

An Alternative Angle

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Abstract

An alternative, but equivalent, form of Riemann Hypothesis.

1 Introduction

The hundreds of years of irresolution and the lack of even a clear and assured direction for attacking the Riemann Hypothesis can very likely be due to disorientation.

A mind open to new routes, in addition to *cul de sac*, may help advance a few steps.

2 Fresh View

Some simple reasoning may deprive the proof of the Conjecture of the need to get any complicated and convoluted mathematics involved. However, an indisputable conclusion must prevail: the imaginary is totally irrelevant (in settling Riemann Hypothesis).

Pick any $t \in \mathbb{R}$ (i.e. t is any real number). If $r + it$ is not root for any $r \in \mathbb{R}$, then the imaginary is obviously irrelevant (in deciding if the complex number $r + it$ is root because it is not).

Unsurprisingly, the imaginary can't be any relevant either when $\zeta(r + it) = 0$, simply because:

1. if $t = 0$, it is *trivially* irrelevant, i.e. no need to consider.
2. if all non-trivial zeros are on the critical line, i.e. with real part 0.5, then $it \neq 0$ but $it \equiv 0$, since no matter what the value of t is, all such (root-related) it are equivalently making $\zeta(r + it) = 0$.
3. if, conversely, it is relevant, meaning its (non-zero) value can make any difference, then the result of $\zeta(0.5 + it)$ must be other than zero (while $\zeta(r + it) = 0$ for some $r \in \mathbb{R}$), implying Riemann Hypothesis is false!

Therefore, Riemann Hypothesis is true IF AND ONLY IF the imaginary (component of ζ roots (i.e. either trivial or non-trivial) is irrelevant.

Formally, the equivalent alternative of Riemann Hypothesis is the following (for arbitrary ζ roots $a + it$ and $b + it$):

$$\sum_{n=1}^{\infty} \frac{1}{n^{a+it}} - \sum_{n=1}^{\infty} \frac{1}{n^{b+it}} = 0 \iff \sum_{n=1}^{\infty} \left(\frac{1}{n^a} + \frac{1}{n^{it}} \right) - \sum_{n=1}^{\infty} \left(\frac{1}{n^b} + \frac{1}{n^{it}} \right) = 0$$

3 Interesting Suggestion

Exactly how and why the imaginary is irrelevant technically may not appear direct to all people, but the irrelevancy should be true regardless.

By the plain reasoning of **Section 2, Fresh View**, the imaginary in a proof of Riemann Hypothesis can be totally ignored. This indicates that theoretically (emphasis: in theory, not in reality) Riemann Hypothesis can be settled even before the introduction of complex numbers, which further suggests that, even before ζ is analytically continued (to the entire complex plane), RH is (theoretically) already true in certain settings other than complex analysis.

The suggestion that Riemann Hypothesis could be theoretically true before the Hypothesis was even proposed may sound incredible, in any manner and in any direction, but has to be (theoretical) reality.

If the world is genuine in any sense of the word, the only thing we need to do, following this alternative approach suggested, is to find this proof in a setting other than analytics.