematics) that the energy density and ‘torsion’ of space in this interaction can result in stable vortex pairs that are connected in the manner of Falaco solitons (Fig. 5). However, being vortices of a 3-D surface in 4-space, the connection is in time, not in space. It is a wormhole.” [12]. See also [13] for additional references.

Exotic statistics

The mark, having spin $1/4$, obeys neither bosonic nor fermionic statistics.

Objects having similar characteristics have been studied in the literature: half-fermions, anyons, quartions, semions, plektons [14].

The study of exotic statistics can be traced back to Giovanni Gentile Jr. in 1940, and has been studied extensively [15].

As already mentioned, we assume that a pair of marks is created from an electron or from a positron, in analogy to the creation of electron-positron pairs from photons.

Our idea to justify the possibility of existence of particles with semifermionic statistics is that, in conjunction with this process of creating a pair of marks from an electron or a positron, a doubling of the dimensions of spacetime also takes place. The idea of variable spacetime dimensions is not new in the literature [16].

The mark can thus be described as an 8D particle, and we guess that this shift to higher dimensions [17] can lead to the possibility of realization of semifermionic statistics.

Superluminality

Since the photon is massless and timeless, in a certain sense it can be said that mass and time appear in conjunction with the creation of the electron-positron pair. We therefore assume that the principle

of dimensional doubling can also be applied to the dimensions of mass and time.

At the mark level, mass and time are no longer scalar but they are complex quantities, made up of a real and an imaginary part.

An imaginary mass suggests the possible presence of tachyonic fields [18] and opens up the possibility of using the advances made by researchers in the field [19].

A quantum of mass

M. H. Mac Gregor in [20] defines “a boson mass quantum $m_b = m_e/\alpha = 70.025$ MeV and a fermion mass quantum $m_f = (3/2) m_b = 105.038$ MeV”, where $m_e = 0.511$ MeV is the electron mass. See also [21].

If to the mark is assigned a mass of $m_e/4\alpha = 17.5$ MeV, a composition of marks has the capability to recover the two quanta of mass defined by Mac Gregor, both as regards the mass and as regards the spin (a composition of semifermions can produce both bosons and fermions).

We argue that the only true bosons are gravitons and photons and that the only true fermion is the electron-positron. The other particles are pseudobosons and pseudofermions, composed of marks (or supermarks), and whose spin is the result of the sum of the spins possessed by the component marks.

In particular, we propose that X17 is composed of 4 marks, which in an “annihilation” process produce an electron-positron pair.

The fact that the mass of X17 is similar to the mass of a mark can be explained as follows: of the 4 component marks, only one has a “real” mass, which is roughly the one measured for the X17 boson.

The other 3 marks possess an “imaginary” mass, totally or almost totally.

Marks in this state can be defined as “dark” ones, and it is to these marks that the title of this article refers.

Braiding and fusion statistics

As a final observation, we note the visual similarity between fig. 2 of [22], regarding braiding and fusion statistics of the semion theory, and fig. 1 of [23], regarding the braiding of anyons.

This formal similarity, in our opinion, suggests that there could be also a substantial connection at the basis of the various proposals for the application of both braiding groups and the theory of anyons to the theory of particles in spacetime dimensions greater than 2+1 [24].

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