# Discriminator Variance Regularization for Wasserstein GAN

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# Abstract

In Wasserstein GAN, it is important to regularize the discriminator to have a not big Lipschitz constant. In this paper, I introduce discriminator variance regularization to regularize the discriminator of Wasserstein GAN to have a small Lipschitz constant. Discriminator variance regularization simply regularizes the variance of the discriminator's output to be small when input is real data distribution or generated data distribution. Intuitively, a low variance of discriminator output implies that the discriminator is more likely to have a low Discriminator Lipschitz constant. regularization does not explicitly regularize the Lipschitz constant of discriminator through differentiation on discriminator but lowers the probability that the Lipschitz constant of the discriminator is high. Discriminator variance regularization is used in Wasserstein GAN with regularization, which suppresses the vibration of GAN. Discriminator variance regularization requires very little additional computing.

### 1. Introduction

In Wasserstein GAN (WGAN) [1], it is important to regularize the discriminator to have a small Lipschitz constant. Several gradient penalty methods [2, 3, 4] were proposed to regularize the Lipschitz constant of the discriminator through differentiation on the discriminator.

In this paper, I introduce discriminator variance regularization (DV regularization) for WGAN to regularize the discriminator to have a small Lipschitz constant. Discriminator variance regularization simply regularizes the variance of the discriminator's output to be low when input is real data distribution or generated data distribution. DV regularization does not explicitly regularize the Lipschitz constant of discriminator through differentiation discriminator but lowers the probability that the Lipschitz constant of the discriminator is high. Also, DV regularization is used together with R1 regularization [8] to prevent vibration of GAN. DV regularization requires very little additional computing.

# 2. Discriminator variance regularization

Assuming discriminator input and output are closed sets, the discriminator of WGAN can be considered a Lipschitz-continuous function. The problem is that the Lipschitz constant of the discriminator may be very large. The large Lipschitz constant of the discriminator causes the gradient to explode and prevents the WGAN from being trained. When training WGAN without regularization terms such as clipping or gradient weight penalty, discriminator output distribution for real data distribution or generated data distribution has an extremely large variance, and the WGAN is hard to be trained.

Intuitively, extremely high discriminator output variance indicates that the discriminator has a large Lipschitz constant. On the other hand, intuitively, if the variance of discriminator output is low, the discriminator would have a small Lipschitz constant. For example, if the discriminator output is constant, the Lipschitz constant of the discriminator is zero. More specifically, the low variance of discriminator output distribution indicates a low probability that the Lipschitz constant of the discriminator is high. Therefore, DV regularization that regularizes variance of discriminator output lowers the probability that the Lipschitz constant of the discriminator is high. In fact, other gradient penalty methods that explicitly regularize the Lipschitz constant of the discriminator are also basically probabilistic methods because training the model is based on Monte Carlo simulation. Therefore, lowering the probability that the Lipschitz constant of the discriminator is high is not illogical.

DV regularization regularizes variance of discriminator output when input is real data distribution or generated data distribution. DV regularization is defined as follows.

$$L_{dv} = var(X) + var(G(Z))$$

DV regularization uses batch distribution to approximate the variance of adversarial values. DV regularization loss for each batch is defined as follows.

$$L_{dv} = sum((a_r - \overline{a_r})^{\circ 2}) + sum((a_g - \overline{a_g})^{\circ 2})$$

The following table explains the terms used in the above equations.

b	Batch size
X	Real data random variable
x	b real data.
Z	Latent random variable
Z	b latent codes.
D	Discriminator
G	Generator
$a_r$	Adversarial values of real data (i.e.,
	D(x)). $b$ -dimensional vector.
$a_g$	Adversarial values of generated data
	(i.e., $D(G(z))$ ). $b$ -dimensional vector
$\overline{a_r}$	Mean of $a_r$
	Mean of $a_g$
vec°2	Element-wise square of example
	vector <i>vec</i>
sum	A function that calculates the sum of
	the input vector
var	A function that calculates the
	variance of a random variable
$L_{dv}$	DV regularization loss
$\lambda_{dv}$	DV regularization loss weight

Simply, DV regularization loss is a sum of the variance of real adversarial values  $a_r$  and fake adversarial values  $a_g$ . DV regularization multiplied by  $\lambda_{dvr}$  and added to discriminator loss.

R1 regularization on the discriminator makes the training stable when the WGAN almost converges. However, R1 regularization alone does not make the discriminator satisfy the Lipschitz condition, so the WGAN is hard to be trained. Therefore, R1 regularization should be used together with an additional regularization term to make the GAN converges stably.

# 3. Experiment results

I used StyleGAN2 [5] architecture with a reduced filter size of convolution for the experiment. The model was trained to generate the CelebA dataset [7] reduced to  $128 \times 128$  resolution. Following hyperparameters were used for model training.

$$batch\ size = b = 32$$
  $r1\ reg\ weight = \lambda_{r1} = 10$   $gp\ reg\ weight = \lambda_{gp} = 10$   $dv\ reg\ weight = \lambda_{dv} = 1$ 

The following figure shows the performance of various regularization methods in WGAN. FID [6] was used for model performance evaluation.

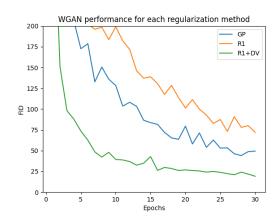


Figure 1. WGAN performance comparison for each regularization method

In figure 1, "GP" represents the gradient penalty method ( $L_{gp} = \left(\left\|\nabla_{\hat{X}}D(\hat{X})\right\|_2 - 1\right)^2$ , where  $\hat{X}$  follows linear interpolation between real data distribution and generated data distribution). "R1" is the R1 regularization method ( $L_{r1} = \left\|\nabla_X D(X)\right\|_2^2$ ). "R1+DV" represents R1 regularization plus DV regularization. One can see the performance of "R1+DV" is the best.

The following figure shows generated image with R1 regularization and DV regularization.



Figure 2. Generated images with R1 regularization and DV regularization

## 4. Conclusion

In this paper, I introduced discriminator variance regularization for Wasserstein GAN regularize the discriminator of Wasserstein GAN to have a small Lipschitz constant. Discriminator variance regularization does not explicitly regularize Lipschitz the constant discriminator through differentiation on discriminator but lowers the probability that the Lipschitz constant of the discriminator is high. Discriminator variance regularization with R1 regularization boosts Wasserstein GAN training.

## 5. References

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